A non-profit organisation supporting students to achieve their best.

Unit 3 and 4 Further Mathematics: Exam 1

Multiple-Choice Question Booklet

Duration: 15 minutes reading time, 1 hour 30 minutes writing time

Structure of book:

Section	Number of questions	Number of questions to be answered	Number of modules	Number of modules to be answered	Number of marks
A	13	13			13
B	54	27	6	3	27
				Total	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are not permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied:
- This question booklet of 25 pages, with a sheet of miscellaneous formulas.

Instructions:
- Detach the formula sheet from this book during reading time.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Section A

Instructions

Answer **all** questions by circling your choice.

Choose the response that is **correct** or that best answers the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Core: Data analysis

The following information relates to questions 1 and 2.

The height of seven tennis players are measured (in centimetres) and shown below.

180, 172, 156, 169, 189, 162, 165

Question 1

The mean height of these tennis players, to one decimal place, is closest to

    A. 169.8
    B. 170.4
    C. 171.2
    D. 170.5
    E. 171.9

Question 2

The median height of these tennis players is

    A. 165
    B. 166
    C. 167
    D. 168
    E. 169

Question 3

Ben is a tutor that spends the whole day teaching students maths. The number of students he helps and the study score that each one receives at the end of the year are two examples of variables. Choose the best description for these two variables.

    A. One is categorical and one is numerical
    B. They are both continuous
    C. They are both discrete
    D. They are both categorical
    E. One is discrete and one is continuous

Question 4

A least-squares regression line with the equation $y = 2.6 + 0.9x$ is fitted to a set of numerical bivariate data. The residual for the point (7, 9) is

    A.   0.1
    B.   3.7
    C.   1.9
    D.   -3.7
    E.   -0.1

The following information relates to questions 5 and 6.

William, a chemical engineer, is analysing the concentration of acid in a number of chemicals that were put through two experiments
        For *Experiment 1* the mean concentration was 81% and the standard deviation was 7.2%
        For *Experiment 2* the mean concentration was 68% and the standard deviation was 4.5%
        Pearson's correlation for the two experiments is 0.74

Question 5

Hydrofluoric acid, one of the chemicals that William is studying, obtained a z-score of 2.5 during *Experiment 1*.
To obtain a z-score of 2 during *Experiment 2* the hydrofluoric acid would need a concentration of

    A.   77%
    B.   73%
    C.   82%
    D.   85%
    E.   78%

Question 6

William wishes to uses a least-squares regression line to predict the concentration of *Experiment 1* from *Experiment 2*. From the statistics given above, the slope of this regression line, correct to two decimal places is

    A.   0.74
    B.   4.63
    C.   1.18
    D.   0.84
    E.   -4.63

Question 7

It is found that there is a correlation between the number of soft drinks purchased from a vending machine and the price of a soft drink from this specific machine. It is found that as the price increases, the number of drinks purchased decreases. The value of the coefficient of determination is 0.89 for this correlation.

The value of Pearson's product-moment correlation coefficient, to two decimal places, for this relationship is

    A.   -0.79
    B.   0.79
    C.   -0.94
    D.   0.83
    E.   -0.83

Question 8

Christina is absent on the day the other 9 students in her class sat a statistics test. No two students achieve the same score. The teacher calculates a five figure summary as well the mean and standard deviation of the scores. After Christina returns to school she sits the statistics test and scores the highest mark in the class. The teacher then recalculates the same statistics for the class as before but makes sure to include Christina's mark.

Which one of the following statistics will not change when Christina's mark is included?

    A.   The mean
    B.   The median
    C.   The lower quartile
    D.   The upper quartile
   E.   The standard deviation

Question 9

A producer of plastic water bottles finds that the radius of the bottles forms a normal distribution with a mean of 3 cm and a standard deviation of 0.2 cm. The maximum radius of a bottle than can be used and sold is 3.2 cm. Any bottles produced that are larger than this must be thrown away. If the factory producing the bottles makes 100000 per year the amount that get thrown away each year is

    A.   50000
    B.   2500
    C.   34000
    D.   16000
    E.   13500

Question 10

Niranjan is working on some data analysis. After seeing the scatterplot below, Niranjan decides to perform a transformation on the data.
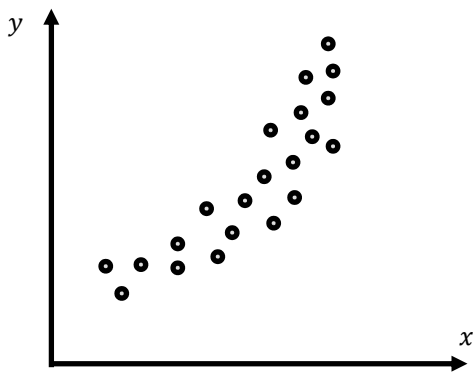
The plot suggests that a useful transformation will be

    A.   A 1/x transformation
    B.   A 1/y transformation
    C.   A log(x) transformation
    D.   A y^2 transformation
    E.   None of the above

Question 11

Which of the following pair of measurements is not affected by outliers?

