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# Units 3 and 4 Further Maths: Exam 1

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Don't look at these solutions until you have attempted the exam.

Found a mistake?

Check the Engage Education website for updated solutions, and then email practiceexams@ee.org.au.

# Section A

## Core: Data analysis

### Question 1
The correct answer is D.

### Question 2
The correct answer is B.

$$\text{Range} = 50 - 8 = 42$$

$$\text{IQR} = 38 - 12 = 26$$

### Question 3
The correct answer is B.

There is an even number of data points. Once the data is put in ascending order median is found by halving the 5th and 6th values. $\frac{22+30}{2} = 26$

### Question 4
The correct answer is C.

$$3\text{rd quartile} - 1\text{st quartile} = 33 - 13 = 20$$

### Question 5
The correct answer is C.

Outliers of this nature positively skew the mean. Their removal will lower the mean.

### Question 6
The correct answer is D.

557.5 g is 3 standard deviations above the mean. The percentage of boxes above this would be 0.15%.

### Question 7
The correct answer is C.

Boxes will be rejected if they are more than one standard deviation below the mean, this is 16% of boxes. As well 2.5% of boxes will be 2 standard deviations above the mean.

In total $1000 \times 18.5\% = 185$ boxes will be rejected.

### Question 8
The correct answer is E.

Both the $y^2$ and $x^2$ transformations are the appropriate for this type of data.

### Question 9
The correct answer is B.

The bottom 50% for school A are stretched over a lengthy interval hence it is a negatively skewed distribution. School B's distribution is roughly symmetrical.

### Question 10
The correct answer is D.

This statement is false since the boxplots only shows that the 41st and 80th top ranked students of school A have are equal with the top 55 ranked students at school B.

### Question 11
The correct answer is B.

Seasonal indices should total the amount of 'seasons' in this case that is 6.

$6 - 5.1 = 0.9$

### Question 12
The correct answer is D.

The linear regression model is $\text{number of kicks} = a \times \text{number of handballs} + b$, where $a = 0.251$ and $b = 159.57$

### Question 13
The correct answer is C.

Performing the linear regression we have $r = 0.370$, which can be interpreted as a positive linear association. Generally Pearson's product-moment coefficient values between 0.25 and 0.5 are considered weak.


# Section B
## Module 1: Number patterns
### Question 1
The correct answer is C.

Using the formula for the sum of the first n terms, with $n = 8, a = 17, d = 7$

$$S_8 = \frac{8}{2} \times [(2 \times 17) + (8 - 1) \times 7]$$

### Question 2
The correct answer is D.

The arithmetic sequence describing the number of stamps added to the original collection is 6, 12, 18, 24, …, 72. So by the end of the 12th week Ben has added 72 extra stamps.

$219 - 72 = 147$

### Question 3
The correct answer is D.

The difference equation needs to multiply the current population $p_n$ by 1.15 in order to have a 15% increase. After the multiplication is performed we must then subtract 300. The initial population is 2,500.

Hence $p_{n+1} = 1.15p_n - 300, with\ p_1 = 2,500$

### Question 4
The correct answer is E.

Year                Kangaroo population

2012                2500
2013                $1.15 \times 2,500 - 300 = 2,575$
2014                $1.15 \times 2,574 - 300 = 2,661.25 \approx 2,661$

### Question 5
The correct answer is C.

Given the two equations:

$$t_3 = a \times r^2 = \frac{4}{3} \qquad\qquad [1]$$

$$t_5 = a \times r^4 = \frac{4}{27} \qquad\qquad [2]$$

Now dividing equation 2 by equation 1:

$$\left[\frac{[2]}{[1]}\right] = \frac{a \times r^4}{a \times r^2} = \frac{\frac{4}{27}}{\frac{4}{3}}$$

$r^2 = \frac{1}{9}$ therefore $r = \frac{1}{3}$. Substituting back into [1] we find that $t_3 = a \times \left(\frac{1}{3}\right)^2 = \frac{4}{3}$

