

The Mathematical Association of Victoria

Trial Exam 2012

FURTHER MATHEMATICS

Written Examination 1

STUDENT NAME:

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

MULTIPLE-CHOICE QUESTION BOOK

Structure of Book

<i>Section</i>	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of modules</i>	<i>Number of modules to be answered</i>	<i>Number of marks</i>
A	13	13			13
B	54	27	6	3	27
					Total 40

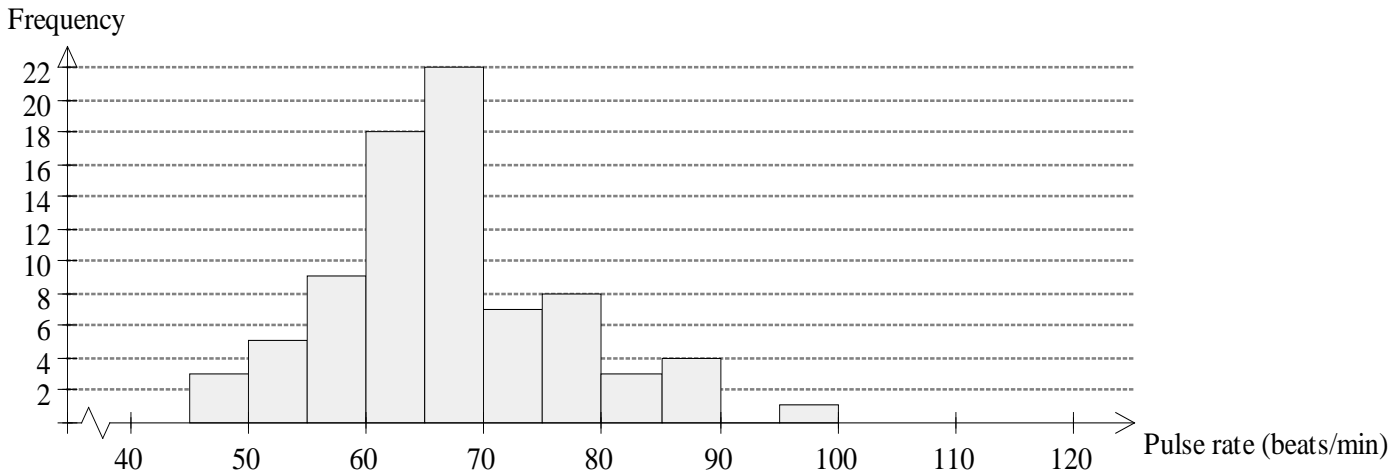
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
 - Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.
- Materials supplied**
- Question and answer book of 40 pages, with a detachable sheet of miscellaneous formulas at the back
 - Answer sheet for multiple-choice questions.
 - Working space is provided throughout the book.
- Instructions**
- Detach the formula sheet from the back of this book during reading time.
 - Unless otherwise indicated, the diagrams in this book are NOT drawn to scale.
- Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

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Core: Data analysis

The following information is relevant to Question 1 and Question 2

The graph below shows the resting pulse rate of 80 people.

**Question 1**

Those who have a pulse rate over 80 beats per minute at rest are regarded as having a health risk.

The percentage of people who have a health risk is

- A. 7
- B. 8
- C. 8.75
- D. 10
- E. 20

Question 2

The mean heart rate is closest to

- A. 52
- B. 57
- C. 62
- D. 67
- E. 72

SECTION A – continued

TURN OVER

Question 3

Parallel boxplots would be an appropriate way to display the relationship between

- A. attitude to becoming a republic (for/against) and gender (male/female)
- B. year level of school children and gender (male/female)
- C. test results (percentage) and class group (group A/group B)
- D. weights of 10 year old children and heights of these children
- E. level of education (primary/secondary/tertiary) reached and country of birth

Question 4

A study of test results for a large population of students revealed a bell-shaped distribution with a mean of 64 and a standard deviation of 12. Which one of the following statements is correct?

- A. No student could have obtained a result below 28
- B. More than 25% of students obtained a result below 52
- C. About 5% of students obtained a result over 88
- D. More than 80% of students obtained a result between 40 and 76
- E. About 99.7% of students obtained a result between 40 and 88

Question 5

The number of occupants in 23 vehicles travelling along a freeway was recorded. The results are summarised in the following table.

Number of occupants	Frequency
1	2
2	3
3	4
4	6
5	7
6	1

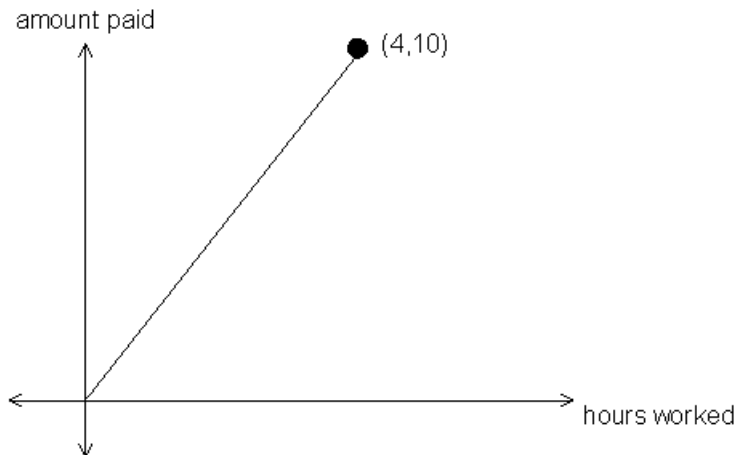
The mean median and mode are all calculated for the number of occupants. The order from lowest to highest for these three values is

- A. mean, median, mode
- B. mean, mode, median
- C. median, mean, mode
- D. median, mode, mean
- E. mode, mean, median

SECTION A – continued

Question 6

A linear regression line is drawn to model the relationship between hours worked and amount paid for a paperboy.

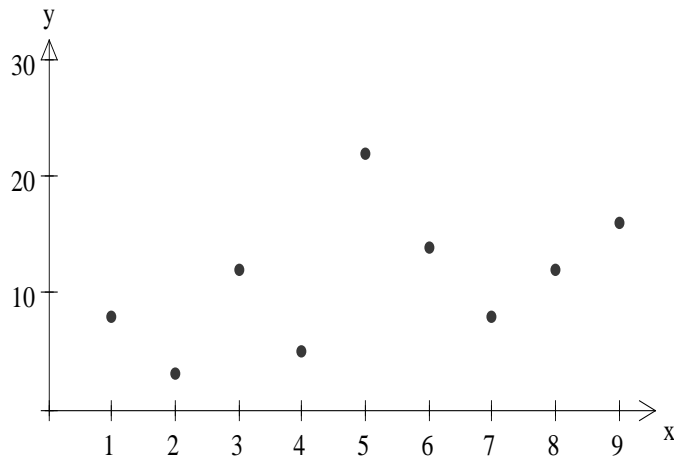


From the graph we can see that for every extra hour worked, the amount paid to the paperboy increases by

