Engage Education Foundation

Units 3 and 4 Further Maths: Exam 2

Practice Exam Question and Answer Booklet

Duration: 15 minutes reading time, 1 hour 30 minutes writing time

Structure of book:

Section	Number of questions	Number of questions to be answered	Number of modules	Number of modules to be answered	Number of marks
A	8	8			36
B			4	2	24
				Total	60

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, rulers, a calculator and bound reference.
- Students are not permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied:

- This question and answer booklet of 30 pages including a formula sheet.

Instructions:

- You must complete all questions of the examination.
- Write all your answers in the spaces provided in this booklet.

Section A – Multiple-choice questions

Instructions

Answer all questions by circling your choice.
Choose the response that is correct or that best answers the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will not be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Questions

Data analysis

Question 1

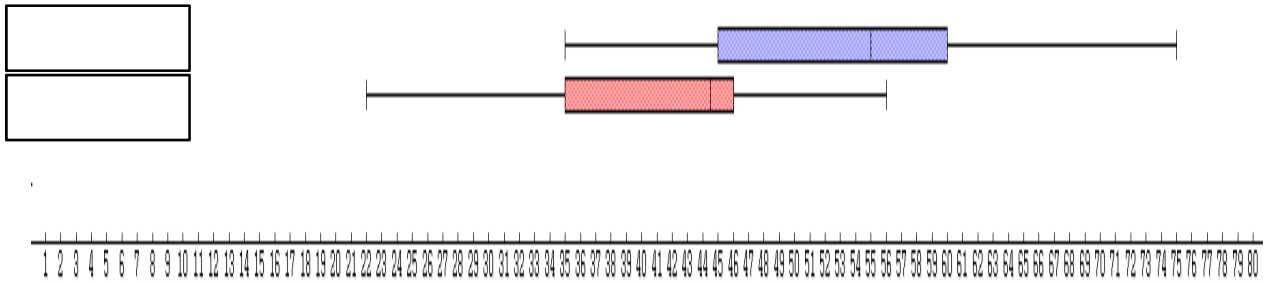
The following table lists the amount of time spent outdoors, in hours, of children from different age groups, 3-6, 7-12 and 13-15.

3-6	7-12	13-15
65	32	12
42	34	10
55	45	34
40	46	60
56	37	32
70	47	23
56	48	12
45	22	28
35	38	22
55	56	21
75	45	18
60	35	16
48	44	15
50	46	4

a. If this study were to be done with only two age groups, name the type of graphs/plots that can be used to represent the information.

1 mark

b. Below is an **incomplete** and **unlabelled** parallel boxplot representing the information presented in the table above. (2 marks)



i. In the boxes provided, label the graphed boxplots with their appropriate age groups.

1 mark

ii. Sketch the remaining boxplot. Show any relevant outlier(s).

1 mark

c. Comment if there is any observable association between time spent outdoors and age groups. Support your statement with specific statistics.

2 marks

Total: 5 marks

Question 2

The heights of the year 10 cohorts at schools A (with 69 students) and B (with 65 students) are recorded and displayed in the back-to-back stemplot below.

School A		School B
	11	2
	12	
7 6 6 5 4 4 3 2	13	1 2 2
9 7 7 7 7 6 6 6 6 6 6 6 5 5 5 5 5 4 4 2 1	14	1 2 3 4 4 5 6
9 9 8 8 8 7 7 6 6 5 4 4 3 3 2 1	15	2 6 7 7 7 8 8 9
9 8 8 7 7 6 5 5 4 3 3 2 1	16	2 2 4 5 5 6 7 8 9 9 9
7 7 7 6 6 5 5 4 3 2 1	17	2 3 5 5 5 5 6 7 7 7 8 9 9 9 9
	18	1 2 3 3 4 5 6 6 7 8 8 8 9
	19	2 3 4 5 5 6
	20	8

Key 17|2 = 172 cm.

a. Using the specific statistics to support your statement, compare the height distribution of the year 10 students at schools A and B.