Solving for a we find $a = \frac{4}{3} \times 9 = 12$

### Question 6
The correct answer is A.

From question 5 we found that $a = 12$ and $= \frac{1}{3}$ , substituting these into the formula for an infinite geometric sum we find $S_\infty = \frac{12}{1-\frac{1}{3}} = 18$

### Question 7
The correct answer is D.

Looking at the options we see that successive terms change signs. So $a$ must be a negative number, and so options A, C or D are left. Solving for b. We have:
$t_2 = -19 = -2 \times 5 + b$
$b = -19 + 10 = -9$

Also, $t_3 = 29 = (-2 \times -19) - 9$ confirms the formula is applicable to the whole equation

### Question 8
The correct answer is D.

Since we see a saw tooth pattern in the graph, the common ration must have a negative value. Furthermore, since the overall pattern is increasing then $a < -1$ is the appropriate choice, as the sequence will not only alternate but also grow in magnitude.

### Question 9
The correct answer is C.

In order to find an answer for the infinite sum of a geometric series, we need $a$ the first term as well as $r$ the common ratio.

Strictly speaking the common ratio, $r$, must satisfy the following condition:

$-1 < r < 1$ In order to be produce a finite answer for a sum to infinity of a geometric sequence, the common ratio must strictly fit within these bounds. As $0 < r < 1$ is also satisfies this condition, the answer is C.

## Module 2: Geometry and trigonometry
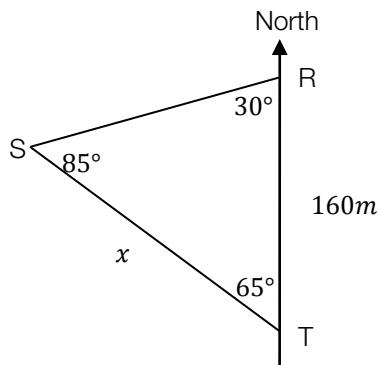
### Question 1
The correct answer is C.

$18° + 56° = 74°$. A basic rule for triangles tells us that the external angle $\angle ACD$ is the sum of the two adjacent angles.
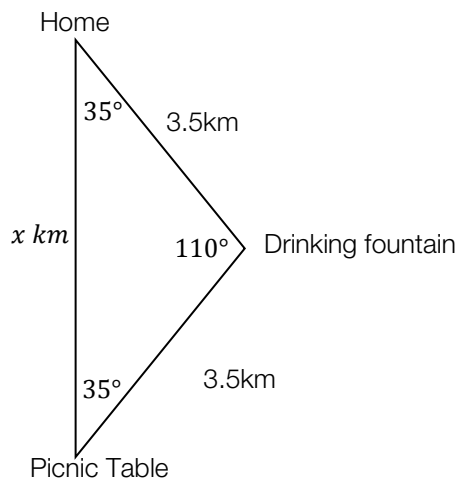
### Question 2
The correct answer is B.

From the information given, we can find the angles of the triangle. As we have two angles and one side, we can use the sine rule.

$\frac{160}{\sin(85°)} = \frac{x}{\sin(30°)}$ , then $x = \sin(30°) \times \frac{160}{\sin(85°)} \approx 80.31m$

### Question 3

Students should be able to construct the following diagram from the information given.



The correct answer is E.

First calculate the length of $x = \sin(110°) \times \frac{3.5}{\sin(35)} \approx 5.73km$. Add this length to the 2 3.5km legs of the trip that we are told about. $\text{total length} = 5.73 + 3.5 + 3.5 = 12.73km$

### Question 4

The correct answer is C.

Area of the above triangle is $\text{Area} = \frac{1}{2} \times 3.5 \times 3.5 \times \sin(110°) \approx 5.76km^2$

### Question 5

The correct answer is B.

