Neap:

Trial Examination 2014

VCE Further Mathematics Units 3 & 4

Written Examination 2

Suggested Solutions

Core

Question 1

a. Difference = 17 600 – 7900 = \$9700 A1

Within \$500 is acceptable

b. Current Philippines GDP = \$3200 per capita

Current Indonesia GDP = \$6300 per capita

Increase required = \$3100 per capita

$\% \text{ required} = \frac{3100}{3200} \times 100 = 97\%$ A1

Within 10% is acceptable if students use correct method

Question 2

a. The total frequency is 40.
Thus the middle numbers will be the 20th and 21st.
There are 10 numbers of 4.5 or less.
There are a further 11 that are 5.0, making a total of 21 of 5.0 or less.
Thus the 20th and 21st values are both 5.0. The median is 5.0. A1

b. In order to determine the quartiles, we divide the numbers into high and low halves. Then we find the middle of each.

For the lower range, there are 20 numbers with the 10th and 11th being values used to find the lower quartile. These are the numbers 4.5 and 5.0. Thus $Q_1 = \frac{5.0 + 4.5}{2} = 4.75$

For the upper range, there are also 20 numbers with the 30th and 31st in the set being used for the upper quartile. Both these values are 6. Thus $Q_3 = \frac{6 + 6}{2} = 6$ M1

Five number summary is 4.0, 4.75, 5.0, 6.0, 7.0.

IQR = 6.0 – 4.75 = 1.25 A1

c. Outliers are those values that are outside of the fences. These fences are determined by moving 1.5 interquartile ranges beyond each of the upper and lower quartiles.

$\text{lower} = Q_1 - 1.5 \times \text{IQR} = 4.75 - 1.5 \times 1.25 = 2.875$

$\text{upper} = Q_3 + 1.5 \times \text{IQR} = 6.0 + 1.5 \times 1.25 = 7.875$

All of the values are between these two boundaries. Thus there are no outliers. A1

Question 3

a. This can be easily answered using a graphing calculator. Those with TI-series calculators can use the *LinReg* function with *Diagnostics* on.
The result is 0.8449 A1

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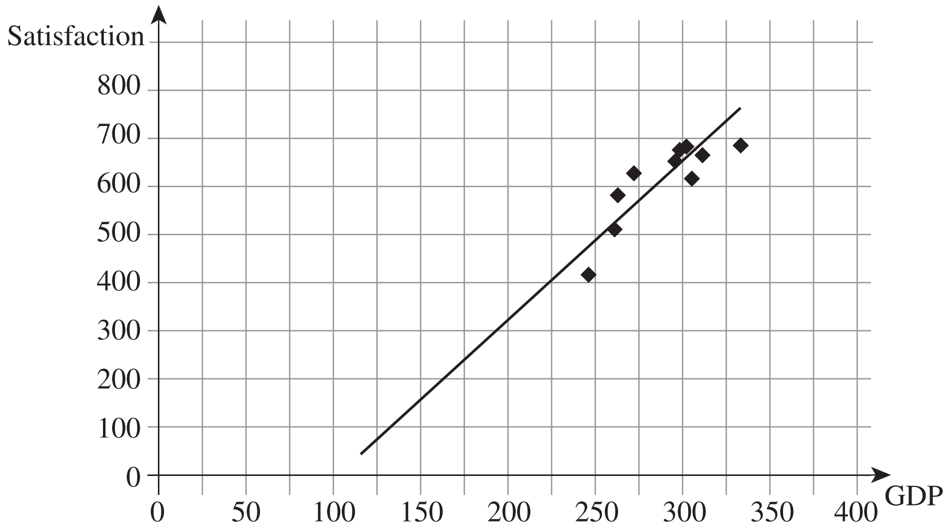
b.



A2

c. It is necessary to square the values of satisfaction before we perform *LinReg* again. This is easily done on the TI-series calculators.

