FURTHER MATHEMATICS

Written Examination 2

TSSM™
Creating VCE Success

(*TSSM's *2015 trial exam updated for the current study design)

SOLUTIONS

Core – Data Analysis

Question 1

a. $Range = 42 - 7 = 35$ hours

1 mark

b. $Q_1 = 12.5, Q_3 = 23.5, IQR = 11$

$Q_1 - 1.5 \times IQR = -4, Q_3 + 1.5 \times IQR = 40$

Hence, 41 and 50 are outliers.

The data is positively skewed with outliers (41 and 50)

2 marks

c. $\frac{13}{23} \times 100 = 56.5\ \%$

1 mark

d. $\bar{x} = 24.48$ hours

1 mark

Question 2

a. Percentage of population living in towns.

1 mark

There is a strong negative linear relationship between the percentage of population living in towns and the percentage of population in farming jobs.

1 mark

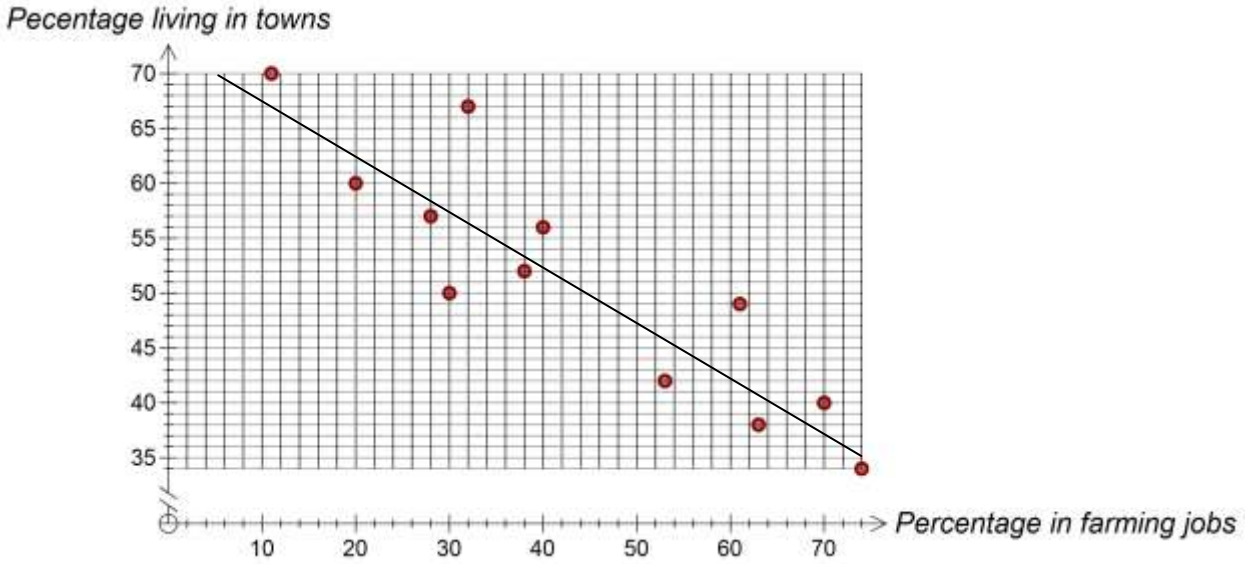
b. Slope – the percentage of population living in towns decreases by 0.4982 with every percent increase in population in farming jobs.
Vertical Intercept – 72.8384% of population was living in the towns when there was no farming job.

2 marks

c. $r^2 = 0.8043$
80.43%

1 mark

d.



1 mark

e.

Predicted value $= 72.8384 - 0.4982 \times 74 = 35.9716$

Actual value (from graph) $= 34$

Residual $= 34 - 35.9716 = -1.97$

1 mark

Question 3

a. $\log_{10}(\text{no of bacteria})$.

1 mark

b. Use CAS to generate the lists.
$\log_{10}(\text{Number of bacteria}) = 4.5941 - 0.0949 \times \text{Day number}$.

2 marks

Total 15 marks

Core – Recursion and financial modelling

Question 4

a. $\text{Increase} = 45 \times 0.1 = \$4.50$

1 mark

b. $\text{Price} = 45 \times 1.1^3 = \$59.90$

1 mark

c. $P_n = 1.1\, P_{n-1}$, $P_0 = 45$

2 marks

d. 2019 . Generate the list on CAS.

2 marks

Question 5

a. $\text{Interest} = 116395 - 75000 = \$41395$

1 mark

b.