Using similar triangles:

$\frac{x}{15.5} = \frac{1.7}{3.5}$. Therefore $x = 15.5 \times \frac{1.7}{3.5} \approx 7.53m$

### Question 6

The correct answer is B.

First calculate the diameter of the circle which is the length of the triangle. $d = \sin(92°) \times \frac{6}{\sin(44°)} \approx 8.632cm$.

$\text{Area of cirlce} = \pi \times \left(\frac{8.632}{2}\right)^2 \approx 59\ cm^2$

### Question 7

The correct answer is D.

Let $r_c$ = radius of the cricket ball and $r_v$ = the radius of the volleyball. We are told that $r_c = \frac{2}{5} r_v$. [1]

Then the ratio of the volumes of the cricket ball with the volleyball is given by the following relationship:

$$\frac{\frac{4}{3} \pi r_c^3}{\frac{4}{3} \pi r_v^3} = \left( \frac{r_c}{r_v} \right)^3 \text{ [2]}$$
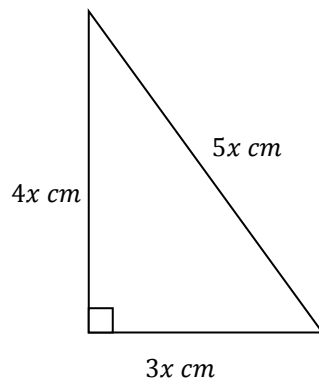
Substituting equation [1] into equation [2], we find:

$$\left( \frac{\frac{2}{5} r_v}{r_v} \right)^3 = \left( \frac{2}{5} \right)^3 = \frac{8}{125}$$

### Question 8

The correct answer is E.

Students should construct the following diagram:-



Students should now find the length of x, from the information provided.

$$\text{Area} = 294 cm^2 = \frac{1}{2} \times 3x \times 4x = 6x^2$$
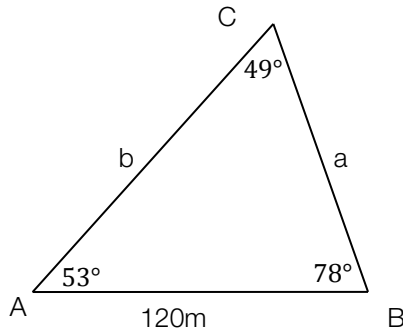$$6x^2 = 294 cm^2$$

$$x^2 = 49 cm^2$$

$$x = 7 cm$$

Total perimeter is $(3 + 4 + 5) \times 7 cm = 84 cm$

### Question 9
The correct answer is D.

Students should construct the following diagram from the bearings given. Note that the inner angles are the complements of the bearings given.



From this we can solve for a and b respectively.

$$a = \sin(53°) \times \frac{120}{\sin(49°)} \approx 126.98m$$

$$b = \sin(78°) \times \frac{120}{\sin(49°)} \approx 155.53m$$

So Jason should begin his swim from point B, and he will swim $155.53 - 126.98 = 28.54m$ less by doing so.

## Module 3: Graphs and Relations
### Question 1
The correct answer is C.

$$\text{Salary} = 1,100 + 75 \times (\text{dishwasher units})$$

$$1925 = 1,100 + 75 \times 11$$

### Question 2
The correct answer is D.

The first equation is $y = 2x$. Hence the gradient of the second line is $m = -2$. The equation of the second line is $y = -2x + 8$

### Question 3
The correct answer is C.
The equation $7y + 15x = -105$

Is the only one that satisfies the points $(-7,0)$ and $(0, -15)$

### Question 4
The correct answer is B.

The extra 5kg costs $107.5 more. Hence $5 \times a = 107.5$

$a = \frac{107.5}{5} = 21.5$. Then $1,515 = \text{base ticket price} + 21.5 \times 10$

$\text{base ticket price} = 1,515 - 215 = 1300$

### Question 5
The correct answer is C.

The original pair given is $(x, y) = (3,3)$. When written as $(x^2, y) = (9,3)$. Option C gives this pairing.