$L_2^{\;2} \rightarrow L_3$

Then the regression is repeated using list 3 instead of list 2, *LinReg L_1,L_3* M1

The result is

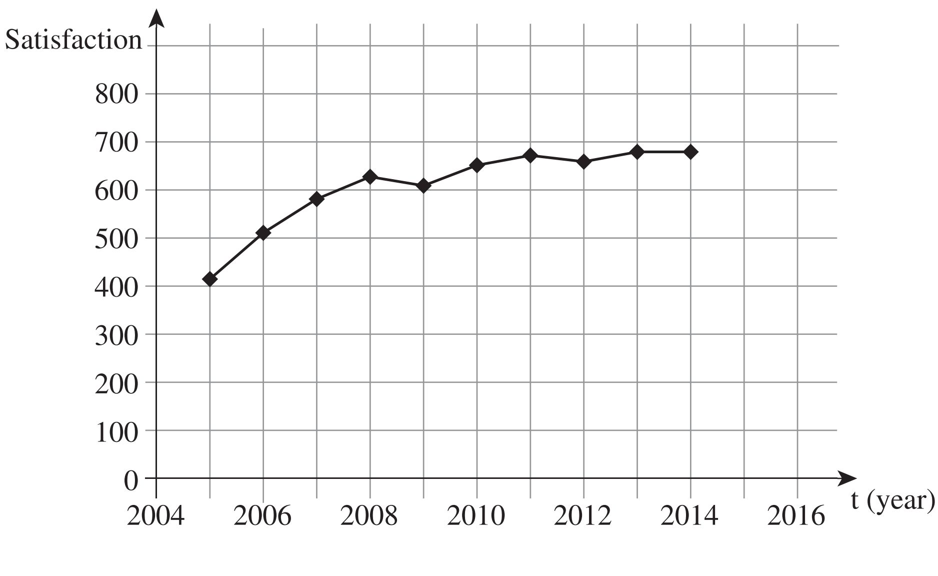
satisfaction = 3065 × GDP – 506 272 A1

d. The calculation required is finding correlation.

$r = 0.8603$

There is an improvement in the correlation and thus the transformed data equation is better. However, the improvement is not very large and the improvement is thus slight. A1

e.



A1

f.

Year	GDP ($)	Satisfaction	Smoothed/Deseasonalised
2005	246	413	–
2006	261	513	502.7
2007	263	582	273.7
2008	272	626	607.7
2009	306	615	631.7
2010	296	654	647.3
2011	299	673	664
2012	312	665	673
2013	302	681	677
2014	334	685	–

A1

g. Deseasonalisation is appropriate when clear seasons exist within the data. That is not the case here and thus 3 point moving mean was used instead. A1

Module 1: Number patterns

Question 1

a. $t_{n+1} = 60 - 0.4(60 - t_n)$, $t_1 = 10$

$t_2 = 60 - 0.4(60 - 10) = 40$ A1

$t_3 = 60 - 0.4(60 - 40) = 52$ A1

Thus there are 40 000 and 52 000 fish in the lake in the years 2015 and 2016 respectively.

b. Using calculator or otherwise we can obtain the next 10 terms.

$t_4 = 56.8$

$t_5 = 58.72$

$t_6 = 59.488$

$t_7 = 59.795$

…

$t_{13} = 59.999$

The pattern is clear. The population is headed toward 60 million. A1

c. $t_2 = 30$

$t_{n+1} = 60 - k(60 - t_n)$

$30 = 60 - k(60 - 10)$ M1

$50k = 30$

$k = 0.6$ A1

d. We need to repeat the process of part **b** but looking for the first term exceeding 57. M1

$t_2 = 30$

$t_3 = 42$

$t_4 = 49.2$

$t_5 = 53.52$

$t_6 = 56.112$

$t_7 = 57.667$

Thus it is in the year 2020 that 95% capacity is reached. A1

Question 2

a. Arithmetic sequence.