- A. \$0.40
- B. \$2.50
- C. \$4
- D. \$10
- E. \$14

SECTION A – continued

TURN OVER

Question 7

When the three median regression line is drawn on the graph above, it will

- A. pass through the left and right median points
- B. have a negative y - intercept
- C. pass through one third of the points on the scatterplot
- D. pass through the average of the three median points
- E. have a y intercept greater than 10

Question 8

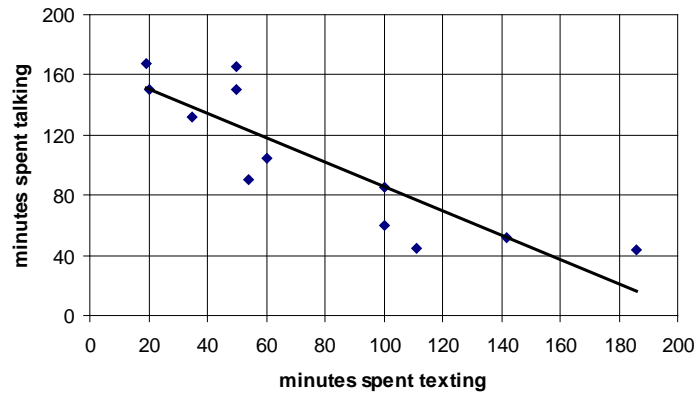
In a data set the lower quartile is 24.3 and the upper quartile is 35.7.
The statement which is not true is that:

- A. the interquartile range is 11.4
- B. 52.8 is an outlier
- C. 75% of the data lie above 24.3
- D. the median can be 30
- E. A value below 7.2 is an outlier

SECTION A – continued

Question 9

An investigation into the weekly mobile phone habits of a sample of year 10 students produced the following results graphed.



A least squares regression line has been fitted to the data and the equation is

$$\text{Minutes spent talking} = -0.8 \times \text{Minutes spent texting} + 166.4$$

The coefficient of determination has the value 0.757

Which one of the following statements is **not true**?

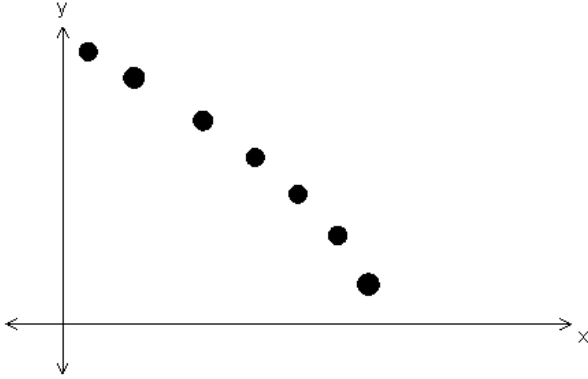
- A. Pearson's correlation coefficient is 0.87
- B. As the time spent texting increases, the time spent talking decreases.
- C. For each increase in 10 minutes of texting, the time spent talking decreases by 8 minutes.
- D. The residual value for the data point (100, 60) is -26.4
- E. 76% of the variation in talking time is explained by the variation in the time spent texting.

SECTION A – continued

TURN OVER

Question 10

A scatterplot is drawn to consider the possible relationship between two variables x and y . The relationship is clearly non-linear.



Two transformations which could each be used to linearise the graph are:

- A. An x^2 transformation and a y^2 transformation
- B. An x^2 transformation and a $\log y$ transformation
- C. A $\log x$ transformation and a y^2 transformation
- D. A $\log x$ transformation and a $\log y$ transformation
- E. A $\frac{1}{x}$ transformation and a $\frac{1}{y}$ transformation

Question 11

The number of family size pizzas sold each day over a week was recorded at Enzo's restaurant.

Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Sales	23	32	18	34	40	48	33

Four-point median smoothing with centring is to be used to smooth the data. The smoothed value for Thursday will be

- A. 32
- B. 33
- C. 34
- D. 35
- E. 36

SECTION A – continued

Question 12

The following table shows seasonal indices at Galland bakery for 2012

Summer	Autumn	Winter	Spring
	0.83		1.06

Possible values for the seasonal indices for summer and winter respectively are

- A. 0.83 and 1.06
- B. 1.06 and 0.83
- C. 0.83 and 1.28
- D. 1.06 and 1.28
- E. 1.28 and 1.06

Question 13

The least squares regression equation for the daily maximum temperature, T , and the income received from the daily ice-cream sales, S , during the month of March is given by the equation

$$S = 3.5T + 148 \text{ where the correlation coefficient is } r = 0.7685$$

The average maximum daily temperature for March was 21°C and the standard deviation was 4°C

The average daily ice-cream sales and the standard deviation for the ice-cream sales, respectively, were closest to

- A. \$221.50 and \$18.22
- B. \$221.50 and \$162
- C. \$221.50 and \$3.07
- D. \$162 and \$2.68
- E. \$148 and \$3.50

**END OF CORE
TURN OVER**

Module 1: Number patterns**Question 1**

The first term of a geometric sequence is -20 and the common ratio is -1.5
The difference between the 2nd and 3rd term is

- A. 112.5
- B. 75
- C. 15
- D. 22.5
- E. -1.5

Question 2

Vasili noticed that he had 85 worms in his worm farm 6 months after it was established and 184 worms 15 months after it was established. If the number of worms in Vasili's worm farm increased by the same amount each month, how many worms were in the farm when it was first established?

- A. 8
- B. 11
- C. 14
- D. 19
- E. 30

Question 3

The difference equation $t_{n+1} = 2t_n - 1$ where $t_1 = k$ generates a sequence with third term of 9 and fourth term of 17.

The value of k is

- A. 3
- B. -7
- C. 4
- D. 5
- E. -2

Question 4

A sequence is generated from the difference equation $T_{n+1} = aT_n + b$ where $T_1 = 1$
Which one of the following is true regarding this sequence?

- A. It is arithmetic if $a = 3$ and $b = 0$
- B. It is arithmetic if $a = 3$ and $b = 1$
- C. It is geometric if $a = 3$ and $b = 0$
- D. It is geometric if $a = 3$ and $b = 1$
- E. It is neither arithmetic nor geometric if $a = 1$ and $b = 3$

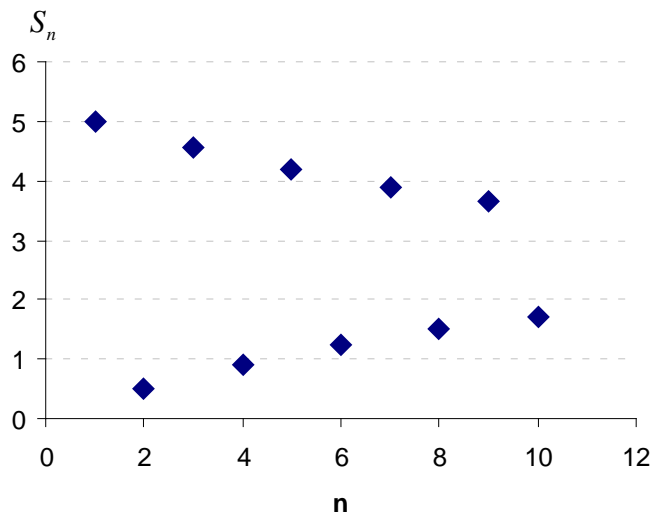
Question 5

The sum of n terms for $n = 1, 2, 3, 4, 5$ in an arithmetic sequence gives 3, 11, 24, 42, 65 respectively.
The common difference for the original arithmetic sequence is

- A. 3
- B. 4
- C. 5
- D. 6
- E. 8

SECTION B - Module 1: Number Patterns – continued

TURN OVER

Question 6

The **sum** of a sequence is shown in the above graph. From the graph we can conclude that the sequence is

- A. Geometric with $a > 0$ and $-1 < r < 0$
- B. Geometric with $a > 0$ and $0 < r < 1$
- C. Geometric with $a > 0$ and $r > 1$
- D. Geometric with $a > 0$ and $r < -1$
- E. Arithmetic with $a > 0$ and $d < 0$

Question 7

Which one of the following sequences reaches a value over 100 first?