2 marks

b. The 5-number summary for school B is as follows:

112 157.5 175 183.5 208

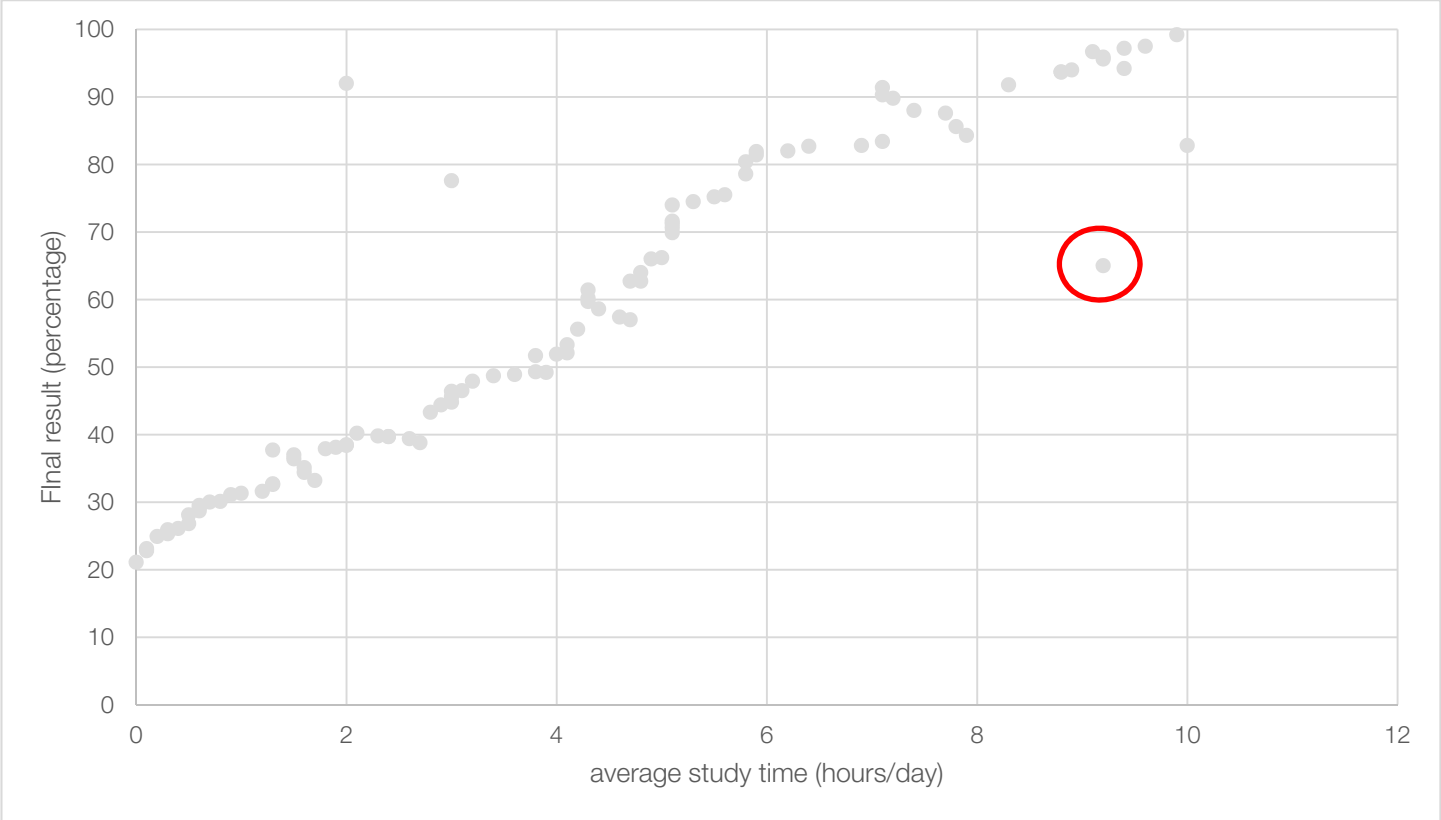
Write down an appropriate calculation and use it to explain whether the height represented by 11|2 is an outlier or not.

2 marks

Total: 4 marks

Question 3

The scatterplot below shows the *final result* and *average study time* (in hours/day) of a sample of high school students in a city.



The equation of the least squares regression line for the data in the scatterplot is

$$\textit{Final results} = 7.8471 \times \textit{average study time} + 25.27$$

a. Draw the least squares regression line on the scatterplot above.