$a = 3.5,\ d = 0.6$

$t_3 = a + (n - 1)d$

$t_3 = 3.5 + 2 \times 0.6 = 4.7$ A1

$t_4 = 3.5 + 3 \times 0.6 = 5.3$ A1

b. $3.5 + (n - 1)0.6 = 9$

$0.6(n - 1) = 5.5$

$n - 1 = \frac{5.5}{0.6}$

$n = 10.17$

It is thus during the 11th year, the year 2023. A1

c. We require the sum of the first 7 terms.

$S_7 = \frac{7}{2}(2 \times 3.5 + 6 \times 0.6) = 37.1$

37.1 tonnes. A1

d. This limitation will not take effect until the 11th year. Until then there is no change.
From the 11th to the 18th years the amount of fish delivered will be 9 tonnes.

First 10 years …

$S_{11} = \frac{10}{2}(2 \times 3.5 + 9 \times 0.6) = 62$ A1

Next 8 years ...

$\text{sum} = 8 \times 9 = 72$

Thus, in total, 134 tonnes will be delivered. A1

e. The 2013 and 2014 orders will remain as before. It is only after that that changes occur.

$a = 4.1,\ r = 1.1$

$S_6 = \frac{4.1(1.1^6 - 1)}{0.1} = 31.634$ M1

The total for the 7 years will be $31.634 + 3.5 = 35.134$, a quantity slightly less than that of the company projection. The difference is 1.97 tonnes. A1

Module 2: Geometry and trigonometry

Question 1

a. $\text{Area} = 0.5 \times 12 \times 15 \times \sin(35)$

$= 51.62$ A1

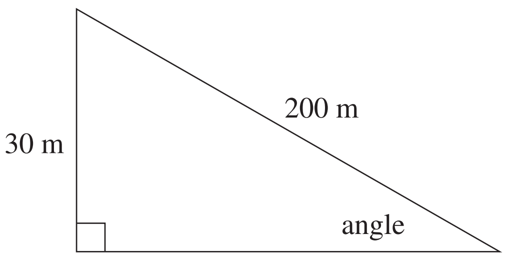
b. $AB^2 = 122 + 152 - 2 \times 12 \times 15\cos(35)$

$AB = 8.608$ A1

Question 2

a. $4 \times 10\ 000 = 400$ m A1

b.



$\sin(\text{angle}) = \frac{30}{200}$

$\text{angle} = 8.62°$ A1

Question 3

a. $\frac{360}{5} = 72$

$180 - 72 = 108$ A1

b. $5 \times (0.5 \times 8 \times 8 \times \sin(72))$ M1

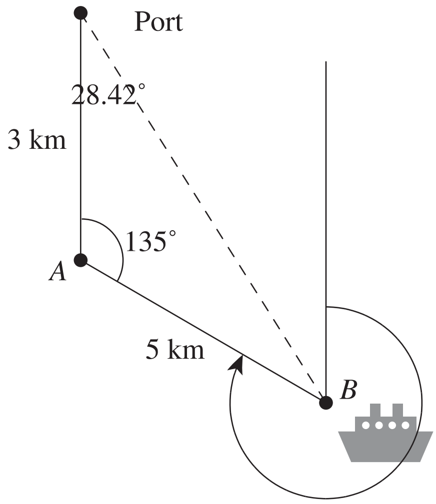
$= 152.169 \text{ cm}^2$ A1

Question 4

a. $\text{dist}^2 = 3^2 + 5^2 - 2 \times 3 \times 5\cos(135)$

$\text{dist} = 7.43$ km A1

b.