- A. 2, 5, 8, ...
- B. -20, -13, -6, ...
- C. 0.2, 0.3, 0.45, ...
- D. 99, 99.05, 99.1, ...
- E. 0.5, -0.6, 0.72, ...

SECTION B - Module 1: Number Patterns – continued

Question 8

For a Fibonacci sequence with $f_8 = a$ and $f_6 = b$, which one of the following statements is **not** true

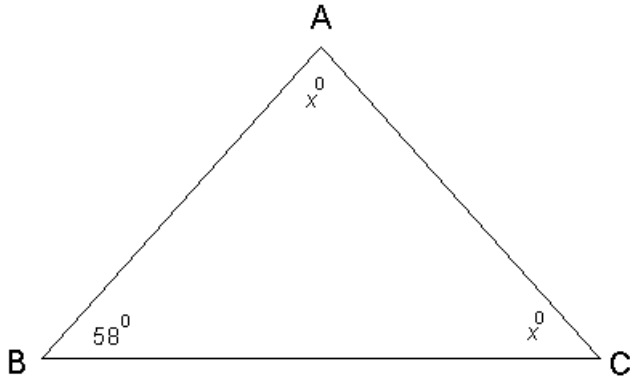
- A. $f_7 = a - b$
- B. $f_9 = 2a - b$
- C. $f_5 = a - 2b$
- D. $f_6 + f_7 = a$
- E. $f_4 + f_5 = 2b$

Question 9

Sally is having a party and is blowing up a balloon with helium. The balloon is initially 15 cm in diameter. The balloon expands 6 cm in diameter in the first second. Each second after that, the balloon's diameter expands by 80% of the previous increase in diameter. If this continues, the balloon will reach a maximum diameter of

- A. 22.5 cm
- B. 30 cm
- C. 45 cm
- D. 49 cm
- E. 75 cm

**END OF MODULE 1
TURN OVER**

Module 2: Geometry and trigonometry**Question 1**

The value of x in the triangle above is

- A. 58
- B. 60
- C. 61
- D. 62
- E. 122

Question 2

The number of sides of a regular polygon with an interior angle of 140° is

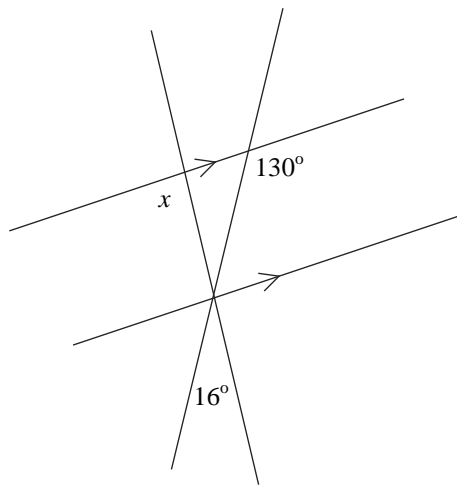
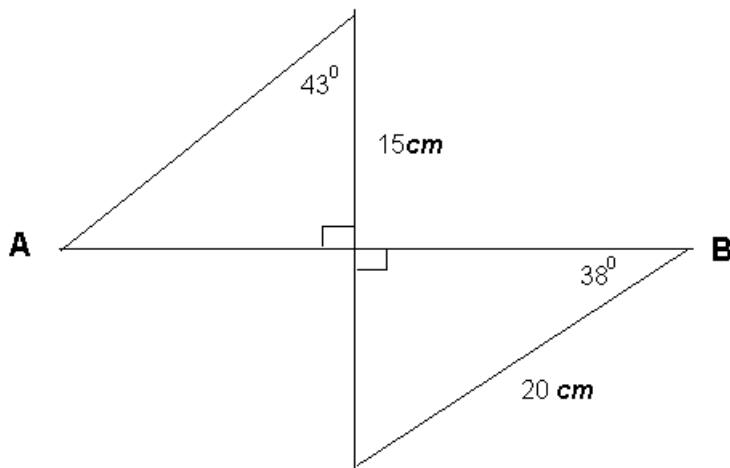
- A. 7
- B. 8
- C. 9
- D. 10
- E. 11

SECTION B - Module 2: Geometry and trigonometry – continued

Question 3

The magnitude of angle x is

- A. 146
- B. 130
- C. 114
- D. 66
- E. 50

**Question 4**

The length of the line AB in the diagram above is given by

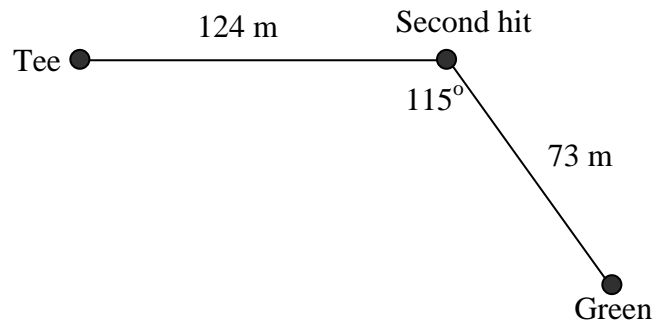
- A. $15 \tan 43^\circ + 20 \cos 38^\circ$
- B. $\frac{15}{\tan 43^\circ} + \frac{20}{\cos 38^\circ}$
- C. $15 \sin 43^\circ + 20 \cos 38^\circ$
- D. $\frac{15}{\sin 43^\circ} + \frac{20}{\cos 38^\circ}$
- E. $15 \cos 43^\circ + 20 \tan 38^\circ$

SECTION B - Module 2: Geometry and trigonometry – continued

TURN OVER

Question 5

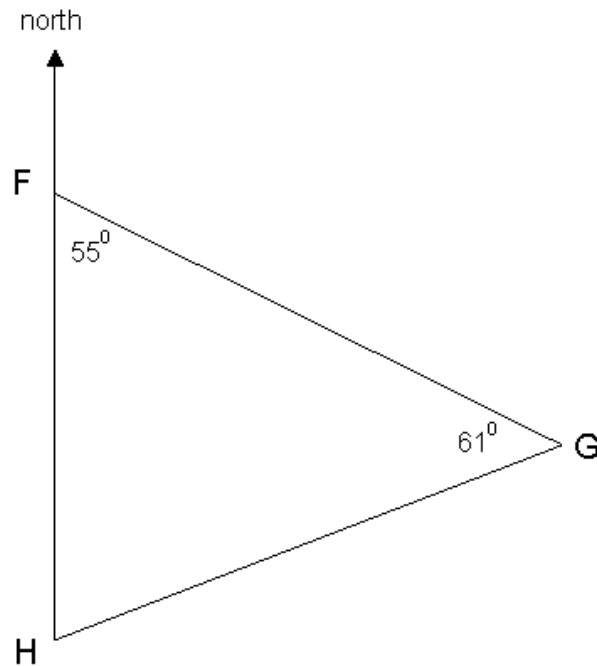
A golf ball is hit from the Tee 124 metres and then hit a second time a further 73 metres to the green as shown on the diagram.