1 mark

b. Interpret the slope of this least squares regression line in terms of the variables *average study time* and *final result*.

1 mark

c. David studies on average 5 hours a day and receives a final score of 85 percent.
Calculate the residual when the above least squares regression line is used to predict his final score.
Write your answer correct to the nearest percent.

1 mark

d. The correlation coefficient, *r*, is equal to 0.9377.
What percentage of the variation in the final result can be explained by the variation in average study time?
Round your answer to nearest percent.

1 mark

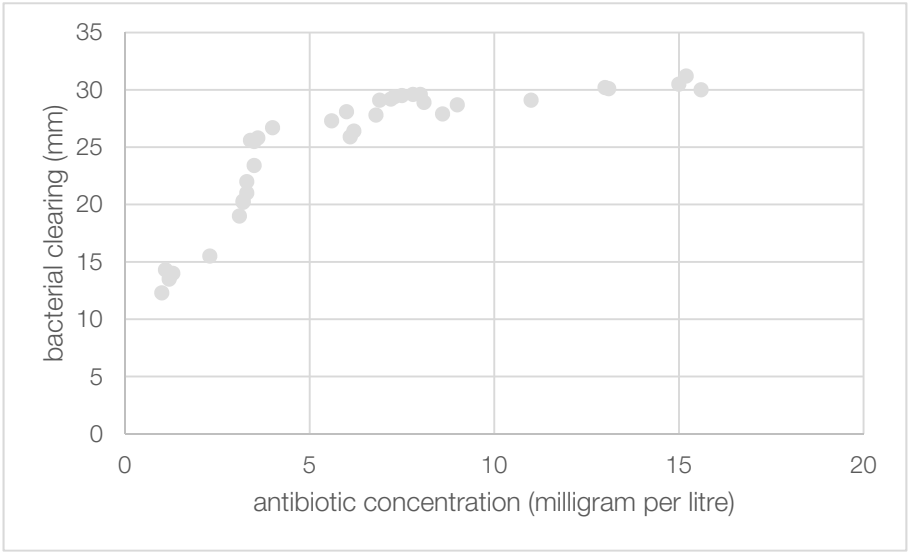
e. A potential outlier (circled in red) has a coordinate of (9.2,65). However, as later investigation finds, the point is a result of a mistake and changed into (9.2, 95).
Describe and explain what happens to the gradient of the least squares regression line after this change.

2 marks

Total: 6 marks

Question 4

The scatterplot and table below show the *bacterial clearing* (measured in millimetres) and the antibiotic concentration used (measured in milligram per litre), for a sample of 35 concentrations.



Antibiotic concentration	Bacterial clearing
1	12.3
1.1	14.3
1.2	13.5
1.3	14
2.3	15.5
3.1	19
3.2	20.2
3.2	20.3
3.3	22
3.3	21
3.4	25.6
3.5	25.5
3.5	23.4
3.6	25.8
4	26.7
5.6	27.3
6	28.1
6.1	25.9
6.2	26.4
6.8	27.8
6.9	29.1
7.2	29.2
7.3	29.4
7.5	29.5
7.8	29.6
8	29.6
8.1	28.9
8.6	27.9
9	28.7
11	29.1
13	30.2
13.1	30.1
15	30.5
15.2	31.2
15.6	30

In this data set, the relationship between the two variables are non-linear and hence, the data will need to be transformed.

Analysts are considering using a log transformation on the explanatory variable.

a. Other than a **log transformation** to the explanatory variable, suggest two other transformations that can be done to linearise the scatterplot.

1 mark

b. The r^2 value of the data transformed by the log transformation applied to the explanatory variable is 0.8845.
By finding the r^2 value of the data transformed by **one** of the transformations suggested in part **a**, explain whether your suggested transformation produces a better linearised data set.

2 marks

Total: 3 marks

Question 5

Patrick receives back his scores for the English and Science tests. His scores are shown below, together with the class mean and standard deviations for each subject.

	Patrick's score	Class average	Class standard deviation
English	76	70	3
Science	67	50	9

a. Using specific statistics to support your answer, in which subject do students' performance show more variability?

1 mark

b. By calculating the respective *z*-scores, comment on which subject Patrick does better in.