$360 - 28.42 = 331.57°$ T — A1

c. $\text{Area} = 0.5 \times 3 \times 5\sin(135)$

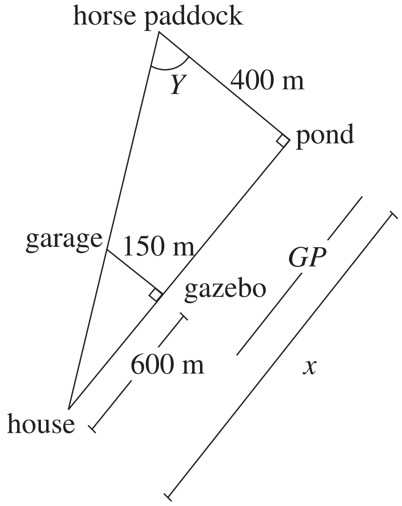
$= 5.3 \text{ km}^2$ — A1

Question 5

a. $\text{dist}^2 = 150^2 + 600^2$

$\text{dist} = 618.5 \text{ m}$ — A1

b.



$\frac{x}{400} = \frac{150}{600}$

$x = 1600$

$GP = 1600 - 600$

$GP = 1000$ — A1

c. $\tan(Y) = \frac{600}{150}$

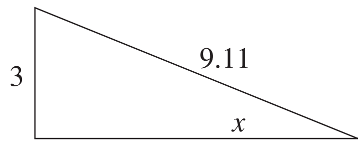
$Y = 75.96°$ — A1

Question 6

a. $AB^2 = 3^2 + 5^2 + 7^2$

$AB = 9.11$ A1

b.



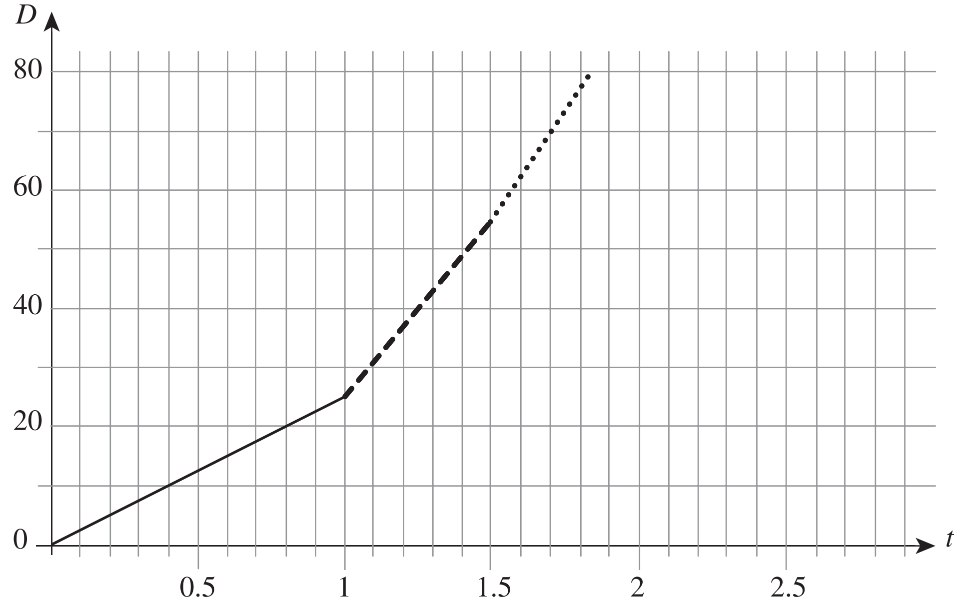
$\sin(x) = \dfrac{3}{9.11}$

$x = 19.23°$ A1

Module 3: Graphs and relations

Question 1

a.



Both line sections correct A1
Both domains correct A1

b. The bus has travelled 55 km after 90 minutes. Thus it is 25 km from Tagaytay Hotel. The time that it will take to complete the journey must be $\frac{25}{75} = \frac{1}{3}$ hours = 20 minutes

The bus will arrive at 6:00 PM + 1 hour and 50 mins. It arrives at 7:50 PM. A1

c. The total distance travelled was 80 km. The total time taken was 110 minutes.