The direct distance from the Tee to the Green is calculated using

- A. $124^2 + 73^2 - 2 \times 124 \times 73 \times \cos(115^\circ)$
- B. $\sqrt{124^2 + 73^2 - 2 \times 124 \times 73 \times \cos(115^\circ)}$
- C. $\frac{124 \sin 115^\circ}{73}$
- D. $\sqrt{124^2 + 73^2}$
- E. $\sqrt{124^2 + 73^2 - 124 \times 73 \times \cos(115^\circ)}$

SECTION B - Module 2: Geometry and trigonometry – continued

Question 6

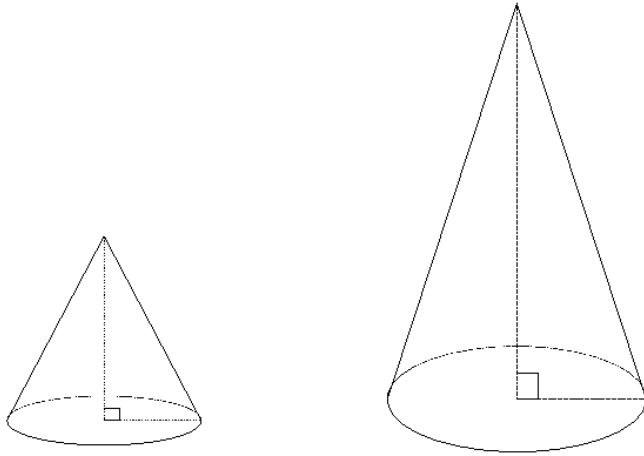
The diagram above shows the relative positions of three points F, G and H.

Which one of the following three figure bearings is of the largest magnitude?

- A. F from G
- B. G from F
- C. G from H
- D. H from G
- E. H from F

SECTION B - Module 2: Geometry and trigonometry – continued

TURN OVER

Question 7

The ratio of the volume of the smaller cone to that of the larger cone is 27:125

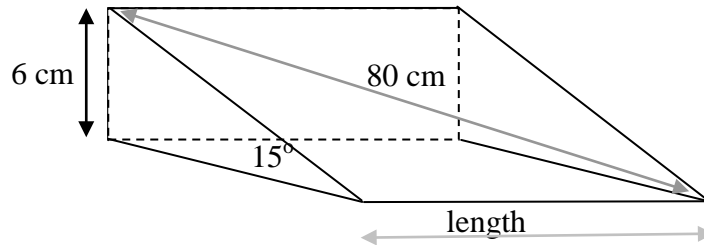
If the surface area of the smaller cone is 90cm^2 , then the surface area of the larger cone in cm^2 is

- A. 125
- B. 150
- C. 250
- D. $416\frac{2}{3}$
- E. $583\frac{1}{3}$

SECTION B - Module 2: Geometry and trigonometry – continued

Question 8

A rectangular desktop has a diagonal length of 80 cm. The desk top is designed to make an angle of 15° with the horizontal and is raised 6 cm at one end.



The length, in centimetres correct to one decimal place, of the rectangular desktop is

- A. 74.2
- B. 76.6
- C. 76.8
- D. 79.5
- E. 80.0

Question 9

A passenger of a car travelling along a straight road notices a beacon at the top of a hill at an angle of elevation of 28° . After travelling 200m the passenger notices that the angle of elevation is now 35° .

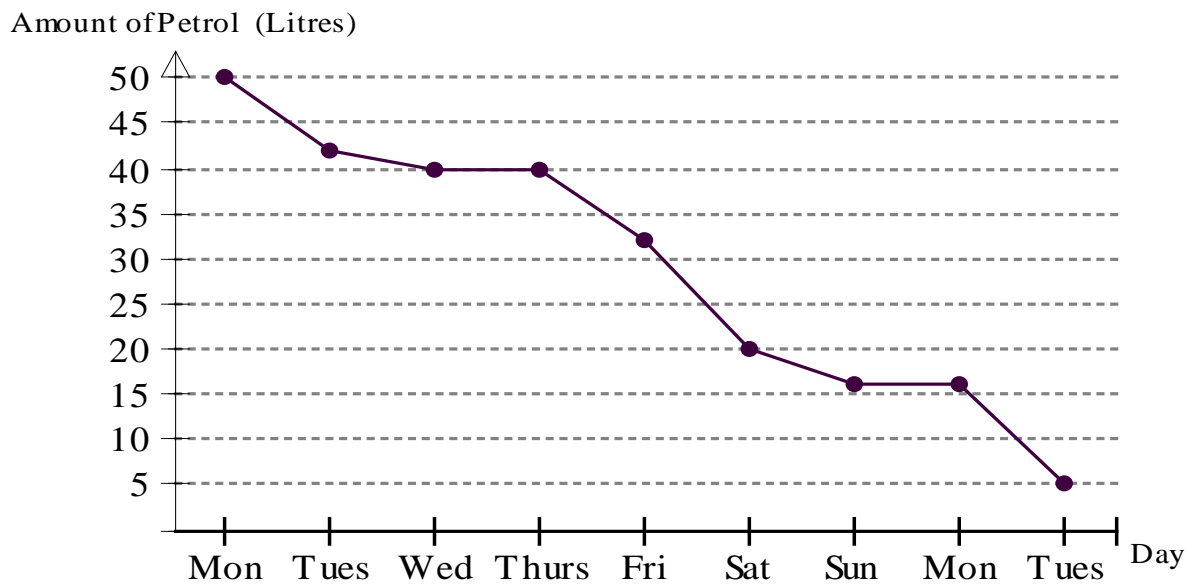
The height, in metres, of the beacon is closest to

- A. 106
- B. 442
- C. 631
- D. 770
- E. 941

**END OF MODULE 2
TURN OVER**

Module 3: Graphs and relations**Question 1**

The number of litres of petrol in the tank of a car is shown on the graph below.



The average rate at which petrol is used, in litres per day, during the time period shown is

- A. 5.0
- B. 5.4
- C. 5.6
- D. 6.3
- E. 6.8

SECTION B - Module 3: Graphs and relations– continued

Question 2

A straight line has the equation $3x - 4y - 2 = 0$. Which one of the following statements is true?

- A. The line has a negative slope
- B. The line has a y intercept of $\frac{1}{2}$
- C. The line is parallel to $y = -\frac{3}{4}x + 3$
- D. The line passes through the point $(3,2)$
- E. The line has the same x intercept as $6x - y - 4 = 0$

Question 3

Three siblings purchased the same type of muesli bars and fruit straps for their weekly recess snack.

