2011 Further Mathematics GA 2: Written examination 1

GENERAL COMMENTS

The majority of students seemed to be well prepared for examination 1 in 2011. The number of students who sat the Further Mathematics examination 1 in 2011 was 30 732.

SPECIFIC INFORMATION

The tables below indicate the percentage of students who chose each option. The correct answer is indicated by shading.

Section A
Core: Data analysis

Question	% A	% B	% C	% D	% E	% No Answer
1	1	0	87	12	0	0
2	1	5	3	5	86	0
3	10	12	45	11	21	1
4	3	3	91	3	1	0
5	9	9	12	15	54	0
6	2	3	4	24	67	0
7	57	9	19	8	7	0
8	58	16	10	8	6	1
9	4	77	12	4	2	0
10	8	10	71	8	2	0
11	10	23	2	62	3	0
12	7	6	34	42	10	0
13	19	8	10	55	8	0

The questions this section were generally well answered, with the notable exception of Question 12.

In Question 12, students were asked to determine the percentage by which an actual sales figure must change to obtain the deseasonalised sales figure given the seasonal index is 0.80.

A possible solution strategy is as follows.

From the formula sheet:

$$\text{seasonal index} = \frac{\text{actual sales}}{\text{deseasonalised sales}}$$

Making the deseasonalised sales the subject of the formula:

$$\text{deseasonalised sales} = \frac{\text{actual sales}}{\text{seasonal index}}$$

or, for the case in Question 12,

$$\text{deseasonalised sales} = \frac{\text{actual sales}}{0.8} = 1.25 \times \text{actual sales}$$

This means that, to obtain the deseasonalised sales for summer, the actual sales figure must be increased by 25%.

Section B
Module 1: Number patterns

Question	% A	% B	% C	% D	% E	% No Answer
1	1	3	91	1	3	0
2	52	2	32	10	4	0
3	8	6	9	72	4	1
4	7	6	79	3	5	0
5	7	14	14	53	12	2
6	7	50	14	9	20	1
7	5	9	17	13	56	1
8	38	6	12	26	17	1
9	12	12	13	54	9	1

The questions in Module 1 (Number patterns) were generally well answered, with the exception of Question 8.

In Question 8, students were asked to find an expression for the sum of the first n terms of a given arithmetic sequence 1, 3, 5 …

A possible solution strategy is as follows.

For this arithmetic sequence $a = 1$ and $d = 2$.

From the formula sheet, the sum of the first n terms of an arithmetic sequence is given by:

$$\begin{aligned} S_n &= \frac{n}{2}\left[2a+(n-1)d\right] \\ &= \frac{n}{2}\left[2\times 1+(n-1)2\right] \\ &= \frac{n}{2}\left[2+2n-2\right] \\ &= \frac{n}{2}\left[2n\right] \\ &= n^2 \end{aligned}$$

Alternatively, students could have observed from the terms of the sequence that the sum of the first one, two and three terms are 1, 1 + 3 = 4 and 1 + 3 + 5 = 9 respectively. These are the first three square numbers so the rule would be $S_n = n^2$.

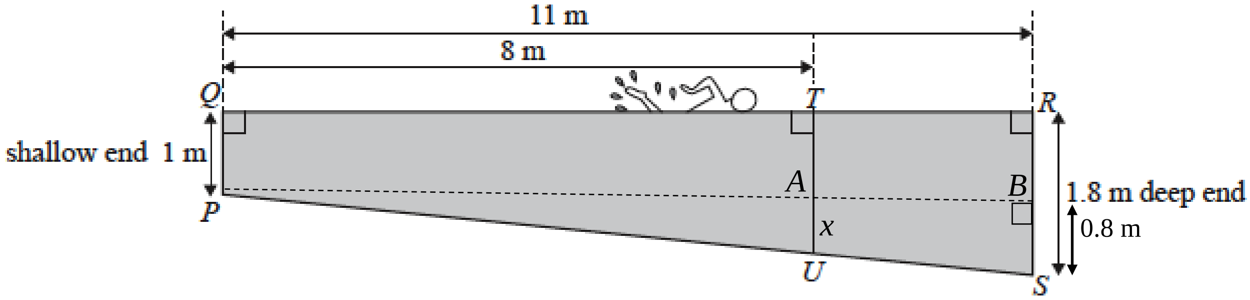
Module 2: Geometry and trigonometry

Question	% A	% B	% C	% D	% E	% No Answer
1	4	2	1	91	2	0
2	71	6	8	3	11	0
3	5	6	72	3	14	0
4	6	8	75	6	4	0
5	6	11	14	58	10	1
6	25	12	20	34	9	1
7	11	54	16	14	4	1
8	4	31	11	40	13	1
9	11	8	18	37	25	1

Module 2 (Geometry and trigonometry) was answered reasonably well; however, Questions 6, 8 and 9 were answered poorly.

Question 6 required students to obtain the length of the longer diagonal of a parallelogram. This involved a routine application of the cosine rule. The most common error was to choose the answer 26.5 cm (option A). This is the correct length of the shortest diagonal in the parallelogram. This suggests that, while the majority of students were able to perform the requisite computations correctly, many did not read the question carefully and identify which computations to perform.