$116395 = 75000\left(1 + \frac{r}{400}\right)^{24}$ or use TVM Solver

$r = 7.4\%$

1 mark

c. $116395\left(1+\frac{7.8}{1200}\right)+480=\$117632$

1 mark

d. Use TVM Solver or generate sequence on CAS.
$131 776

2 marks

Question 6

a. Use TVM Solver – N = 36, I = 6.95, PV = 200000, Pmt = -1350, PpY = 12, CpY = 12
$192352.48

2 marks

b. Use TVM solver
28 years and 3 months.

2 marks

Total 15 marks

Module 1: Matrices

Question 1

a. 3×4

1 mark

b. 3

1 mark

c. There were 20 pasties sold on Tuesday.

1 mark

d. The number of columns in matrix C is the same as the number of rows in matrix N.

1 mark

e. $CN = \begin{bmatrix} 99.70 & 75.50 & 40.90 & 85.80 \end{bmatrix}$

This matrix represents the sales collected on each of the four days.

2 marks

Question 2

a. $T = \begin{bmatrix} 0.90 & 0.15 & 0.25 \\ 0.075 & 0.80 & 0.25 \\ 0.025 & 0.05 & 0.5 \end{bmatrix}$

2 marks

b. 15%

1 mark

c. Need to consider only those people who change from C, not those who stay shopping at C the following week

$0.025 \times 240 + 0.05 \times 195 = 15.75$

16 customers change their preference to shop C

1 mark

d. $T^2 \times S_1 = \begin{bmatrix} 337.95 \\ 238.875 \\ 83.175 \end{bmatrix}$

This matrix shows the number of customers buying their bakery products from each of the three shops after 2 weeks.

2 marks

Question 3

a. The revenue collected in week 2.

1 mark

b.

$$\begin{bmatrix} 270 & 182 \\ 235 & 205 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{41}{2516} & \frac{-91}{6290} \\ \frac{-47}{2516} & \frac{27}{1258} \end{bmatrix}$$

2 marks

Total 15 marks

Module 2: Networks and decision mathematics

Question 1

a. $a = 35,\ b = 0$

2 marks

b. The last column does not have a zero entry.

1 mark

c. $1 \to B,\ 2 \to D,\ 3 \to C,\ 4 \to A \quad d = 75 + 65 + 90 + 45 = 275$ km

$1 \to D,\ 2 \to C,\ 3 \to B,\ 4 \to A \quad d = 80 + 55 + 95 + 45 = 275$ km

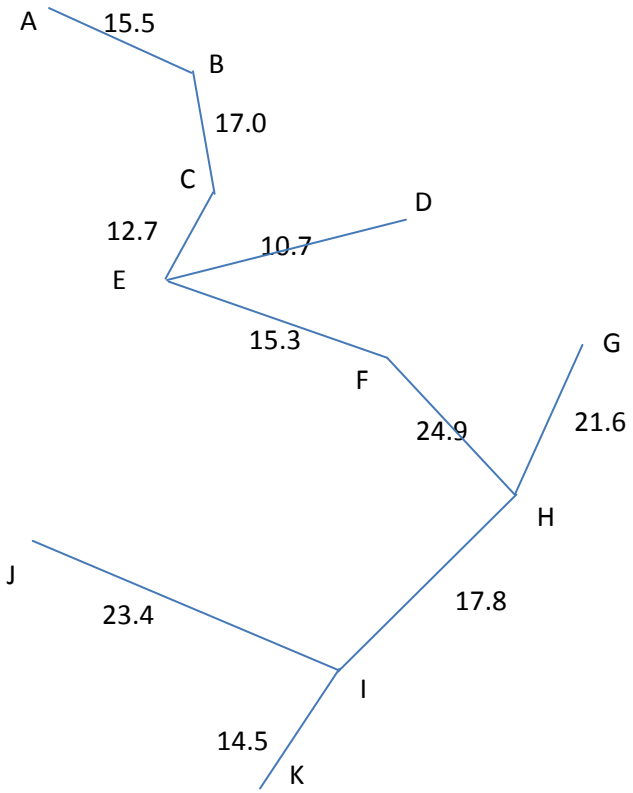
1 mark

d. 80 + 55 + 95 + 45 = 275 km

1 mark

Question 2

a.



2 marks

b. 173.4 km

1 mark

Question 3

a.

Activity	Time (in hours)	Depends on
A	3	-
B	5	-
C	2	A
D	3	A
E	3	B, D
F	5	C, E
G	1	C
H	2	F, G

2 marks

b. A - D - E - F - H

2 marks

Question 4

a. Euler trail has to start at B or C as they are the only vertices with odd degree.