Katherine purchased eight muesli bars and six fruit straps for \$14.12.

Michael purchased twelve muesli bars and five fruits straps for \$17.90

Zoe purchases three muesli bars and two fruit straps. The change she receives from a \$10 note will be

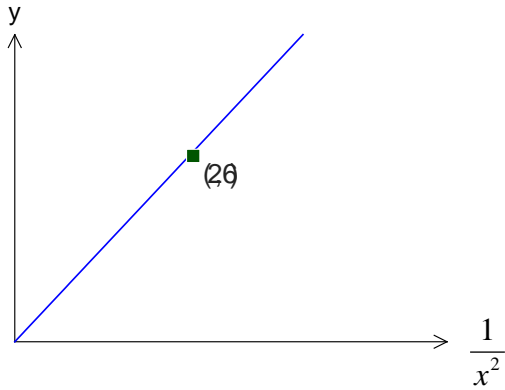
- A. \$4.75
- B. \$4.90
- C. \$5.00
- D. \$5.10
- E. \$5.25

SECTION B - Module 3: Graphs and relations– continued

TURN OVER

Question 4

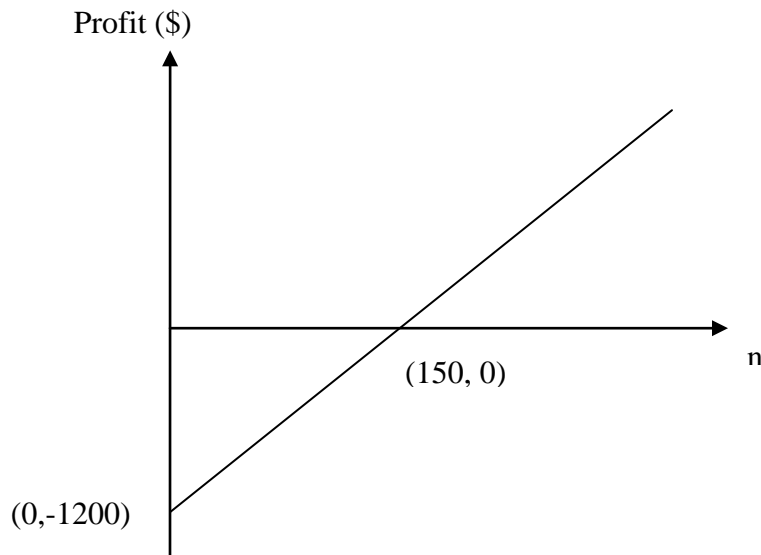
The straight line graph shown is obtained by plotting y against $\frac{1}{x^2}$. It passes through the point with coordinates $(2, 6)$. The rule giving y in terms of x is



- A. $y = \frac{3}{x^2}$
- B. $y = \frac{3}{2x^2}$
- C. $y = \frac{4}{x^2}$
- D. $y = \frac{1}{3x^2}$
- E. $y = \frac{24}{x^2}$

The following information is relevant to Questions 5 and 6

The profit made from selling n T-Shirts is shown on the graph below.



Each T-Shirt was sold for \$35

Question 5

To break even, the number of T-shirts that need to be sold is

- A. 1200
- B. 150
- C. 115
- D. 35
- E. 8

Question 6

The Cost for n T-shirts is given by the equation

- A. Cost = $8n - 1200$
- B. Cost = $27n - 1200$
- C. Cost = $1200 - 43n$
- D. Cost = $1200 + 27n$
- E. Cost = $1200 - 8n$

SECTION B - Module 3: Graphs and relations– continued

TURN OVER

Question 7

The daily cost of public transport is determined from the first time to the last time a travel card is used.

The charges are shown below

$$\text{Adult Charge} = \begin{cases} \$5.00 & 0 < \text{hours} \leq 3 \\ \$8.00 & 3 < \text{hours} \leq 7 \\ \$16.00 & \text{hours} > 7 \end{cases}$$

$$\text{Children under ten} = \$0.50 \times \text{number of hours}$$

Sarah and John travel on public transport with their two children aged 10 and 7 to the city at 9.00 am.

John returns home on public transport with the children at 11.30 am while Sarah stays in the city to go shopping before returning home at 3.30 pm.

The total charge for transport was

- A. \$15.50
- B. \$18.50
- C. \$19.25
- D. \$27.25
- E. \$33.75

Question 8

Taxi company A charges \$4 per kilometre and \$3 for the initial charge.

Taxi company B charges \$5 per kilometre for the first 6 kilometres and then \$3 per kilometre after that.

The two rules which determine the charge, C_A and C_B , from the distance travelled, x , for Company A and Company B respectively is given by

A. $C_A = 4 + 3x$ and $C_B = \begin{cases} 5x, & 0 \leq x \leq 6 \\ 3x, & x > 6 \end{cases}$

B. $C_A = 4x + 3$ and $C_B = \begin{cases} 5x, & 0 \leq x \leq 6 \\ 3x, & x > 6 \end{cases}$

C. $C_A = 4x + 3$ and $C_B = \begin{cases} 5x, & 0 \leq x \leq 6 \\ 3x + 30, & x > 6 \end{cases}$

D. $C_A = 4x + 3$ and $C_B = \begin{cases} 5x, & 0 \leq x \leq 6 \\ 3x + 12, & x > 6 \end{cases}$

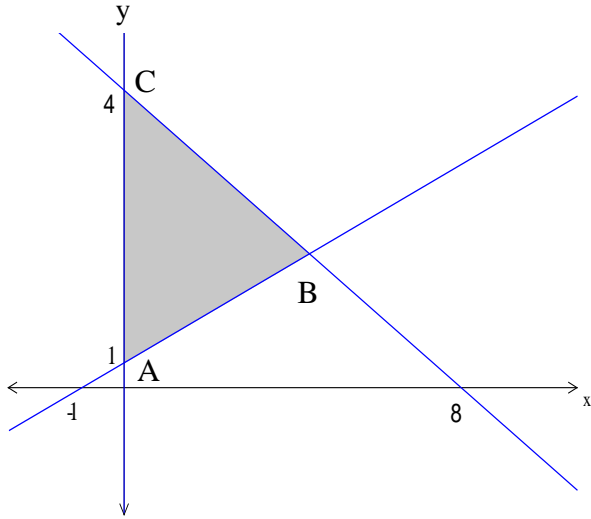
E. $C_A = 4 + 3x$ and $C_B = \begin{cases} 5x, & 0 \leq x \leq 6 \\ 3x + 12, & x > 6 \end{cases}$

SECTION B - Module 3: Graphs and relations– continued

TURN OVER

Question 9

In the feasible region shown, the maximum value occurs only at the point B where the objective function is given by $M = kx + y$.



The value of k is

- A. $k < 1$
- B. $k > \frac{1}{2}$
- C. $k < \frac{1}{2}$
- D. $k = 1$
- E. $k = \frac{1}{2}$

END OF MODULE 3

Module 4: Business-related mathematics**Question 1**

Part of Karen's bank statement is shown below.