2 marks

Total: 3 marks

Question 6

Investigation of stock price trend of a company across the seasons from 2010 to 2012 enables analysts to work out a least squares regression line that represents the relationship between **deseasonalised** *stock price* and *time*.

$$deseasonalised\ stock\ price = 1.21 \times time + 14.35$$

The *time* variable in this equation has been coded using Summer 2010 = 0, Autumn 2010 = 1, and so on.

a. Given that the seasonal index for Winter is 0.84, predict the **actual** stock price of the company during Winter 2012. Give your answer correct to 2 decimal places.

2 marks

b. Give the time code for Spring 2020. Explain whether using the above equation to predict the stock price for the company will be reliable.

1 mark

Total: 3 marks

Recursion and financial modelling

Question 7

Jacob buys a car on a buy-back scheme. Under this scheme, Jacob can opt to sell his car back to the dealer through either flat rate depreciation or unit cost depreciation.

a. Under flat rate depreciation, the value of Jacobs's car, in dollars, after n years, V_n, can be modelled using the recurrence relation below.
$V_n = V_{n-1} - 6000, V_0 = 36000$

i. Write the general rule to find the value of V_n in terms of n.

1 mark

ii. Hence, or otherwise, find the time it takes for Jacob's car to depreciate to half its original value.

1 mark

b. The unit cost method can also be used to depreciate the value of Jacob's car. Through this method, Jacob's car depreciates by \$0.75 per kilometre travelled.

i. How many kilometres does the car have to have travelled for it to be depreciated to half its original value?

1 mark

ii. Jacob travels, on average, 7500 kilometres per year. Explain whether he should opt for the buy-back scheme under flat rate or unit cost depreciation?

1 mark

Total: 4 marks

Question 8

Anna is considering investing an amount of $1000. She has an option of depositing it in an interest savings account or an annuity for 3 three years

a. The savings account offers an annual interest rate of 4%, compounding monthly.

i. Calculate the effective annual interest rate of this account. Give your answer correct to 2 decimal places.

1 mark

ii. Calculate the amount of interest Anna will have earned by the end of the third year. Give your answer correct to 2 decimal places.

2 marks

b. The other investment strategy is to deposit the money in an annuity, at an annual interest rate of 4% (compounding every 6 months), where Anna would have to withdraw $80 at the end of each 6 months after the interest has been compounded.
The amortisation table below represents the details of this annuity.

Withdrawal number	Withdrawn amount	Interest earned	Principal reduction	Balance of annuity
0	0	0	0	1000
1	80	20	60	940
2	80	18.8	61.2	878.8
3	80	17.58	62.42	
4	80	16.33	63.67	752.71
5	80		64.95	687.76
6	80	13.76	66.24	621.52
7	80	12.43	67.57	553.95
8	80	11.08	68.92	485.03
9	80	9.7		414.73
10	80	8.29	71.71	343.02
11	80	6.86	73.14	269.88
12	80	5.4	74.6	195.28

i. The shaded boxes are missing numbers. Complete theses boxes. Write your answer correct to 2 decimal places.

3 marks

ii. Calculate the total amount of interest Anna would earn by the end of the 3rd year using this strategy. And hence, explain which strategy Anna should use to invest her money.

2 marks

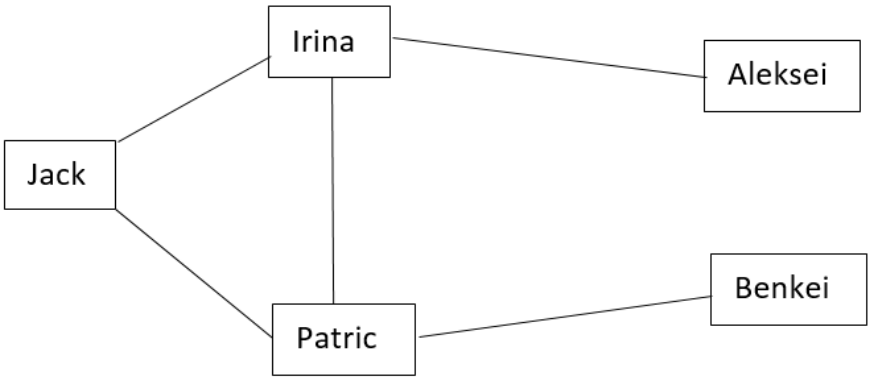
Total: 8 marks

Section B – Modules

Module 1 – Matrices

Question 1

The ability to communicate with each other within a specific group of five people speak a variety of languages is summarised in the diagram below.



a. Construct a matrix, L, representing thus situation.