    A. Mean & Interquartile Range
    B. Mean & Range
    C. Median & Standard Deviation
    D. Median & Interquartile Range
    E. Mean & Standard Deviation

The following information relates to questions 12 and 13.

Charles runs a florist in Toorak and finds that his sales follow a seasonal trend shown by the seasonal indices in the table below:

Summer	Autumn	Winter	Spring
1.11	0.78		1.62

Question 12

The seasonal index for winter is

    A. 1.00
    B. 0.79
    C. 1.17
    D. 3.51
    E. 0.49

Question 13

Charles sells 23 bouquets of flowers during summer and wants to improve on his sales during the next season, autumn. Using the seasonal indices given above, the **minimum** number of bouquets that Charles must sell in autumn to outperform his sales in summer is

    A. 17
    B. 24
    C. 33
    D. 12
    E. 34

Section B

Instructions

Select **three** modules and answer **all** questions within the modules by circling your choice.

Choose the response that is **correct** or that best answers the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Module 1: Number patterns

Question 1

The sixth term of the geometric sequence 7, -28, 112, … is

- A. 28672
- B. -7168
- C. 5694
- D. -1792
- E. 1792

Question 2

For the arithmetic sequence -16, -11, -6, -1, 4, … The values of a, the first term and d, the common difference are

- A. $a = 16, d = 5$
- B. $a = -16, d = -5$
- C. $a = -16, d = -4$
- D. $a = -16, d = 5$
- E. $a = -16, d = 4$

Question 3

The amount of chlorine in a certain swimming pool increases by 4% every week. If the pool had 300 ml of chlorine in the first week, in which week will it first have more than 1000 ml?

- A. 20th week
- B. 21st week
- C. 31st week
- D. 32nd week
- E. Never

Question 4

The first-order difference equation $t_{n+1} = 4t_n - 14$ has a first term of 12. The fifth term of the sequence generated by this difference equation is

- A. 1882
- B. 494
- C. 474
- D. 7514
- E. 3245

Question 5

For the Fibonacci-type sequence ….. 19, 30, 49, 79, 128, …. The first two terms could be

  A. $f_1 = 3, f_2 = 2$
  B. $f_1 = 3, f_2 = 5$
  C. $f_1 = 2, f_2 = 3$
  D. $f_1 = 5, f_2 = 3$
  E. $f_1 = 4, f_2 = 2$

Question 6

The first two terms of a geometric sequence are 375 and 75. The **sum of the next four terms** is

  A. 468.72
  B. 43.6
  C. 468
  D. 18.72
  E. 93.6

Question 7

Eloise opens a savings account at her local bank. She initially places $50 into the account which earns an interest rate of 7.2% per annum. She decides to deposit $25 into her account at the start of each month and the interest is calculated and added to the balance at the end of each month. A difference equation which would describe this scenario is

  A. $t_{n+1} = 1.006t_n + 25, t_0 = 50$
  B. $t_{n+1} = 1.072t_n + 25, t_0 = 50$
  C. $t_{n+1} = 1.072t_n + 50, t_0 = 25$
  D. $t_{n+1} = 1.072t_n + 26.5, t_0 = 50$
 E. $t_{n+1} = 1.006t_n + 25.15, t_0 = 50$

Question 8

The second-order difference equation that can used to generate the sequence 1, 2, 4, 8, 16, is

  A. $t_n = t_{n-1} + t_{n-2}$, where $t_1 = 1, t_2 = 2$
  B. $t_n = 2t_{n-1} + t_{n-2}$, where $t_1 = 1, t_2 = 2$
  C. $t_n = t_{n-1} + 2t_{n-2}$, where $t_1 = 1, t_2 = 2$
  D. $t_n = 2t_{n-1}$ where $t_1 = 1$
 E. $t_n = t_{n-1} + 2$, where $t_1 = 1$