Date	Transaction details	Withdrawals	Deposits	Balance
28 Aug		150.00		326.36
07 Sep	Deposit – cash		580.00	906.36
15 Sep	Withdrawal – ATM	200.00		706.36
23 Sep	Withdrawal – funds transfer	171.60		534.76
30 Sep	Closing balance			534.76

Karen receives interest on this account calculated at the rate of 3.4% per annum on the minimum monthly balance. In September, the interest Karen received was

- A. \$2.57
- B. \$2.00
- C. \$1.52
- D. \$0.92
- E. \$0.50

Question 2

Sam invested \$6 000 in an account that earned simple interest of 7.5% per annum. The term of the investment is chosen so that the total amount in the account is at least \$8 250. The minimum term of the investment is

- A. 5 months
- B. 18 months
- C. 51 months
- D. 52 months
- E. 60 months

SECTION B - Module 4: Business related mathematics– continued

TURN OVER

Question 3

Vickie and Rob both invest \$1000. Vickie earns 10% per annum interest compounding annually and Rob earns 10% per annum simple interest. After three years the interest earned by Vickie will have exceeded the interest earned by Rob by

- A. \$30
- B. \$31
- C. \$33
- D. \$300
- E. \$310

Question 4

The cost of a mathematics tuition session is \$50 to which a 10% charge for GST is added. Andrew used this service on a number of occasions this year. The total cost to Andrew for the year could have been

- A. \$500
- B. \$1000
- C. \$1100
- D. \$1200
- E. \$1500

Question 5

A photocopier when new was valued at \$9,400. The value of the photocopier depreciates at a rate of 12 cents per 100 pages printed. After it has copied a total of 5 million pages, the value of the photocopier is

- A. \$8 800
- B. \$8 200
- C. \$6 000
- D. \$3 400
- E. \$1 200

SECTION B - Module 4: Business related mathematics– continued

Question 6

Douglas received 8% discount on a shirt marked at \$150 and 6% discount on a jacket marked at \$250. The percentage discount received by Douglas for the two items combined is

- A. 6.25%
- B. 6.75%
- C. 7%
- D. 7.25%
- E. 14%

Question 7

Giulia invested \$2000 for two years at an interest rate of 7.2% per annum compounding monthly. The value of Giulia's investment after the two years is given by

- A. 2000×1.006^2
- B. 2000×1.006^{24}
- C. 2000×1.072^2
- D. 2000×1.072^{24}
- E. 2000×1.72^2

Question 8

A digital television was purchased for \$3299 on June 1 2011

On June 1 of each year after that its value depreciates by 25% using the reducing balance method.

The value of the television will be below \$500 for the first time on June 1 in the year

- A. 2017
- B. 2018
- C. 2019
- D. 2020
- E. 2021

SECTION B - Module 4: Business related mathematics– continued

TURN OVER

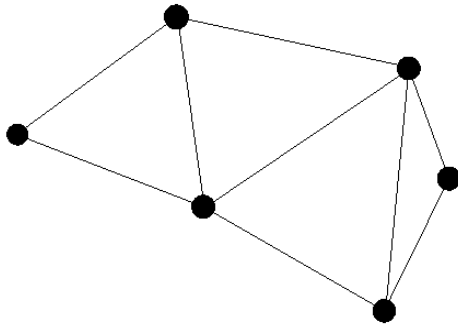
Question 9

Chris needs to raise at least \$1000 for his world challenge trip at the end of the year. At the **beginning** of April Chris invests \$800 at 6% per annum compounding monthly. After interest is paid at the **end** of each month, Chris adds another \$60 to his investment.

The first time the investment will exceed \$1000 is at the end of

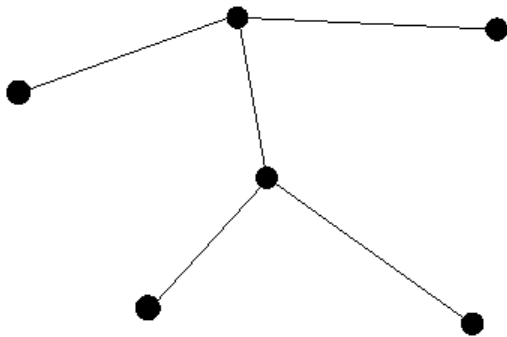
- A. May
- B. June
- C. July
- D. August
- E. September

END OF MODULE 4

Module 5: Networks and decision mathematics**Question 1**

Which one of the following statements concerning the degrees of the vertices in the network above is true?

- A. All vertices are of even degree
- B. All vertices are of odd degree
- C. Four vertices are of odd degree and two are of even degree
- D. Four vertices are of even degree and two are of odd degree
- E. Three vertices are of even degree and three are of odd degree

Question 2

The minimum number of edges that would need to be added to the graph above so that it would contain an Euler circuit is

- A. 0
- B. 1
- C. 2
- D. 3
- E. 4

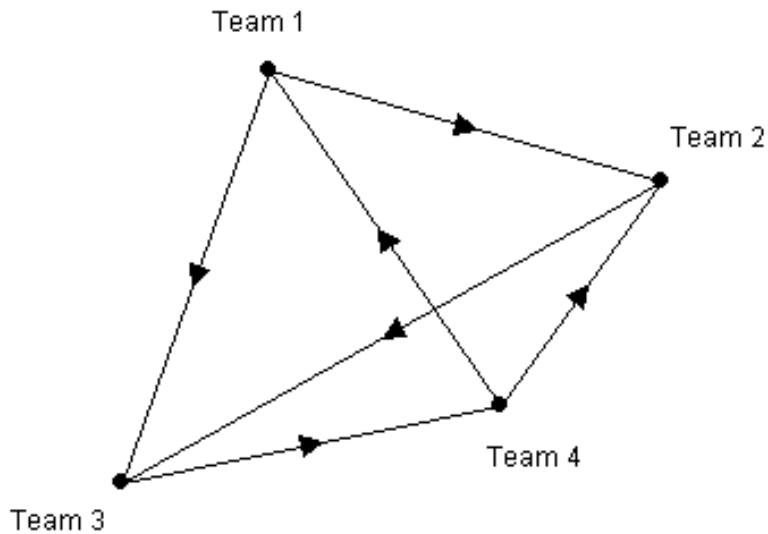
SECTION B - Module 5: Networks and decision mathematics– continued

TURN OVER

Question 3

Four debating teams, 1, 2, 3 and 4 competed once against each other.

The results are displayed in the following directed graph where an arrow from Team 1 to Team 2 shows that Team 1 defeated Team 2.



The two teams with the most victories are to meet in the final.

This final will be between

- A. Team 1 and Team 3
- B. Team 1 and Team 4
- C. Team 2 and Team 3
- D. Team 2 and Team 4
- E. Team 3 and Team 4

Question 4

The five positions on a basketball court are centre, power forward, small forward, shooting guard and point guard.

Archie, Bob, Callum, Dave and Eddie are a group of five basketballers.