1 mark

b. Calculate $L + L^2$, and hence, determine which pair of people would require more than one intermediate to communicate with each other?

2 marks

Total: 3 marks

Question 2

A coffee brand has three shops A, B and C in a city. Regular customers will either stay at their shops, change to another shop (A, B or C), or switch completely to another brand (O). Once they do switch to another brand, they do not go back.

Originally, there are 1000 customers going to shop A, 600 to shop B and 500 to shop C.

From day to day, this situation can be represented by the transition matrix T below

$$T = \begin{array}{c} \\ A \\ B \\ C \\ O \end{array} \begin{array}{c} \begin{array}{cccc} \text{Today} & & & \\ A & B & C & O \end{array} \\ \begin{bmatrix} 0.4 & 0.2 & 0.15 & 0 \\ 0.3 & 0.6 & 0.15 & 0 \\ 0.2 & 0.15 & 0.6 & 0 \\ 0.1 & 0.05 & 0.1 & 1 \end{bmatrix} \end{array} \begin{array}{l} A \\ B \\ C \\ O \end{array} \quad \text{Tomorrow}$$

a. Use the information in the transition matrix T to

i. Determine the number of customers who will leave shop A on the next day.

1 mark

ii. Determine the number of customers who will leave the brand permanently on the next day.

1 mark

b. The initial state matrix is denoted as S_1, and S_n represents the state matrix describing the customer numbers of the shops on the n^{th} days. Using the rule $S_n = T \times S_{n-1}$, determine

i. S_3.

1 mark

ii. The number of customers who go the shop B on the 4th day.

1 mark

c. The transition matrix T predicts that, in long term, all of the customers of the brand will be lost to different brand.

i. On which day will all of the original customers have switched to another brand (that is when the number of customers in every shop first becomes smaller than zero)?

1 mark

ii. What is the largest number of customers that is predicted to go to shop B on any day?

1 mark

d. The coffee brand decides to attract more customers through marketing while also rearranging them around the shops. How many people, if added to or removed from the three shops each day, will ensure that the number of customers in each shop does not change from day to day.

2 marks

e. The rule $S_n = T \times S_{n-1}$ does not take into account the fact that every day, half of the customers going to each shop will introduce and attract **one** of their friends to their respective shop every day.
The new rule can be rewritten as

$$S_n = T \times S_{n-1} + M \times S_n$$

Given that M is a 4 x 4 matrix, determine it.

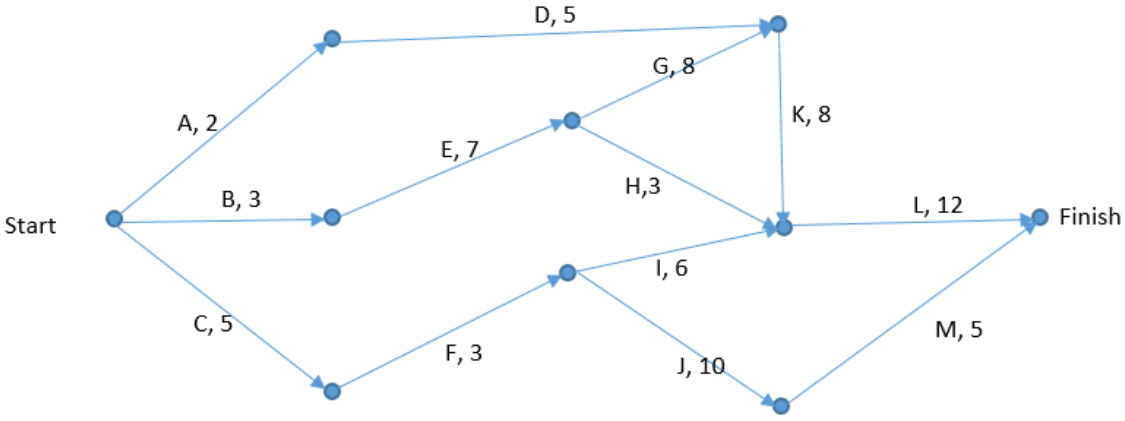
1 mark

Total: 9 marks

Module 2 – Networks and decision mathematics

Question 1

A group project involves 13 activities (A-M). The order of activities and amount of time needed to finish each task (in hours) are summarised in the network as below.