Question 9

The number of people (y) attending a sports event on each subsequent game week (x) is shown on the graph below. You may assume that the weekly increase in people attending is the same.
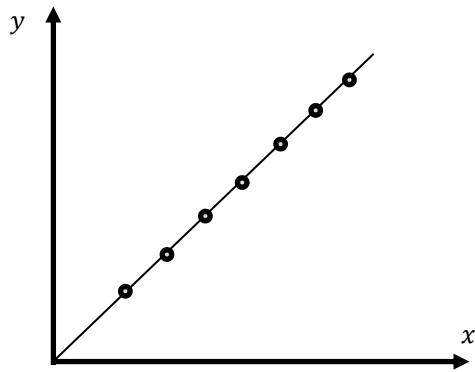
The number of people attending the sports event appears to follow

  A. A geometric sequence
  B. An arithmetic sequence
  C. A Fibonacci-like sequence
  D. An infinite geometric sequence
  E. None of the above

Module 2: Geometry and Trigonometry
Question 1
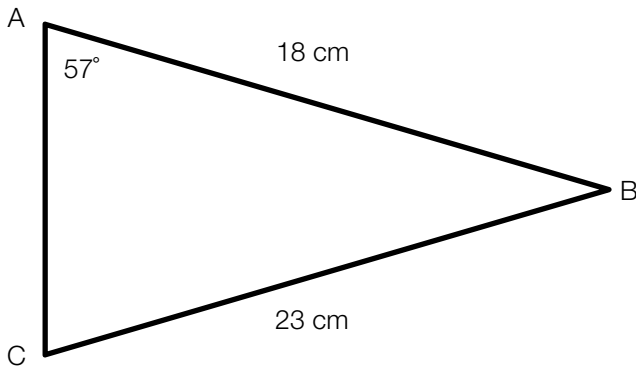
The area of triangle *ABC* is closest to

    A. 196 cm^2
    B. 205 cm^2
    C. 218 cm^2
    D. 229 cm^2
    E. 235 cn^2

Question 2
Flavia watches the soccer from her seat in AAMI Park. To get there from the ground she must climb 240 steps each of height 15 cm. When seated she looks down onto the soccer pitch at an angle of depression of 72°. If a player kicks the ball right to where Flavia is seated in a straight line the distance it travels will be closest to

    A. 37.85 m
   B. 33.42 m
    C. 35.31 m
    D. 29.50 m
    E. 34.23 m

Question 3
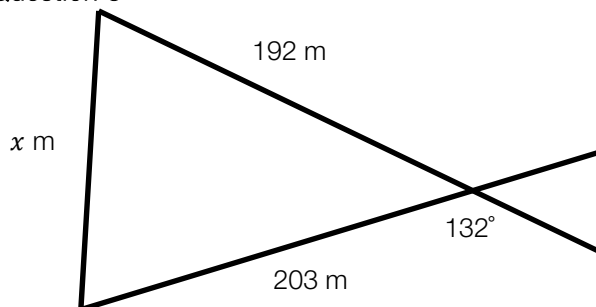
The value of x in the diagram above, to two decimal places, is

    A. 208.23 m
    B. 198.17 m
    C. 150.83 m
    D. 274.89 m
    E. 160.97 cm

The following information relates to questions 4, 5 and 6.

The contour map below is a bird's eye view of two hills located in a park. The local council decides to place a ramp from point A to point B in order to let people walk up the hill more easily.
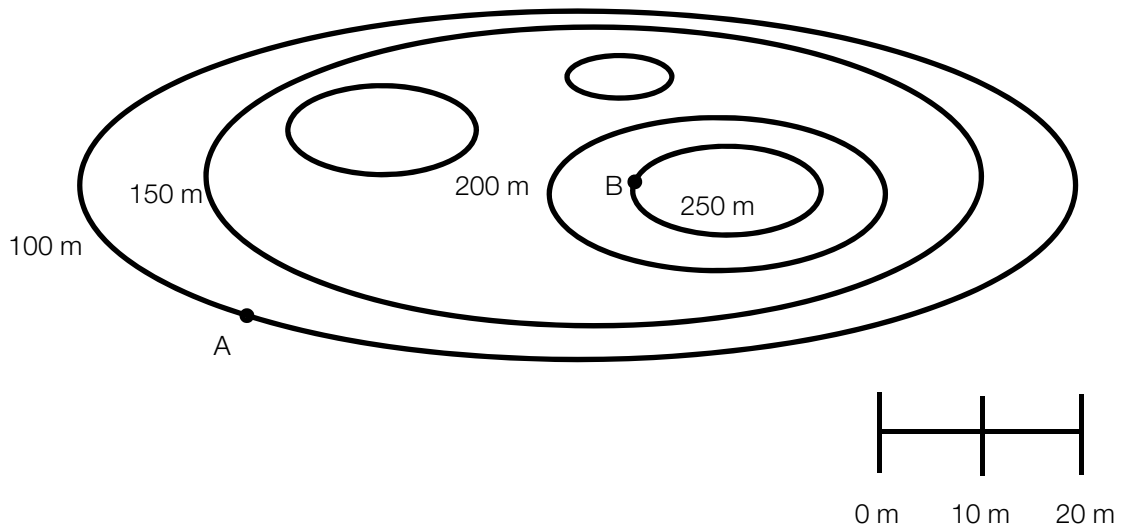
Question 4