### Question 6
The correct answer is E.

At $x = 10$ Equation 1 $(10, 23\frac{1}{3})$. Equation 2 $(10, -2.5)$. Equation 1 has the larger y value.

### Question 7
The correct answer is C.

$\text{Cost} = 1,000 + 0.20 \times \text{number of ice creams sold}$

As each ice cream costs 20 cents to make, Alan only makes 80 cents profit for each one sold. But we must also consider that it cost Alan $2,000 initially to buy the van. Hence:
$\text{Profit} = -2000 + 0.80 \times \text{number of ice creams sold}$

### Question 8
The correct answer is D.

To break even is the same as solving when he makes a profit of $0. This occurs when $0 = -2000 + 0.80 \times \text{number of ice creams sold}$

$\frac{2000}{0.8} = 2500 \text{ ice creams}$

### Question 9
The correct answer is D.

Take $\frac{1}{x} = a$, to be a new variable. Then the equation of the line given in the graph is $y = \frac{4}{9} \times a$, since it passes through the origin, it has a y-intercept of 0.

Substituting $a = \frac{1}{x}$. We recover $y = \frac{4}{9} \times \frac{1}{x} = \frac{4}{9x}$

## Module 4: Business-related mathematics
### Question 1
The correct answer is D.

### Question 2
The correct answer is A.

$SI = \frac{Prt}{100}$

### Question 3
The correct answer is B.

$$38000 - 10000 = 0.25 \times km$$

### Question 4
The correct answer is A.

$$\text{Estelle's interest} = \frac{2000 \times 6.5 \times 4}{100} \qquad \text{Jo's interest} = \left[2000 \times \left(1 + \frac{6.5}{100}\right)^4\right] - 2000$$

### Question 5
The correct answer is C.

$$(102 \times 1.1) \times 3 = total\ cost$$

### Question 6
The correct answer is E.

$$\text{Price on Friday } \therefore \text{Saturday} = 58.00 \times 1.25 \times 0.90 \times 0.75 \times 1.20$$

### Question 7
The correct answer is B.

Use finance solver.

### Question 8
The correct answer is B.

Use finance solver.

### Question 9
The correct answer is A.

Through using finance solver find the FV for 10 years, then use this value as the principle to work out the months further taken to pay the loan. Divide this value by 12 to put in years and add ten to get total loan life.

## Module 5: Networks and decision mathematics
### Question 1
The correct answer is B.

$$\text{edges of a connected graph} = \text{number of vertices} - 1$$

### Question 2
The correct answer is B.

Simple trial and error.

### Question 3
The correct answer is B.

$$V + F = E - 2$$

### Question 4
The correct answer is D.

### Question 5
The correct answer is E.

cut capacity $= 5 + 8 + 4$

### Question 6
The correct answer is B.

### Question 7
The correct answer is E.

### Question 8
The correct answer is B.

### Question 9
The correct answer is D.

In order to have a Eularian path, there must be only two vertices of odd degree.

## Module 6: Matrices

### Question 1
The correct answer is C.

$5 + d = 7$

### Question 2
The correct answer is C.

### Question 3
The correct answer is D.

$$[6 \quad 8] \times \begin{bmatrix} 0.75 \\ 0.52 \end{bmatrix} = [6.5]$$

### Question 4
The correct answer is D.

$$\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ 24 \end{bmatrix} \therefore \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix} \times \begin{bmatrix} 9 \\ 24 \end{bmatrix}$$

### Question 5
The correct answer is E.

### Question 6
The correct answer is C.

$M(NP) = (2 \times 2) \therefore (2 \times 4) \times [(x \times y)(5 \times 2)] = (2 \times 2) \therefore x = 4, y = 5$

### Question 7
The correct answer is B.

### Question 8
The correct answer is B.

### Question 9
The correct answer is A.

$$SS = T^{50} \times S_1$$