The information regarding each activity is summarised in the table below

Activity	**Earliest starting time**	**Latest starting time**
A	0	11
B	0	0
C	0	12
D	2	
E	3	3
F	5	17
G		10
H	10	23
I	8	
J	8	23
K	18	
L		26
M	18	

a. Complete the table by filling in the missing information in the shaded boxes.

3 marks

b. Determine the minimum time of completion for the project.

1 mark

c. Determine the critical path.

1 mark

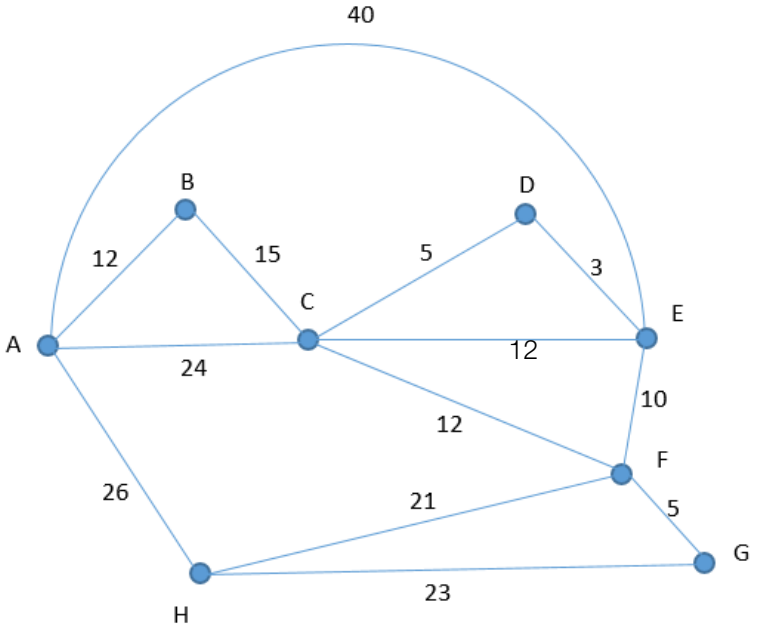
d. Reduction of completion time costs \$50 per hour for activity B, and \$65 per hour for activity L. Reduction of completion time of the whole project results in an extra \$70 per hour. Given that the maximum reduction time for activities B and L is 2 hours, determine the maximum extra earning from the project.

1 mark

Total: 6 marks

Question 2

The train system connecting 8 cities (A-H) and the distance each train travels, in kilometres, are summarised in the network below.



a. Explain why an Eulerian trail is present in this network.

1 mark

b. Given that both a Hamiltonian path and an Eulerian trail are present in the network.

i. Write down the order of vertices that make up the Eulerian trail.

1 mark

ii. Write down the order of vertices that make up the Hamiltonian path.

1 mark

c. By using the Dijkstra algorithm, or otherwise, determine the shortest path to go from A to G.

1 mark

Total: 4 marks

Question 3

A charity sets out to provide a clean water system for each of the four poor communities, A, B, C and D. Four companies, M, N, O and P offer to build the system for them. The cost in dollars building one system from each company for each community is summarised the table below.

Community	A	B	C	D
M	1400	1900	1800	1100
N	1600	1300	1300	2000
O	1700	1500	1800	1600
P	1750	1900	1200	1800

Using the Hungarian algorithm, or otherwise, complete the following table summarising the assignment of the companies to the communities so as to minimise the fund needed for the project.