The length of the ramp, assuming that it would be entirely straight, is closest to

    A. 204 m
    B. 102 m
    C. 192 m
    D. 155 m
    E. 176 m

Question 5

The average slope of the ramp from A to B will be

    A. 1.92
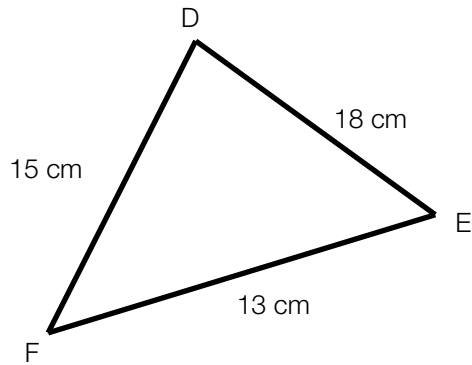   B. 1.29
    C. 2.42
    D. 0.27
    E. 3.75

Question 6

The angle of elevation from A to B is closest to

    A. $15°$
    B. $29°$
    C. $53°$
    D. $75°$
    E. $90°$

Question 7

A cartographer creates a map of a nearby island. The map is 40 cm in length while the actual area covered by the map is 25 km in length. The scale factor is
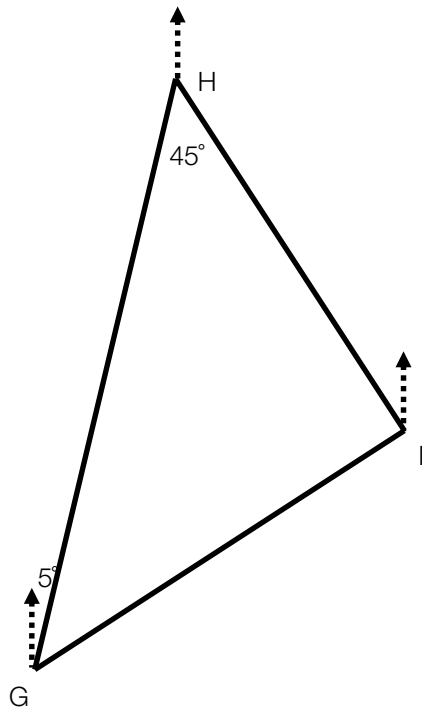
   A.   1: 62.5
   B.   1: 625
   C.   1: 6,250
   D.   1: 62,500
   E.   1: 625,000

Question 8

Using Heron's formula to calculate the area of triangle *DEF* above, the correct calculation is

   A. $A = 23\sqrt{(23-15)(23-18)(23-13)}$
   B. $A = \sqrt{23(15-23)(18-23)(13-23)}$
   C. $A = \sqrt{23 \times (8) \times (5) \times (10)}$
   D. $A = 23\sqrt{(8) \times (5) \times (10)}$
   E. $A = 23\sqrt{(8) \times (15) \times (10)}$

Question 9

The bearing of point H from point G is 005° and the angle GHI in the triangle GHI is 45°. The diagram above shows this situation.

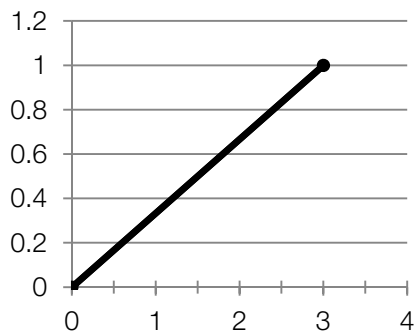The bearing of point H from point I is

A. 95°
B. 40°
C. 320°
D. 140°
E. 175°

Module 3: Graphs and relations

Question 1

The equation of the straight line that passes through the points (–2, –2) and (5, –1) is:

    A. $\;-7y = x - 12$
    B. $\;7y = x + 12$
    C. $\;x - 7y = 12$
    D. $\;x + 7y = 12$
    E. $\;-x - 7y = 12$

Question 2

Determine the slope of the graph below.

    A. $\;m = 1$
    B. $\;m = 3$
    C. $\;m = {}^1\!/_2$
    D. $\;m = {}^1\!/_3$
    E. $\;m = -{}^1\!/_3$

Question 3

When the bathplug is pulled out, water starts to drain at a constant rate of 1.25 litres per second until the bathtub is empty.

