The Mathematical Association of Victoria
FURTHER MATHEMATICS
SOLUTIONS: Trial Exam 2015
Written Examination 2

CORE (15 marks)

Question 1

a) i) Number of cars towed is discrete A1

ii) Rainfall A1

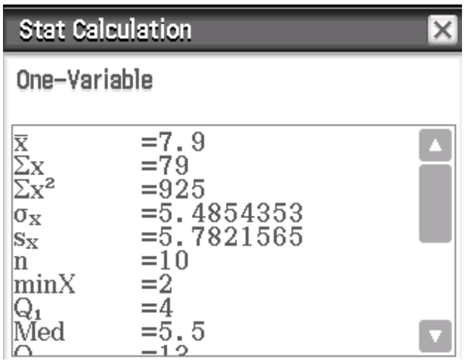
b) Upper Boundary

$$\begin{aligned} &= Q_3 + 1.5 \times IQR \\ &= 13 + 1.5 \times (13 - 4) \\ &= 13 + 1.5 \times 9 \\ &= 13 + 13.5 \\ &= 26.5 \end{aligned}$$

A1

25.5 is not an outlier since $25.5 < 26.5$ A1

c) mean = 7.90 and s = 5.78 A1



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Stat Calculation
One-Variable
x̄          =7.9
Σx         =79
Σx²        =925
σx         =5.4854353
sx         =5.7821565
n          =10
minX       =2
Q₁         =4
Med        =5.5
```

d)

$$m = \frac{r \times S_y}{S_x} = \frac{0.7984 \times 15.47}{5.78} = 2.137$$

A1

$$c = \overline{y} - m\overline{x} = 34.9 - 2.137 \times 7.9 = 18.018$$

A1

Number of cars towed $= 2.137 \times \text{rainfall} + 18.018$

CORE continued

e) $\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{2.137}{1} = \frac{21.37}{10}$ 21 cars A1

f) $42 = 3.137 \times \text{rainfall} + 18.018$

$\text{rainfall} = \frac{42 - 18.018}{2.137} = 11\,\text{mm}$

A1

g) The highest rainfall in the data set is 20 mm. This means that 25.5 mm is a case of **extrapolation** and therefore unreliable since the model may not continue beyond the data set. (Award mark for extrapolation) A1

Question 2

a) log (rainfall) or (Number of cars towed)^2 Accept log(x) or y^2 A1

b) (i) $\text{Number of cars towed} = \frac{-108.96}{13} + 57.02 = 48.638... \approx 49\,\text{cars}$ A1

(ii) At $\frac{1}{13} = 0.08$

$\text{residual} = y_{actual} - y_{predicted} = 51 - 49 = 2$

A1

(iii) Point plotted at (0.08, 2) A1

c) Shows a **random** distribution of points or **no pattern** A1

Module 1: Number Patterns (15 marks)

Question 1

a) $d = 3 - 2.4 = 2.4 - 1.8 = 0.6$

Therefore

$$\begin{aligned} t_5 &= a + 4d \\ &= 1.8 + 4 \times 0.6 \\ &= 4.2 \end{aligned}$$

A1

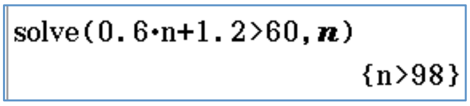
b)

$$\begin{aligned} A_n &= a + (n-1)d \\ &= 1.8 + (n-1)0.6 \\ &= 1.8 + 0.6n - 0.6 \end{aligned}$$

$\therefore \quad A_n = 0.6n + 1.2$ (must be in simplified form) A1

c)

$$\begin{aligned} A_n &> 60 \\ 0.6n + 1.2 &> 60 \\ 0.6n &> 58.8 \\ n &> \frac{58.8}{0.6} \\ n &> 98 \end{aligned}$$



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solve(0.6·n+1.2>60,n)
                                {n>98}
```
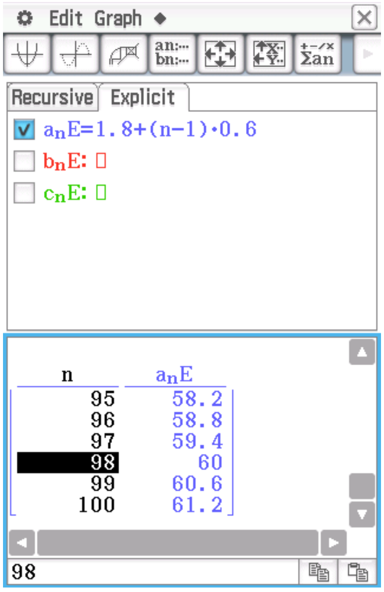