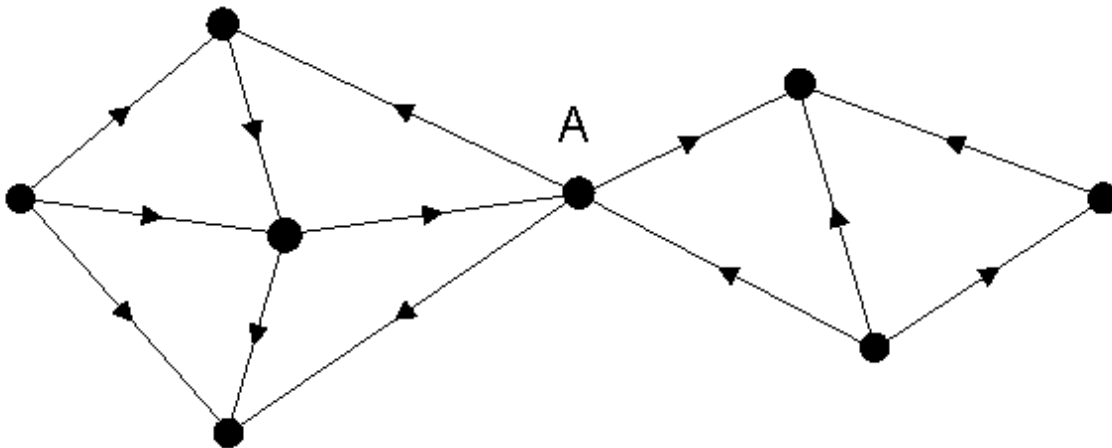
Each of these players is capable of playing in more than one position.

A suitable way of illustrating which of the positions can be allocated to each player is with a

- A. a simple graph
- B. a bipartite graph
- C. a planar graph
- D. a connected graph
- E. a complete graph

Question 5

The following directed graph contains 8 vertices, one of which is labelled Vertex A.



The number of other vertices that cannot be reached from Vertex A is

- A. 1
- B. 2
- C. 3
- D. 4
- E. 5

Question 6

The number of edges, faces and vertices for five connected graphs, A, B, C, D and E are shown in the following table.

Graph	Edges	faces	vertices
A	5	4	3
B	7	5	4
C	10	10	5
D	11	7	6
E	13	8	7

The only graph which is **not** planar is

- A. Graph A
- B. Graph B
- C. Graph C
- D. Graph D
- E. Graph E

Question 7

A network graph with five vertices is connected with the minimum number of edges.
The minimum number of **extra** edges needed to make this a complete graph is

- A. 5
- B. 6
- C. 10
- D. 14
- E. 16

Question 8

Four water polo teams, Antelopes, Beavers, Cougars and Dingoes play each once in round robin competition. The results are summarised as follows:

Antelopes defeated Beavers

Beavers defeated Cougars and Dingoes

Cougars defeated Antelopes

Dingoes defeated Antelopes and Cougars

The number of **two-step** dominances for each of the four teams is

A.

Team	2 step dominances
Antelopes	1
Beavers	2
Cougars	1
Dingoes	2

B.

Team	2 step dominances
Antelopes	2
Beavers	3
Cougars	1
Dingoes	2

C.

Team	2 step dominances
Antelopes	3
Beavers	5
Cougars	2
Dingoes	4

D.

Team	2 step dominances
Antelopes	2
Beavers	3
Cougars	2
Dingoes	3

E.

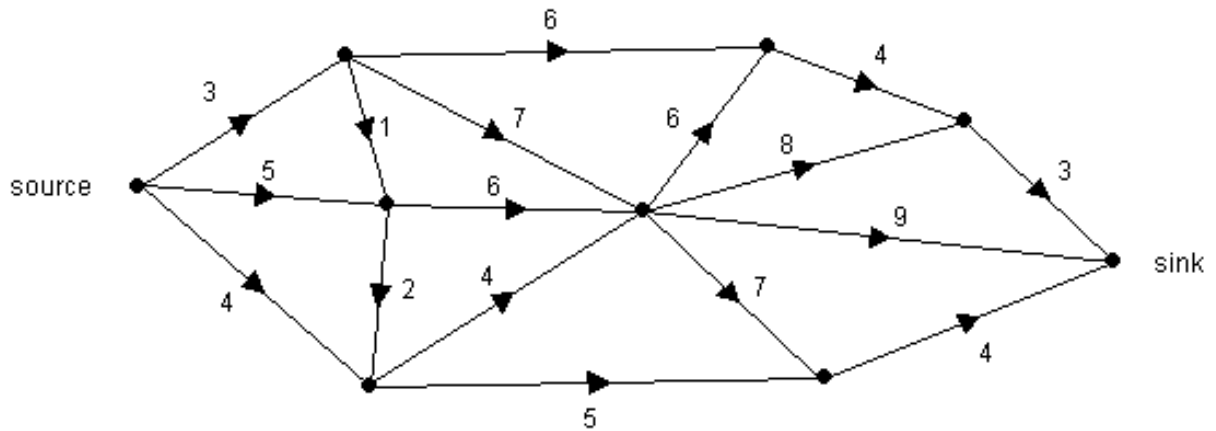
Team	2 step dominances
Antelopes	2
Beavers	2
Cougars	1
Dingoes	3

SECTION B - Module 5: Networks and decision mathematics– continued

TURN OVER

Question 9

Consider the directed graph below.



The maximum number of edges whose weight could contribute to the capacity of **any** cut through this network is

- A. 3
- B. 4
- C. 5
- D. 6
- E. 7

END OF MODULE 5

Module 6: Matrices**Question 1**

$$\begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix}$$

The determinant of the matrix above is equal to

- A. -2
- B. 3
- C. 5
- D. -5
- E. 6

Question 2

C and D are 2×2 matrices.

If $CX = D$ then

- A. $X = \frac{D}{C}$
- B. $X = \frac{C}{D}$
- C. $X = DC^{-1}$
- D. $X = C^{-1}D$
- E. $X = (CD)^{-1}$

SECTION B - Module 6 : Matrices– continued

TURN OVER

Question 3

If matrix A has order 3×4 and $AB = \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}$ then Matrix B has the order

- A. 3×4
- B. 4×3
- C. 4×1
- D. 1×4
- E. 1×3

Question 4

The inverse of the matrix $\begin{bmatrix} 2 & 6 \\ 1 & 4 \end{bmatrix}$ is determined.

An element that does not appear in this inverse is

- A. -3
- B. 3
- C. 1
- D. 2
- E. $-\frac{1}{2}$

Question 5

Consider the 2×2 matrix $A = \begin{bmatrix} x & 4 \\ 6 & y \end{bmatrix}$

For which of the following values of x and y will A^{-1} exist?

- A. $x = 6, y = 4$
- B. $x = 4, y = 6$
- C. $x = 3, y = 8$
- D. $x = 2, y = 12$
- E. $x = 5, y = 5$

SECTION B - Module 6 : Matrices– continued

Question 6

A frictionless system of pulleys is loaded with 87 kg and 29 kg weights.