If the bathtub initially held 80 litres of water, how many seconds will it take before the bathtub is empty?

    A. 63
    B. 64
    C. 65
    D. 66
    E. 67

Question 4

A region is defined by the following inequalities:

$$y \leq 10$$
$$y - 9 \leq -2x$$

Which of the following coordinates do **not** lie within this region?

A. (0, 10)
B. (–2, 5)
C. (0, 8)
D. (2, 5)
E. (0, 0)

Question 5

At Freaky Fred's Fruit Shop it costs $8.20 to buy 7 nectarines and 2 mangoes. It costs $8.90 to buy 3 nectarines and 5 mangoes.

How much does it cost to buy 1 nectarine and 2 mangoes?

A. $2.40
B. $2.90
C. $3.40
D. $3.90
E. $4.40

Question 6

The cleaner's charge is given by the rule $C(\$) = 30 + 30t$, where t represents the number of hours worked.

Which of the following statements is false?

A. After working 1 hour, the cleaner receives $60
B. After working 2 hours, the cleaner receives $120
C. After working 4 hours, the cleaner receives $150
D. After working 6 hours, the cleaner receives $210
E. After working 9 hours, the cleaner receives $300

Question 7

The following graph represents the cost, in dollars, of hiring one laptop.
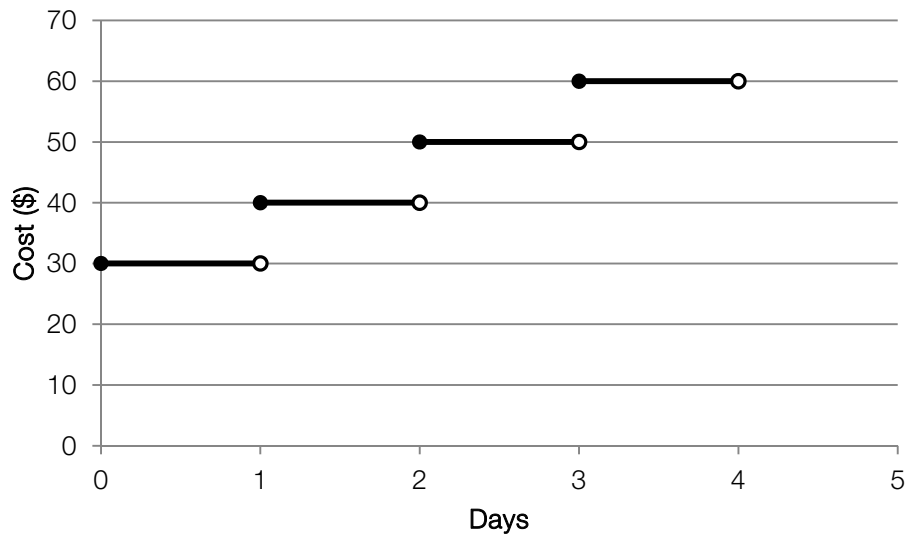
How much does it cost to hire 2 separate laptops for 3 days?

    A. $50
    B. $60
    C. $100
    D. $110
    E. $120

Question 8

Stuart runs a T-Shirt factory. There is a fixed cost of $50 as well as a cost of $2 to manufacture each T-Shirt. Each T-Shirt sells for $15.

If Stuart produces 65 T-Shirts, how many of them must he sell in order to break even?

    A. 10
    B. 12
    C. 34
    D. 50
    E. 65

Question 9

Which of the following statements regarding the equation $-6y = 12x + 6$ is **not** true?

    A. A point on the line is (1,–3)
    B. The y-intercept occurs at (0,–1)
    C. The line does not pass through the origin
    D. The gradient of the line is 12
    E. The x-intercept occurs at (–0.5,0)

Module 4: Business-related mathematics

Question 1
$6000 is invested at a simple interest rate of 5% per annum. After 4 years the investment is worth

    A. $900
    B. $960
    C. $6900
    D. $7200
    E. $9600

Question 2
$2000 is invested at a compound interest rate of 3% per annum. After 5 years the investment is worth

    A. $2251.02
    B. $2318.55
    C. $2388.10
    D. $2459.75
    E. $2533.54

The following information relates to questions 3, 4 and 5.
Dave decides to buy a new phone that costs $850. His hire-purchase agreement requires him to pay an initial deposit of $150 and then pay monthly instalments of $40 for the next two years.