☑ a_nE=1.8+(n−1)·0.6
☐ b_nE:
☐ c_nE:

n	a_nE
95	58.2
96	58.8
97	59.4
98	60
99	60.6
100	61.2

98

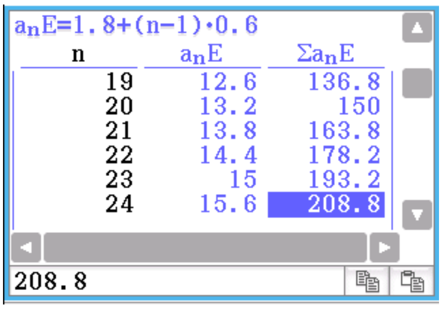
The leak will exceed 1 litre per minute in the **99th** hour. A1

d)

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{24} = \frac{24}{2}\left[2 \times 1.8 + (24-1)0.6\right]$$

$$= 208.8$$



a_nE=1.8+(n−1)·0.6

n	a_nE	Σa_nE
19	12.6	136.8
20	13.2	150
21	13.8	163.8
22	14.4	178.2
23	15	193.2
24	15.6	208.8

208.8

A total of **208.8 litres** of water will leak in the first 24 hours. A1

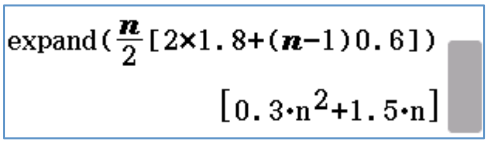
Module 1: Number Patterns continued

e)

$$\begin{aligned} S_n &= \frac{n}{2}[2a+(n-1)d] \\ &= \frac{n}{2}[2\times 1.8+(n-1)\times 0.6] \\ &= \frac{n}{2}[3.6+0.6n-0.6] \\ &= \frac{n}{2}[3+0.6n] \\ &= 1.5n+0.3n^2 \end{aligned}$$

Award 1 mark for correct method if answers are incorrect

(M1)



```
expand(n/2[2×1.8+(n−1)0.6])
                    [0.3·n^2+1.5·n]
```

Therefore $a = 1.5$ and $b = 0.3$ A2

Question 2

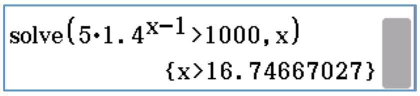
a) $\frac{t_2}{t_1} = \frac{7}{5} = 1.4$ and $\frac{t_3}{t_2} = \frac{9.8}{7} = 1.4$ (both calculations must be shown)

Since $\frac{t_2}{t_1} = \frac{t_3}{t_2} = 1.4$ then this follows a geometric sequence A1

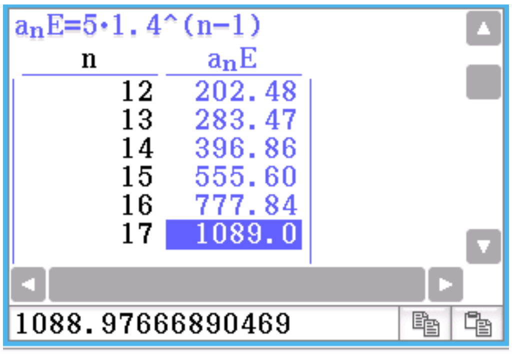
b) $B_n = 5\times 1.4^{n-1}$ A1

c)

$$\begin{aligned} &B_n > 1000 \\ &5\times 1.4^{n-1} > 1000 \\ &n > 16.74667... \end{aligned}$$



```
solve(5·1.4^(x−1)>1000,x)
              {x>16.74667027}
```



```
anE=5·1.4^(n−1)
     n      anE
    12    202.48
    13    283.47
    14    396.86
    15    555.60
    16    777.84
    17    1089.0
1088.97666890469
```