The tensions t_1 and t_2 in the ropes and the acceleration, a , of the 29 kg weight are found by solving the following system of equations.

$$t_1 - 2t_2 = 0$$

$$t_1 - 3a = 87$$

$$t_2 + 2a = 29$$

The matrix that can be set up to find the tensions and acceleration is

A.
$$\begin{bmatrix} 1 & -2 \\ 1 & -3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \\ a \end{bmatrix} = \begin{bmatrix} 0 \\ 87 \\ 29 \end{bmatrix}$$

B.
$$\begin{bmatrix} 1 & -2 & 0 \\ 1 & -3 & 0 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \\ a \end{bmatrix} = \begin{bmatrix} 0 \\ 87 \\ 29 \end{bmatrix}$$

C.
$$\begin{bmatrix} t_1 \\ t_2 \\ a \end{bmatrix} = \begin{bmatrix} 1 & -2 & 0 \\ 1 & 0 & -3 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 87 \\ 29 \end{bmatrix}$$

D.
$$\begin{bmatrix} 1 & -2 & 0 \\ 1 & -3 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \\ a \end{bmatrix} = \begin{bmatrix} 0 \\ 87 \\ 29 \end{bmatrix}$$

E.
$$\begin{bmatrix} 1 & -2 & 0 \\ 1 & 0 & -3 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \\ a \end{bmatrix} = \begin{bmatrix} 0 \\ 87 \\ 29 \end{bmatrix}$$

SECTION B - Module 6: Matrices– continued

TURN OVER

The following information is relevant to Questions 7 and 8

The town of Beanville has a large population of coffee drinkers who all buy a coffee every morning. Two coffee bars, Mario's and Sienna's operate with Mario's initially attracting 90% of customers.

It is known that 60% of those who buy a coffee from Mario's on a given day will return to Mario's the following day and 20% of those who buy from Sienna's on a given day switch to Mario's the following day.

Question 7

The daily percentage of customers at Mario's in the long term will be closest to

- A. 33%
- B. 40%
- C. 67%
- D. 90%
- E. 100%

Question 8

If Sienna's initially had 90% of customers rather than Mario's then the daily percentage of customers at Mario's in the long term will be

- A. unaltered
- B. equal to that of Mario's in the original problem
- C. slightly higher than in the original problem
- D. slightly lower than in the original problem
- E. considerably higher than in the original problem

SECTION B - Module 6 : Matrices– continued

Question 9

When an election was first announced six hundred people were surveyed and it was found that 300 people intended to vote for Labour, 200 for Liberal and 100 for another party.

By election day these numbers are expected to change according to the following transition matrix

$$\begin{bmatrix} 0.6 & 0.1 & 0.1 \\ 0.2 & 0.8 & 0.3 \\ 0.2 & 0.1 & 0.6 \end{bmatrix}$$

The total number of votes from this group that are expected to change by election day is

- A. 120
- B. 150
- C. 180
- D. 200
- E. 240

END OF MULTIPLE - CHOICE QUESTION BOOKLET

FURTHER MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Further Mathematics Formulas

Core: Data analysis

standardised score:
$$z = \frac{x - \bar{x}}{s_x}$$

least squares line:
$$y = a + bx \quad \text{where } b = r \frac{s_y}{s_x} \quad \text{and} \quad a = \bar{y} - b\bar{x}$$

residual value:
$$\text{residual value} = \text{actual value} - \text{predicted value}$$

seasonal index:
$$\text{seasonal index} = \frac{\text{actual figure}}{\text{deseasonalised figure}}$$

Module 1: Number patterns

arithmetic series:
$$a + (a + d) + \dots + (a + (n - 1)d) = \frac{n}{2} [2a + (n - 1)d] = \frac{n}{2} (a + l)$$

geometric series:
$$a + ar + ar^2 + \dots + ar^{n-1} = \frac{a(1 - r^n)}{1 - r}, \quad r \neq 1$$

infinite geometric series:
$$a + ar + ar^2 + ar^3 + \dots = \frac{a}{1 - r}, \quad |r| < 1$$

Module 2: Geometry and trigonometry

area of a triangle:
$$\frac{1}{2} bc \sin A$$

Heron's formula:
$$A = \sqrt{s(s - a)(s - b)(s - c)} \quad \text{where } s = \frac{1}{2}(a + b + c)$$

circumference of a circle:
$$2\pi r$$

area of a circle:
$$\pi r^2$$

volume of a sphere:
$$\frac{4}{3} \pi r^3$$

surface area of a sphere:
$$4\pi r^2$$

volume of a cone:
$$\frac{1}{3} \pi r^2 h$$

volume of a cylinder:
$$\pi r^2 h$$

volume of a prism:
$$\text{area of base} \times \text{height}$$

volume of a pyramid:
$$\frac{1}{3} \text{area of base} \times \text{height}$$

Pythagoras' theorem: $c^2 = a^2 + b^2$

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos C$

Module 3: Graphs and relations

Straight line graphs

gradient (slope): $m = \frac{y_2 - y_1}{x_2 - x_1}$

equation: $y = mx + c$

Module 4: Business-related mathematics

simple interest: $I = \frac{PrT}{100}$

compound interest: $A = PR^n$ where $R = 1 + \frac{r}{100}$

hire purchase: effective rate of interest $\approx \frac{2n}{n+1} \times \text{flat rate}$

Module 5: Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Module 6: Matrices

determinant of a 2×2 matrix: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

inverse of a 2×2 matrix: $A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$

END OF FORMULA SHEET

MULTIPLE CHOICE ANSWER SHEET

STUDENT NAME:

Circle the letter that corresponds to each correct answer.

SECTION A		SECTION B							
Compulsory		Answer THREE different modules.							
		Show EACH MODULE answered by shading the appropriate box and WRITING in the box below							
		<input type="checkbox"/>	Number patterns		<input type="checkbox"/>	Number patterns		<input type="checkbox"/>	Number patterns
		<input type="checkbox"/>	Geometry and trigonometry		<input type="checkbox"/>	Geometry and trigonometry		<input type="checkbox"/>	Geometry and trigonometry
		<input type="checkbox"/>	Graphs and relations		<input type="checkbox"/>	Graphs and relations		<input type="checkbox"/>	Graphs and relations
		<input type="checkbox"/>	Business-related maths		<input type="checkbox"/>	Business-related maths		<input type="checkbox"/>	Business-related maths
	Core: Data Analysis	<input type="checkbox"/>	Networks and decision maths		<input type="checkbox"/>	Networks and decision maths		<input type="checkbox"/>	Networks and decision maths
1	A B C D E	<input type="checkbox"/>	Matrices		<input type="checkbox"/>	Matrices		<input type="checkbox"/>	Matrices
2	A B C D E								
3	A B C D E		MODULE		MODULE		MODULE		MODULE
4	A B C D E								
5	A B C D E	1	A B C D E	1	A B C D E	1	A B C D E	1	A B C D E
6	A B C D E	2	A B C D E	2	A B C D E	2	A B C D E	2	A B C D E
7	A B C D E	3	A B C D E	3	A B C D E	3	A B C D E	3	A B C D E
8	A B C D E	4	A B C D E	4	A B C D E	4	A B C D E	4	A B C D E
9	A B C D E	5	A B C D E	5	A B C D E	5	A B C D E	5	A B C D E
10	A B C D E	6	A B C D E	6	A B C D E	6	A B C D E	6	A B C D E
11	A B C D E	7	A B C D E	7	A B C D E	7	A B C D E	7	A B C D E
12	A B C D E	8	A B C D E	8	A B C D E	8	A B C D E	8	A B C D E
13	A B C D E	9	A B C D E	9	A B C D E	9	A B C D E	9	A B C D E