Question 3
The flat interest rate per annum that Dave is paying with this arrangement is closest to

    A. 19.26%
    B. 18.57%
    C. 15.43%
    D. 20.14%
    E. 22.92%

Question 4
The effective interest rate per annum that Dave with this arrangement is

    A. 35.65%
    B. 36.98%
    C. 37.14%
    D. 40.14%
    E. 33.42%

Question 5
If Dave decides to go with a different hire-purchase agreement with the same initial deposit but instead he decides to pay twice as much each month for a period of only one year what is the difference in the cost of the phone between the original agreement and this one?

    A. -$960
    B. -$150
    C. $0
    D. $150
    E. $960

Question 6

For the past five years, Nick's property has been valued each year. In the first year the total value increased by 2% and then increased a further 12% in the second year. In the third year the property decreased in value by 7%, it then increased by 6% in the fourth year and then eventually decreased by 4% in the final year. Compared to the initial value, the value of Nick's property is now closest to

- A.   1% lower
- B.   6% higher
- C.   5% lower
- D.   8% higher
- E.   The same

Question 7

Callum buys a new computer for his business for $3000. The value of the computer is determined using the reducing balance deprecation method with a rate of 16.7% depreciation per annum. If the scrap value of the computer is $500, the computer will be able to be written off during the

- A.   8th year
- B.   9th year
- C.   10th year
- D.   11th year
- E.   12th year

Question 8

Anna has $246000 that she wishes to invest in a perpetuity in order to receive a regular monthly payment from the interest. If the interest rate is 6.1% per annum, the amount Anna should expect to receive each month is

- A. $125.05
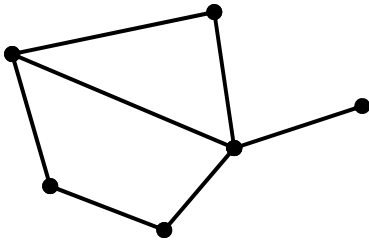- B. $1250.50
- C. $1500.60
- D. $15006
- E. $20500

Question 9

A home loan of $180000 is taken out at an interest rate of 8.9% pa compounding monthly. To be fully repaid in 15 years, equal monthly payments will be closest to

- A. $17983.21
- B. $2329.34
- C. $1945.63
- D. $2270.43
- E. $1814.99

Module 5: Networks and decision mathematics
Question 1

The number of vertices with an even degree in the above network is:

    A. 0
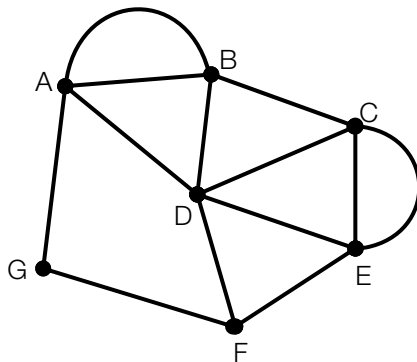   B. 1
    C. 3
    D. 4
    E. 6

Question 2
A connected planar graph has 8 edges.

This graph could have:

    A. 8 vertices and 4 faces
    B. 4 vertices and 6 faces
    C. 6 vertices and 3 faces
    D. 7 vertices and 1 face
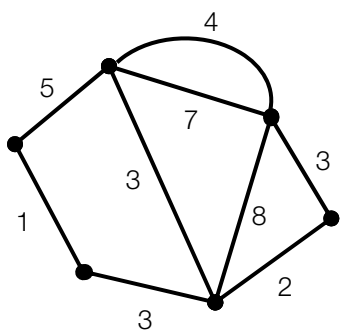   E. 9 vertices and 5 faces

Question 3

A Hamiltonian Path for the graph above that begins at F is:

    A. G-A-B-D-C-E-F
    B. F-G-A-B-D-C-E-F
    C. F-E-C-E-D-F-G-A-B-A-D-B-C-D
    D. F-E-C-D-B-A-G
    E. A Hamiltonian Path does not exist
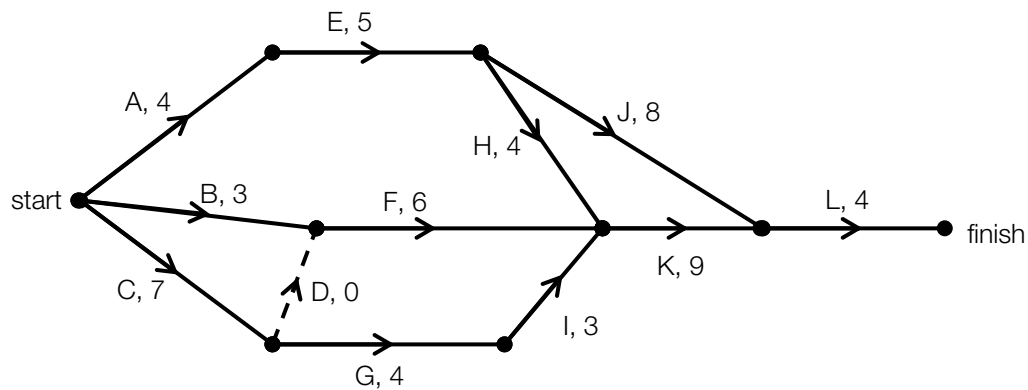
Question 4

A minimal spanning tree for the network above will include the edge that has the weight of:

    A.   1

    B.   4

    C.   5

    D.   7

    E.   8

The following information relates to questions 5 and 6.