It will first exceed 1000 litres in the **17th** hour. A1

Module 1: Number Patterns continued

Question 3

a) $\frac{3}{20}\times 100 = 15\%$ A1

b) $a = 1 - 0.15 = 0.85$ A1

c)

$$S_\infty = \frac{a}{1-r}$$

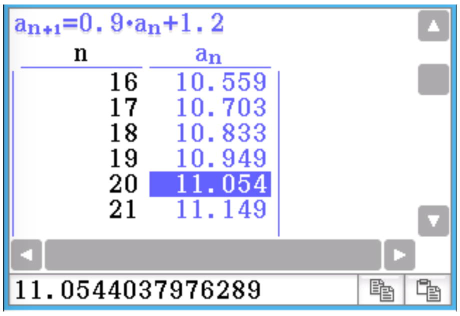
$$= \frac{20}{1-0.85}$$

$$= 133.33...$$

The total volume of water that will leak is 133 litres A1

Question 4

a)



```
an+1=0.9·an+1.2
       n     an
      16   10.559
      17   10.703
      18   10.833
      19   10.949
      20   11.054
      21   11.149
11.0544037976289
```

$L_{20} = 11.05...$ **11 litres** of water will leak in the 20th hour A1

b) If $L_n = 12$ then $L_{n=1} = 0.9\times 12 + 1.2 = 12$.
Therefore the leak will not exceed 12 litres. A1

c) If $L_n = 15$ and $L_{n+1} = 15$
then $c\times 15 + 1.2 = 15$
Solve for c

$c = 0.92$ A1

Module 2: Geometry and Trigonometry (15 marks)

Question 1

a) Length of support = $\sqrt{15^2+10^2}=18$ cm A1

b) 180 – 30 – 73 = 77 A1

c) $BC=\dfrac{\sin(30^o)\times 180}{\sin(73^o)}$ A1

d) BC = 0.94 m = 94 cm A1

e) Area = $\frac{1}{2}\times 94\times 180\sin(77)=8243$ cm^2 = 0.82 m^2 A1

f) *ABC* and *ADE* are similar triangles

AD = 1.8 – DB = 1.8 – 0.2 = 1.2 metres

$$\frac{DE}{BC}=\frac{AD}{AB}$$ M1

$$\begin{aligned}DE&=\frac{AD\times BC}{AB}\\&=\frac{1.2\times 94}{1.8}\\&=62.6666....\\&=0.63\end{aligned}$$

DE is 0.63 metres A1

Question 2

$$SA_{\text{paint}}=\frac{1}{16}\times SA_{\text{swing}}$$

$$\therefore k^2=\frac{1}{16}$$

$$\therefore k=\frac{1}{4}$$

$$\therefore k^3=\frac{1}{64}$$ A1

$$\begin{aligned}V_{\text{paint}}&=k^3\times V_{\text{swing}}\\&=\frac{1}{64}\times 19000\\&=297\text{ cm}^3\end{aligned}$$ A1

Module 2: Geometry and Trigonometry continued

Question 3

a) Area of playground

= circular area + rectangular area

$$= \pi(5)^2 + 10 \times 15$$
$$= 228.5398...$$
$$\approx 229 \text{ m}^2$$ A1

b) Entire Area = $\pi(5.23)^2 + 10.46 \times 15$

$= 242.83166...\text{m}^2$

Base Area of concrete = Entire Area – Area of playground

= 242.83166 – 229

= 13.83…. M1

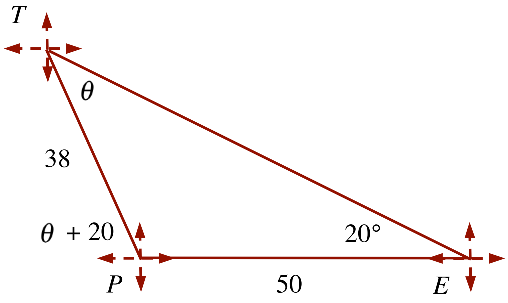
Volume = Base Area x height =18.83…x 0.4 = 5. 53 ≈ 6 m^3 A1