Average speed is thus $\frac{80}{110} \times 60 = \frac{480}{11} = 43.6$ km/h A1

d. The first line of the hybrid function is complete.
The second line gives the location on the second stage. This stage starts at the point (1, 25) so we substitute this value into the equation.

$25 = 60 \times 1 + a$

$a = -35$ A1

b is the time that the second phase of the journey ends. This is the 1.5 hour mark and thus $b = 1.5$ A1

d is the time that the bus reaches Tagaytay Hotel. We know that this is when $t = \frac{11}{6}$ and

thus $d = \frac{11}{6}$ A1

Question 2

a. $y \leq x$ A1

$x \geq 4$ A1

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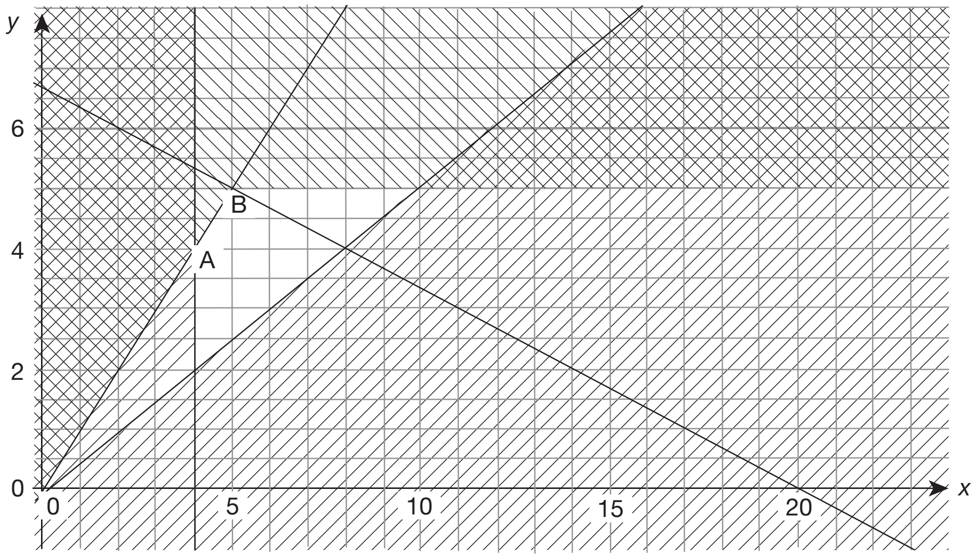
b.



A1

c. The point A is where the line $x = 4$ crosses $y = x$.
Clearly the x-value is 4. The y-value is 4.
Thus $A(4, 4)$. A1

At B, $x + 3y = 20$ and $y = x$.
Thus $B(5, 5)$ A1

d. Check profits at all 4 corners.

At (4,4), $P = 4 \times 50 + 4 \times 200 = 1000$

At (5,5), $P = 5 \times 50 + 5 \times 200 = 1250$

At (4,2), $P = 4 \times 50 + 2 \times 200 = 600$

At (8,4), $P = 8 \times 50 + 4 \times 200 = 1200$ M1

Thus the best option is to have an average of 5 cave and 5 volcano tours. A1

This will give a profit of $1250. A1

Module 4: Business-related mathematics

Question 1

a. $747 – $339 = $408 A1

b. $\text{SI} = \dfrac{339 \times 2}{12 \times 100}$

$\text{SI} = \$0.57 \ (0.565)$ A1

Question 2

a. **i.** $\$340\ 000 \times \dfrac{20}{100} = \$68\ 000$ A1

ii. $\$340\ 000 - 2 \times \$68\ 000 = \$204\ 000$ A1

iii. $\$340\ 000 - 4 \times \$68\ 000 = \$68\ 000$

4 years A1

b. **i.** 22% A1

ii. $\$340\ 000 \times 0.78^4 = \$125\ 851 \ \ (125\ 851.1904\ldots)$ A1

iii. After 1 year:

flat rate $BV = \$272\ 000$

reducing balance $BV = \$265\ 200$ M1

After 2 years:

flat rate $BV = \$204\ 000$

reducing balance $BV = \$206\ 856$

At the end of two years. A1

Question 3

a. $\dfrac{\$8775}{3} = \$2925$ A1

b. $r = \dfrac{8775 \times 100}{65\ 000 \times 3}$

$r = 4.5\%$ A1

c. After 3 years:

$A = 65\ 000 \times \left(1 + \dfrac{4.1}{4 \times 100}\right)^{4 \times 3}$

$A = \$73\ 461.48$

After 2 years:

$A = 65\ 000 \times \left(1 + \dfrac{4.1}{4 \times 100}\right)^{4 \times 2}$

$A = \$70\ 525.18$ M1

$73\ 461.48 - 70\ 525.18$

$= \$2936.29$

$\approx \$2936$ A1

Question 4

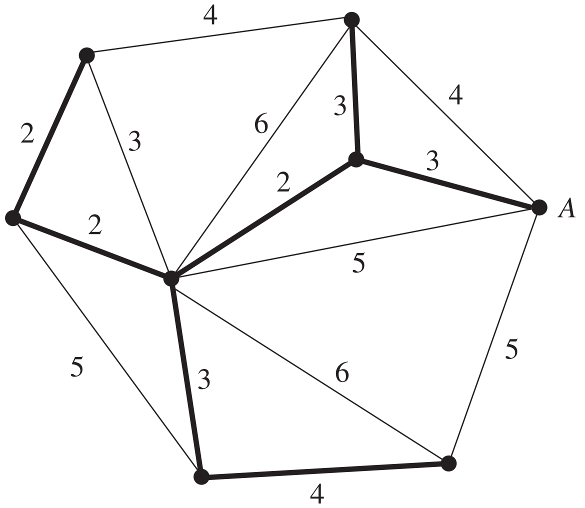
a.

$N =$	$N =$	
$I\% = 5.3$	$I\% = 5.3$	
$PV = 48\ 000$	$PV = 48\ 000$	
$Pmt = -1450$	$Pmt = -1700$	
$FV = 0$	$FV = 0$	
$P_PY = 12$	$P_PY = 12$	
$N = 35.876\ldots$	$N = 30.227\ldots$	
36 repayments in total	31 repayments in total	M1
5 fewer repayments		A1

Module 5: Networks and decision mathematics

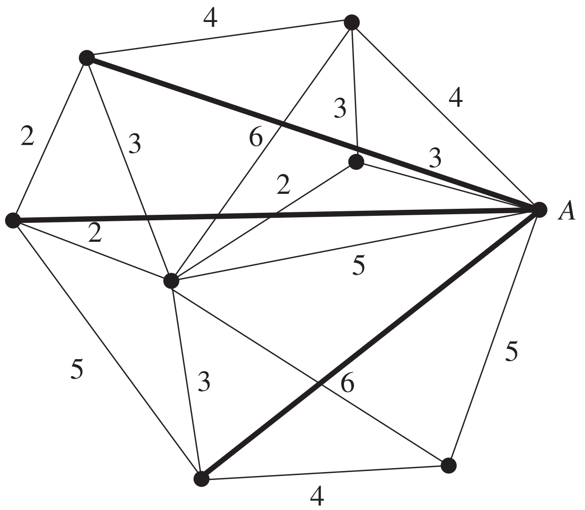
Question 1

a.