The following network shows the activities that are needed to complete a building project and their completion times (in weeks).

Question 5

Which of the statements about the building project is **not** true?

    A.   It will take a minimum of 29 weeks to complete
    B.   Activity J may start later than activity G
    C.   Activity I is on the critical path
    D.   Activities A, B and C all have the earliest start time
    E.   The earliest start time for activity G is week 7

Question 6

What is the critical path for the above network?

    A.   A-E-J-L
    B.   A-E-H-K-L
    C.   C-G-I-K-L
    D.   B-F-I-K-L
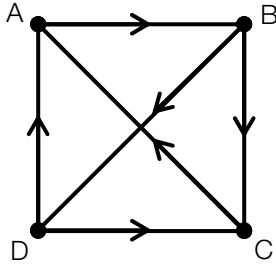   E.   L-K-H-E-A

Question 7
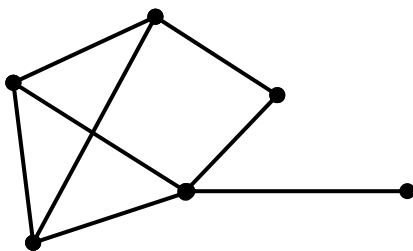
The graph above has:

    A.   1 face
    B.   2 faces
    C.   3 faces
    D.   4 faces
    E.   5 faces

The following information relates to questions 8 and 9.

There are four finalists in a checkers competition. In 2011, each competitor will play each other. The results from the competition are shown in the directed graph below. An arrow from D to A indicates that A was defeated by D.

Question 8

It was decided that in 2011, the winner would be the finalist with the highest number of two-step dominances. The winner in 2011 was:

    A. Finalist A
    B. Finalist B
    C. Finalist C
    D. Finalist D
    E. It was a draw between finalists C and D

Question 9

In 2012 an extra 2 finalists, E and F, were allowed to compete in the checkers competition. The number of extra games to be played in 2012 will be:

    A. 6
    B. 15
    C. 21
    D. 9
    E. 2

Module 6: Matrices

Question 1

The order of the matrix $[2 \quad 3]$ is:

    A. 2 x 3
    B. 3 x 2
    C. 1 x 2
    D. 2 x 1
    E. 1 x 1

Question 2

The matrix $\begin{bmatrix} 14 & 7 & 7 \\ 7 & 14 & 7 \\ 7 & 7 & 14 \end{bmatrix}$ is equivalent to:

    A. $\begin{bmatrix} 7 & 7 & 14 \\ 7 & 14 & 7 \\ 14 & 7 & 7 \end{bmatrix}$

    B. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

    C. $2\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$

    D. $7\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$

    E. $7\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix}$

Question 3

What is the determinant of the matrix $\begin{bmatrix} 4 & 8 \\ -3 & 3 \end{bmatrix}$?

    A. −24
    B. 12
    C. 36
    D. 48
    E. −48

Question 4

$6\begin{bmatrix} -3 & -1 \\ 1 & 2 \end{bmatrix} - 2\begin{bmatrix} 4 & 2 \\ 8 & -5 \end{bmatrix}$ equals:

    A. $\begin{bmatrix} -26 & -10 \\ 10 & 22 \end{bmatrix}$

    B. $\begin{bmatrix} -10 & -2 \\ 22 & 2 \end{bmatrix}$

    C. $2\begin{bmatrix} -13 & -5 \\ -5 & 11 \end{bmatrix}$

    D. $3\begin{bmatrix} -13 & 5 \\ -5 & 11 \end{bmatrix}$

    E. $2\begin{bmatrix} -13 & -5 \\ 5 & 11 \end{bmatrix}$

Question 5

$$S_3 = \begin{bmatrix} 100 \\ 20 \end{bmatrix}$$

$$T = \begin{bmatrix} .8 & .4 \\ .2 & .6 \end{bmatrix}$$

Therefore, which of the following matrices is S_4?