Question 8 required students to find the depth of a swimming pool with a trapezoidal cross-section at a distance of 8 metres from the shallow end of the pool (see the diagram below).



The most common mistake in answering this question was to assume that the relationship between similar triangles also applies to trapeziums. Making this assumption leads 1.31 metres (option B), which was incorrect.

Similar triangles could be used to answer the question in part by drawing in the dotted line to create the similar triangles *PBS* and *PAU* as shown above, where $AU = x$ and $BS = 0.8$ m.

We can then write:

$$\frac{x}{0.8} = \frac{8}{11} \quad \text{or} \quad x = 0.8 \times \frac{8}{11} = 0.58, \text{ correct to two decimal places}$$

Thus, the depth of the swimming pool at a distance of 8 metres from the shallow end of the pool is

depth = $TU = TA + AU = 1.00 + 0.58 = 1.58$ metres, correct to two decimal places

Question 9 proved challenging for many students. In this question, students were asked to determine the area of triangle *ABC*, given that the area of triangle *ABD* is 100 cm^2.

The most efficient way to solve this problem was to use scaling. The problem was to determine the linear scale factor *k*. Because the triangles *ABC* and *ABD* are similar it is clear that $k = \frac{40}{24}$.

Thus, the area of triangle $ABC = k^2 \times$ area of triangle $ABD = \left(\frac{40}{24}\right)^2 \times 100 = 277.777\ldots \text{ cm}^2 \approx 278 \text{ cm}^2$

Some students obtained the answer 278 cm^2, but then added in an area of 100 cm^2 to obtain the answer 378 cm^2 (option E). This could occur if they thought that the similar triangles were *ABD* and *DCB* rather than *ABD* and *ABC.*

Module 3: Graphs and relations

Question	% A	% B	% C	% D	% E	% No Answer
1	2	97	1	0	0	0
2	13	5	56	14	11	0
3	68	12	9	6	4	1
4	64	9	20	5	2	1
5	6	65	18	7	3	1
6	5	7	6	75	7	0
7	5	8	16	13	59	1
8	10	19	59	6	6	0
9	15	18	10	18	38	1

Module 3 (Graphs and relations) was generally well done, with the exception of Question 9.

Question 9 was the third in a series of three questions relating to the solution of a linear programming problem. Students were required to choose a rule for an objective function that would ensure the value of objective function M was maximised for all the points lying between and including the points A and B.

If an objective function is maximised for all points lying on the line between and including the points A and B, then it must have the same value at points A and B. Thus, of the rules given, the only rule that satisfies this criterion is $M = 5x + 5y$ (option E).

Another way of looking at this problem is to recognise that, for an objective function to be maximised for all points lying on the line between and including the points A and B, the family of lines associated with the objective function must be parallel to the line $x + y = 9$. This implies that its equation must be in the form $W = ax + ay$. Again, the rule $M = 5x + 5y$ is the only one of the rules given that satisfied this criterion.

Module 4: Business-related mathematics

Question	% A	% B	% C	% D	% E	% No Answer
1	5	6	3	8	77	0
2	11	2	4	81	1	0
3	7	4	4	3	82	1
4	10	11	15	8	56	1
5	20	11	16	7	44	1
6	11	18	21	43	6	1
7	7	33	16	40	4	1
8	15	12	15	13	43	1
9	15	44	20	12	8	1

Module 4 (Business and related mathematics) was generally well answered.

In Question 7, many students chose option D: $15\,000(1+\frac{r}{100})^{36} - 15\,000(1+\frac{r}{100})^{24}$ and this suggested that these students had used an appropriate method of solution, but had used the yearly interest r rate rather than the monthly interest rate $\frac{r}{12}$.

Module 5: Networks and decision mathematics

Question	% A	% B	% C	% D	% E	% No Answer
1	30	61	3	1	5	0
2	9	5	74	3	9	0
3	9	63	7	9	11	1
4	69	3	4	3	20	0
5	2	12	2	3	81	0
6	82	3	4	5	5	0
7	17	8	38	33	4	0
8	12	8	11	56	13	0
9	8	30	37	20	4	0

The questions in Module 5 (Networks and decision mathematics) were generally well answered, with the exception of Questions 7 and 9.

To answer Question 7 correctly, students needed to realise that it was the latest starting time for activity *M*, 24 minutes (option D), and not the earliest starting time of 18 minutes (option C) that determined the time that Caleb could speak on the phone before commencing activity *M*.

In Question 9, ‘The path could have included vertex Q more than once’ is the only true statement. Each of the other statements can be demonstrated to be false.

Module 6: Matrices

Question	% A	% B	% C	% D	% E	% No Answer
1	2	1	0	2	95	0
2	92	1	4	1	2	0
3	4	13	67	10	5	1
4	17	57	9	8	8	1
5	5	9	68	16	2	1
6	7	15	10	18	48	1
7	58	6	13	8	15	1
8	10	8	9	49	24	1
9	8	10	45	25	12	0

Module 6 (Matrices) was generally very well done.