A1

b. 3



A1

Question 2

a. C and F all other teams must play 1 more game. A1

b.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array} \begin{array}{c} A\,B\,C\,D\,E\,F \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

A2

c. The top row represents how many games team A has won. The total is 1. A1

d. A complete graph. A1

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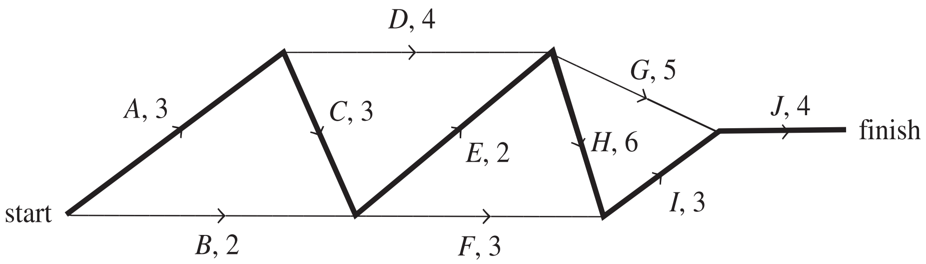
Question 3

a. *D* and *E* A1

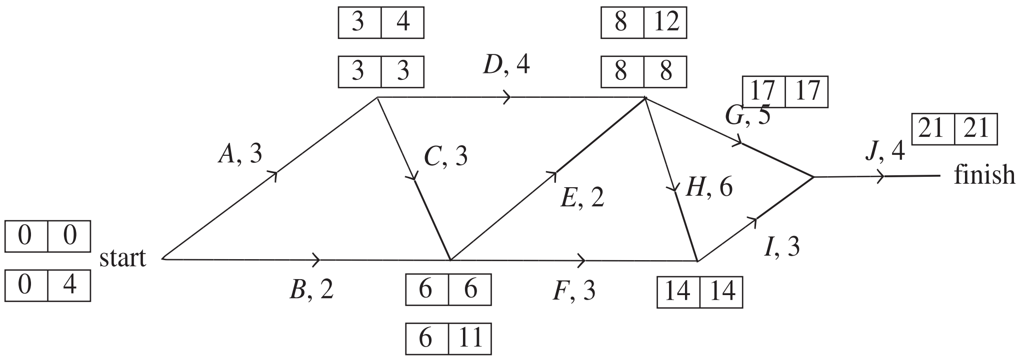
b.



A1

Minimum completion time is 21. A1

c.



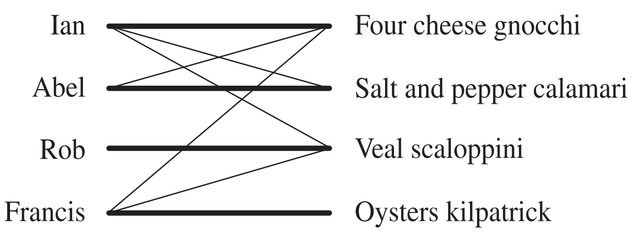
F slack time $11 - 6 = 5$ A1

Question 4

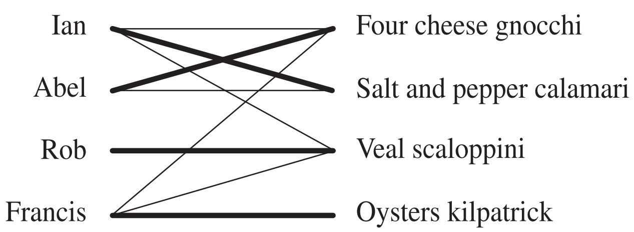
a. $\begin{bmatrix} 0 & 0.9 & 1.5 & 3.5 \\ 0 & 0.9 & 2.4 & 4.3 \\ 0 & 1 & 2.1 & 4 \\ 0 & 0.3 & 1.5 & 3.3 \end{bmatrix}$ A1

$\begin{bmatrix} 0 & 0.6 & 0 & 0.2 \\ 0 & 0.6 & 0.9 & 1 \\ 0 & 0.7 & 0.6 & 0.7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ A1

b.



OR



A1

c. $2.3 + 2.8 + 4.2 + 5.7 = 15$ min either way A1

Module 6: Matrices

Question 1

a. The element in the transition matrix that concerns us is the element in row 2, column 1. Row 2 gives the outcome in the next year for those obtaining achievement awards. Column 1 relates to those who were state selections in the earlier year.