Question 4

a) 10 – 2 = 8 m A1

b) $\theta = \tan^{-1}\left(\frac{8}{38}\right) = 12^\circ$ A1

c)



$$\frac{\sin(\theta)}{50} = \frac{\sin(20)}{38}$$
$$\theta = \sin^{-1}\left(\frac{50 \times \sin(20)}{38}\right)$$
$$= 27^\circ$$

Bearing = 270 + 27 + 20 = 317° A1

Note: An alternative answer of 083° is accepted for the use of the ambiguous case

Module 3: Graphs and relations (15 marks)

Question 1

a) $C = 15x + 500$ A1

b) $R = 35x$ A1

c) $A(0,500)$ A1

Point B is where Revenue = Cost
$15x + 500 = 35x$
$x = 25$

When $x = 25$, $R = C = 875$
Point B $(25,875)$ A1

d) Profit = Revenue - Cost
Solve $650 = 35x - (15x + 500)$ M1
$x = 57.5$

Therefore minimum number of guests will be 58 A1

Question 2

a) He must prepare at least twice as many cold food hampers as he does hot food hampers. A1

b) Highlight the sides of the triangle with vertices $(0,0)$, $(0,20)$ and $(5,10)$ A1

c) Maximum value x can take when $y = 8$ is 4 A1

d) $P = 60x + 20y$ A1

e) Maximum profit is \$500 A1

Occurs at point $(5,10)$ therefore 5 hot hampers and 10 cold hampers A1

f) $P = 60x + 30y$
Solution is all integer points between $(0,20)$ and $(5,10)$ M1

$(0,20), (1,18), (2,16), (3,14), (4,12), (5,10)$ A1

g) Substitute any of the 6 points. Profit = \$600 A1

Module 4 Business related mathematics (15 marks)

Question 1

a) Ronith pays $352 \times 1.1 = \$387.20$ A1

b) Price before GST is added is $\frac{352}{1.1} = \$320$ A1

Question 2

a) Cost is $2500 \times 1.03^5 = \$2898$ A1

b) Depreciation is \$2000 over 4 years therefore \$500 per year M1

Annual depreciation rate is $\frac{500}{2500} \times 100 = 20\%$ A1

c) Solve $500 = 2500 \times (1 - \frac{r}{100})^4$ M1

$r = 33.13\%$ A1

d) As the reducing balance rate is calculated on the depreciated value rather than the original value of the equipment. A1

Question 3

a) $P = \frac{100Q}{r}$

where $r = \frac{4.8}{12} = 0.4$ M1

$P = \frac{100 \times 2000}{0.4}$

$= \$500{,}000$ A1

b) He will receive the monthly payment indefinitely as a perpetuity only pays the interest obtained and the principal remains the same. A1

Module 4 continued

Question 4

a. $R = 1 + \frac{r}{100}$ where r is the interest rate per period.

$R = 1 + \frac{\frac{4.8}{12}}{100} = 1.004$ A1

b. $A = 20000 \times 1.004^{72}$

$= \$26,660$ A1

c. Interest without added deposits = \$6660 to nearest dollar

Finance Solver
$\text{N} = 72$
$\text{I}(\%) = 4.8$
$\text{PV} = -20000$
$\text{Pmt} = -50$
$\text{FV} = ?$
$\text{PpY} = 12$
$\text{CpY} = 12$
Future value is \$30822.22 M1

Interest gained is \$10822 to nearest dollar
Additional interest gained is $10822 - 6660 - (72 \times 50) = \$562$ A1

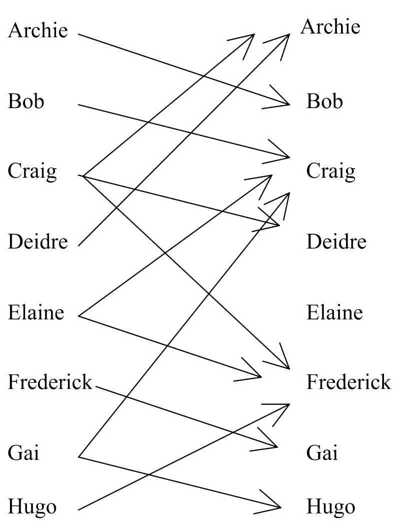
Module 5: Networks and decision mathematics (15 marks)