A. $\begin{bmatrix} 88 \\ 32 \end{bmatrix}$

B. $\begin{bmatrix} 32 \\ 88 \end{bmatrix}$

C. $\begin{bmatrix} 20 \\ 100 \end{bmatrix}$

D. $\begin{bmatrix} 66 \\ 54 \end{bmatrix}$

E. $\begin{bmatrix} 54 \\ 66 \end{bmatrix}$

Question 6

The matrix equation $\begin{bmatrix} 1 & 0 & 2 \\ 3 & 4 & 3 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 53 \\ 26 \end{bmatrix}$ can be used to evaluate which set of simultaneous equations?

A. $x + y + 2z = 10$
$3x + 4y + 3z = 53$
$y + 2z = 26$

B. $x + 2z = 10$
$3x + 4y + 3z = 53$
$y + 2z = 26$

C. $x + 2z = 10$
$3x + 4y + 3z = 10$
$y + 2z = 53$

D. $2x + z = 10$
$3x + 4y + 3z = 53$
$2x + y = 26$

E. None of the above.

Question 7

Let $A = \begin{bmatrix} 8 & 4 \\ 2 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 10 \end{bmatrix}$, and $C = \begin{bmatrix} 7 \\ 4 \end{bmatrix}$.

Which of the following matrix products is not defined?

A. AC
B. CB
C. AB
D. BC
E. BAC

Question 8

If $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 2 & 8 \\ m & 4 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 2 & 8 \end{bmatrix}$, the value of m is

    A. 4
    B. 6
    C. 10
    D. 21
    E. 22

Question 9

In a certain village there are two bakeries: Crumby's and Baker's Joy.

- 85% of villagers who shop at Crumby's on a given week will shop there the next week.
- 25% of villagers who shop at Baker's Joy on a given week will shop there the next week.

Considering this, the transition matrix that represents this information is:

    A. $\begin{bmatrix} .85 & .25 \\ .15 & .75 \end{bmatrix}$
    B. $\begin{bmatrix} .85 & .75 \\ .15 & .25 \end{bmatrix}$
    C. $\begin{bmatrix} .85 & .15 \\ .25 & .75 \end{bmatrix}$
    D. $\begin{bmatrix} .15 & .75 \\ .85 & .25 \end{bmatrix}$
    E. $\begin{bmatrix} .75 & .15 \\ .85 & .25 \end{bmatrix}$

Formula Sheet

Core: Data analysis

Standardised score:

$$z = \frac{x - \bar{x}}{s_x}$$

Least squares line:

$y = a + bx$ where $b = r\frac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$

Residual value:

residual value = actual value − predicted value

Seasonal index:

$$\text{seasonal index} = \frac{\text{actual figure}}{\text{deseasonalised figure}}$$

Module 1: Number patterns

Arithmetic series:

$$a + (a + d) + \cdots + (a + (n-1)d) = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a + l)$$

Geometric series:

$$a + ar + ar^2 + \cdots + ar^{n-1} = \frac{a(1 - r^n)}{1 - r}, r \neq 1$$

Infinite geometric series:

$$a + ar + ar^2 + ar^3 + \cdots = \frac{a}{1 - r}, |r| < 1$$

Module 2: Geometry and trigonometry

Area of a triangle:

$$\frac{1}{2}bc \sin A$$

Heron's formula:

$A = \sqrt{s(s - a)(s - b)(s - c)}$, where $s = \frac{1}{2}(a + b + c)$

Circumference of a circle:

$2\pi r$

Area of a circle:

πr^2

Volume of a sphere:

$\frac{4}{3}\pi r^3$

Surface area of a sphere:

$4\pi r^2$

Volume of a cone:

$\frac{1}{3}\pi r^2 h$

Volume of a cylinder:

$\pi r^2 h$

Volume of a prism:

area of base × height

Volume of a pyramid:

$\frac{1}{3}$ area of base × height

Pythagoras' theorem:

$c^2 = a^2 + b^2$

Sine rule:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine rule:

$c^2 = a^2 + b^2 - 2ab \cos C$

Module 3: Graphs and relations

Straight line graphs

Gradient (slope):

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Equation:

$$y = mx + c$$

Module 4: Business-related mathematics

Simple interest:

$$I = \frac{PrT}{100}$$

Compound interest:

$$A = PR^n, \text{ where } R = 1 + \frac{r}{100}$$

Hire purchase:

$$\text{effective rate of interest} \approx \frac{2n}{n+1} \times \text{flat rate}$$

Module 5: Networks and decision mathematics

Euler's formula:

$$v + f = e + 2$$

Module 6: Matrices

Determinant of a 2 x 2 matrix:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

Inverse of a 2 x 2 matrix:

$$A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}, \text{ where } \det A \neq 0$$