2 marks

b. B-E-D-F-G-C-D-B-A-C or the reverse path from C to B.

1 mark

Total 15 marks

Module 3: Geometry and measurement

Question 1

a. $\frac{1}{2}(1 + 1.2) \times 0.8 = 0.88 \text{ m}^2$

1 mark

b. Side length $= \sqrt{0.8^2 + 0.1^2} = 0.806$

Perimeter $= 1 + 0.806 + 0.806 + 1.2 = 3.81$ m

2 marks

Question 2

a. Distance $= \sqrt{1.2^2 + 0.806^2 - 2 \times 1.2 \times 0.806 \times \cos(83°)} = 1.362$ m

2 marks

b. $0.88 \times 0.015 \times n = 3.3$
$n = 250$ table tops

1 mark

Question 3

a. $360° - (180° - 124°) = 304°$

1 mark

b. $\cos(56°) = \frac{x}{25}$
$x = 13.98\ km = 13980$ m

1 mark

c. $BP = \sqrt{32^2 + 25^2 - 2 \times 32 \times 25 \times \cos(94°)} = 42.0$ km

1 mark

d. $\frac{\sin(PBA)}{25} = \frac{\sin(94°)}{42}$
Angle $PBA = 36.43°$
Bearing of P from $B = 180° + 38° + 36.43° = 254°T$

3 marks

Question 4

a. $\sqrt[3]{9} : \sqrt[3]{4}$

1 mark

b. SA of smaller bowl $= \frac{\sqrt[3]{4}}{\sqrt[3]{9}} \times 24$

$4\pi r^2 = \frac{\sqrt[3]{4}}{\sqrt[3]{9}} \times 24$

$r = 1.2$ cm

2 marks

Total 15 marks

Module 4: Graphs and relations

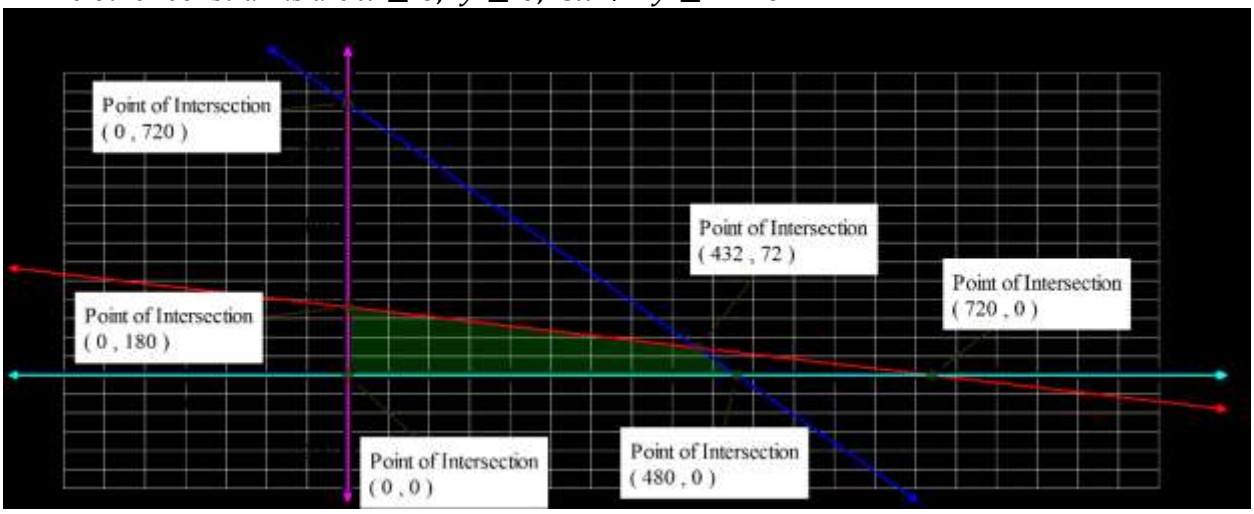
Question 1

a. $2x + 8y <= 1440 \ \ or \ \ x + 4y \leq 720$

1 mark

b.