Question 1

a) Elaine A1

b) Craig and Frederick A1

c)



A1

Question 2

a) B, C, D, F, G A1

b) Each vertex is visited without returning to the starting point therefore Hamiltonian Path A1

c) ABCDHGFE A1

d) Because this would be an Euler circuit and this only exists if the degree of every vertex is even. Only A and E are of even degree. A1

e) 3 A1

The three new roads would connect any 3 pairs of vertices with odd degree.

Module 5 continued

Question 3

a) 11 A1

Along path B-C-F

b) B-C-F-H-I A1

c) 19 hours A1

d) Earliest start time $= 4$. Latest start time $= 7$. M1

Float time = Latest start time − Earliest start time $= 7 - 4 = 3$ A1

e) A, D and G A1

All have a float time of 5 hours.

f) A directed arrow from the end of A to the end of E labelled J, 2 or

A directed arrow from the end of B to the end of E labelled J, 2 A1

Module 6: Matrices (15 marks)

Question 1

a) Drakes defeated Beavers A1

b) No side defeated Crabs A1

c) Column 1 must contain all zeros
Row 1 must contain three ones after the initial zero

Two answers are possible

A1

$$R_{2015} = \begin{array}{cc} \begin{array}{cccc} A & B & C & D \end{array} & \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} & \begin{array}{c} A \\ B \\ C \\ D \end{array} \end{array} \qquad R_{2015} = \begin{array}{cc} \begin{array}{cccc} A & B & C & D \end{array} & \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} & \begin{array}{c} A \\ B \\ C \\ D \end{array} \end{array}$$

Question 2

a) $\begin{bmatrix} 0.5 \\ 0.3 \\ 0.2 \end{bmatrix} \begin{array}{c} Y \\ N \\ U \end{array}$ A1

b) 10% of those who say “yes” in any particular week change to “no” the next week. A1

c) $0.3 \times 250 + 0.5 \times 150 + 0.6 \times 100 = 210$ A1

d) $k = 500$ A1

$n = 2$ A1

e) $500 \times \begin{bmatrix} 0.7 & 0.3 & 0.4 \\ 0.1 & 0.5 & 0.2 \\ 0.2 & 0.2 & 0.4 \end{bmatrix}^{19} \begin{bmatrix} 0.5 \\ 0.3 \\ 0.2 \end{bmatrix} = \begin{bmatrix} 270.8 \\ 104.2 \\ 125 \end{bmatrix}$

125 are undecided A1

Module 6 continued

Question 3

a) $$\begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} b \\ w \end{bmatrix}=\begin{bmatrix} 46.50 \\ 49.50 \end{bmatrix}$$ A1

b) The determinant of $\begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}=14$

As the determinant is non-zero there will be a unique solution. A1

c)

$$\begin{bmatrix} b \\ w \end{bmatrix}=\begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}^{-1}\begin{bmatrix} 46.50 \\ 49.50 \end{bmatrix}$$

$$=\begin{bmatrix} 6.00 \\ 7.50 \end{bmatrix}$$

Total for 1 beer and 1 wine is \$13.50 A1

Question 4

a)

$$\begin{bmatrix} J \\ C \end{bmatrix}=\begin{bmatrix} \frac{1}{25} & -\frac{7}{150} \\ -\frac{1}{50} & \frac{2}{75} \end{bmatrix}\begin{bmatrix} 16250 \\ 12750 \end{bmatrix}$$

$$=\begin{bmatrix} 55 \\ 15 \end{bmatrix}$$

Each jumper costs \$55 and each cap costs \$15 A1

b) Determine the inverse of $\begin{bmatrix} \frac{1}{25} & -\frac{7}{150} \\ -\frac{1}{50} & \frac{2}{75} \end{bmatrix}$

The inverse is $\begin{bmatrix} 200 & 350 \\ 150 & 300 \end{bmatrix}$ M1

The original equations are:

$200J+350C=16250$ and $150J+300C=12750$ A1