Thus the result is 35%. A1

b. Year 12 is 3 years into the future. Thus it would require 3 transition matrix applications. M1

$$\begin{bmatrix} S_{12} \\ a_{12} \\ p_{12} \end{bmatrix} = \begin{bmatrix} 0.15 & 0.05 & 0.80 \\ 0.35 & 0.80 & 0.10 \\ 0.50 & 0.15 & 0.10 \end{bmatrix}^3 \begin{bmatrix} 41 \\ 50 \\ 25 \end{bmatrix} = \begin{bmatrix} 28 \\ 60 \\ 27 \end{bmatrix}$$ A1

c. This question requires us to consider the inverse matrix. M1

$$T^{-1} = \begin{bmatrix} -0.243 & -0.430 & 2.374 \\ -0.056 & 1.439 & -0.991 \\ 1.299 & -0.009 & -0.383 \end{bmatrix}$$ A1

This inverse matrix will take the proportions one year backward instead of forward.

Thus $\begin{bmatrix} S_8 \\ a_0 \\ p_8 \end{bmatrix} = \begin{bmatrix} -0.243 & -0.430 & 2.374 \\ -0.056 & 1.439 & -0.991 \\ 1.299 & -0.009 & -0.383 \end{bmatrix} \begin{bmatrix} 41 \\ 50 \\ 25 \end{bmatrix} = \begin{bmatrix} 28 \\ 45 \\ 43 \end{bmatrix}$ A1

d. Now we need to consider several years into the past.

$$\begin{bmatrix} S_8 \\ a_0 \\ p_8 \end{bmatrix} = \begin{bmatrix} -0.243 & -0.430 & 2.374 \\ -0.056 & 1.439 & -0.991 \\ 1.299 & -0.009 & -0.383 \end{bmatrix}^8 \begin{bmatrix} 41 \\ 50 \\ 25 \end{bmatrix} = \begin{bmatrix} 176.08 & -22.79 & -134.88 \\ -86.19 & 60.37 & -48.83 \\ -88.88 & -36.58 & 184.71 \end{bmatrix} \begin{bmatrix} 41 \\ 50 \\ 25 \end{bmatrix} = \begin{bmatrix} 2708 \\ -516 \\ -181 \end{bmatrix}$$ A1

The results have stopped making sense. It would be impossible to have a negative number of students in any category. Thus it is clear that the matrix fails to operate successfully in all past year applications. A1

Question 2

a. The matrix C is 3×1. Thus we need to pre multiply by a 1×3 so that the product is 1×1. A1

Thus matrix A is $\begin{bmatrix} 10 & 25 & 30 \end{bmatrix}$ A1

b. The matrix must give the proportions of cost remaining, not the reduction itself. A1

$$\begin{bmatrix} 0.90 & 0 & 0 \\ 0 & 0.95 & 0 \\ 0 & 0 & 0.85 \end{bmatrix}$$

$$\begin{bmatrix} 0.90 & 0 & 0 \\ 0 & 0.95 & 0 \\ 0 & 0 & 0.85 \end{bmatrix} \begin{bmatrix} 25 \\ 10 \\ 5 \end{bmatrix} = \begin{bmatrix} 22.50 \\ 9.50 \\ 4.25 \end{bmatrix}$$ A1

c. $\begin{bmatrix} 1 & -0.05 & -0.05 \\ 0 & 0.92 & 0 \\ 0 & -0.10 & 1 \end{bmatrix}$ A2

Each error results in 1 lost mark.
The key is to remember that rows refer to the new price and columns to the old price.

d. $\begin{bmatrix} 1 & -0.05 & -0.05 \\ 0 & 0.92 & 0 \\ 0 & -0.10 & 1 \end{bmatrix}\begin{bmatrix} 25 \\ 10 \\ 5 \end{bmatrix} = \begin{bmatrix} 24.25 \\ 9.20 \\ 4.00 \end{bmatrix}$ A1