The other constraints are $x \geq 0, \ y \geq 0, \ 3x + 2y \leq 1440$



3 marks

c. Profit $(0, 180) = 3060$
Profit $(480, 0) = 4800$
Profit $(432, 72) = 5544$
Max Profit $= \$5544$

1 mark

d. Wood screws $= 432$ boxes
Metal screws $= 72$ boxes

1 mark

Question 2

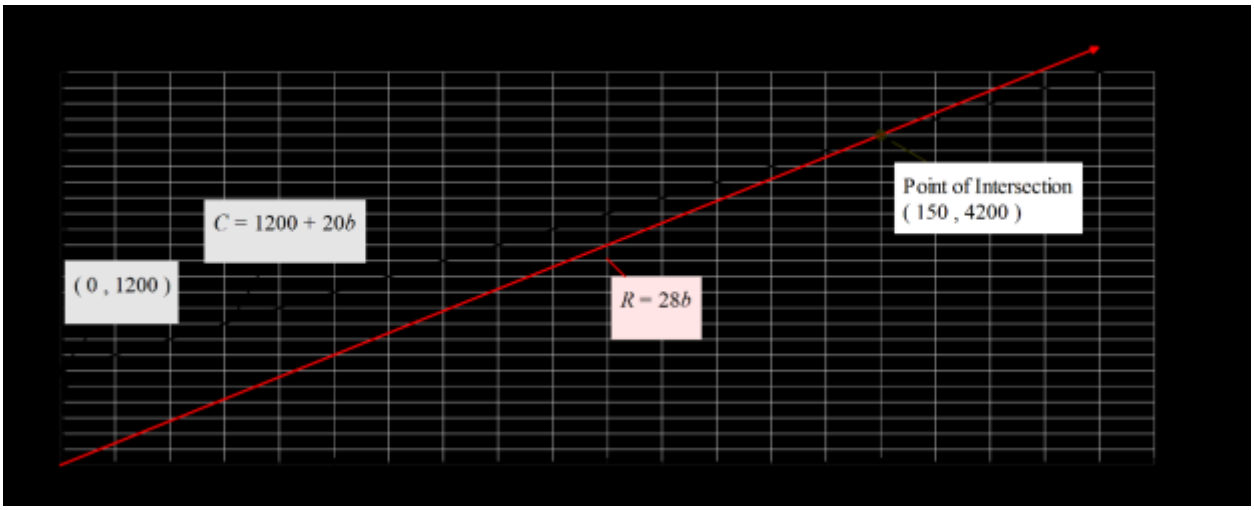
a. It will cost the manufacturer \$20 to produce the metal screw boxes.

1 mark

b. $28b - (1200 + 20b) = 8b - 1200$

1 mark

c.

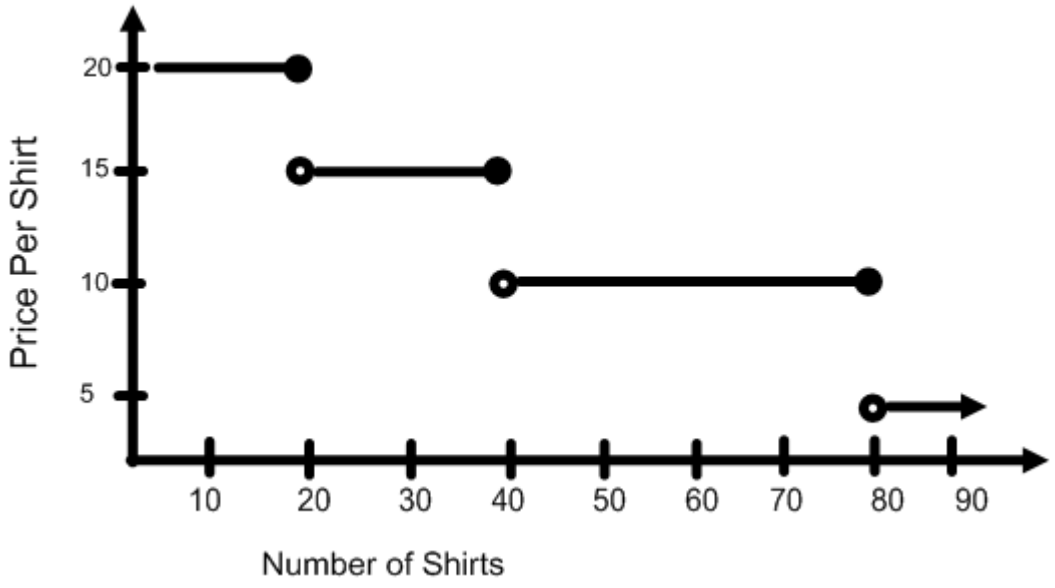


Break-even point is (150, 4200)

3 marks

Question 3

a.



2 marks

b. $750

1 mark

c. $a = 20 \ \ and \ \ b = 80$

1 mark

Total 15 marks