Company	Community
M	
N	
O	
P	

2 marks

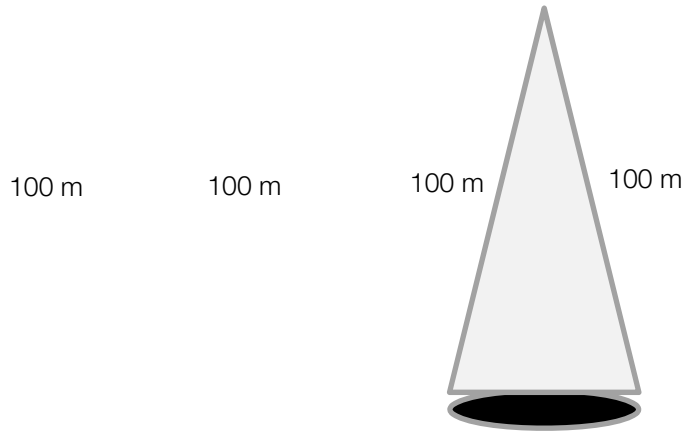
Module 3 – Geometry and measurement

Question 1

A group of architects are planning to build an artistic sculpture made out of marble and metals.

Its overall shape is of a cone (in lighter colour), and its base is partly spherical (on darker shade).

The sculpture cross-section at its centre (to right) has the shape of a circle sector.



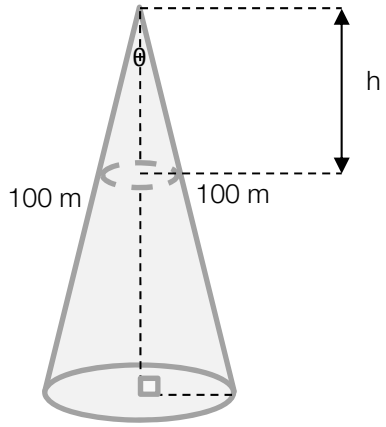
a. The architects wish to make the sculpture so that its central cross-section is one twelfth of the full circle. How large should angle be for this to be achieved? Write your answer correct to the nearest degree.

1 mark

b. Consider the cross-section on the right. Calculate the area of the segment shaded with the darker colour. Round your answer to nearest square metre.

2 marks

c. Now consider the cone-shaped portion of the sculpture.



i. Calculate the angle . Give your answer correct to the nearest degree.

1 mark

ii. Hence, or otherwise, calculate the volume of the cone-shaped portion of the sculpture. Give your answers correct to 2 decimal places.

2 marks

iii. The top portion of the cone, with length h m, is going to be made of marble. The architect would like 30% of the volume of the cone to be made out of marble. Calculate h for this to happen. Give your answer correct to the nearest metre.

2 marks

d. The architects would like to build the sculpture at a location so that it can be seen clearly from the observatory. The observatory is currently 250 m away on a bearing of 65° from a monument. It is planned that the sculpture will be built on a bearing of 20° from the observatory. What should be the bearing and the distance of the sculpture from the monument to ensure that the monument, the observatory and the sculpture, together make an isosceles triangle? Write your answers correct to 1 decimal place.

2 marks

e. The architects are collaborating with their colleagues abroad to work on this sculpture. The cities they are in has the coordinates, respectively, of (43°N,26°W) and (43°N,15°E). Determine

i. The distance between them. Give your answer correct to the nearest kilometre.

1 mark

ii. The time difference between them. Given your answer correct to the nearest minute.

1 mark

Total: 12 marks

Module 4 – Graphs and relations

Question 1

Graham, a scientist, is attempting to treat his mice of a bacterial infection by combining the two mixtures of antibiotics, A and B, into a medicine. The amount of amoxicillin (*amox*) and azithromycin (*azi*) in each millimetre of A and B is shown in the table below.

	1 mL of A	1 mL of B
Amox	3.5 mg	3.5 mg
Azi	1 mg	4 mg

a. How many milligrams of amox are there in 20 mL of A?

1 mark

b. Give a combination of mixtures, i.e. the volume of each mixture, that will give 10 mg of amox.

1 mark

Let x be the number of millilitres of A in Graham's medicine.
Let y be the number of millilitres of B in Graham's medicine.

Inequalities 1 to 4 represent the amox and azi requirements for Graham's mice.

Inequality 1 $x \geq 0$

Inequality 2 $y \geq 0$

Inequality 3 $3.5x + 3.5y \geq 140$

Inequality 4 $x + 4y \geq 100$

In order for the antibiotics to be absorbed effectively, at least 150 mg of chemical X has to be present in the medicine as well.

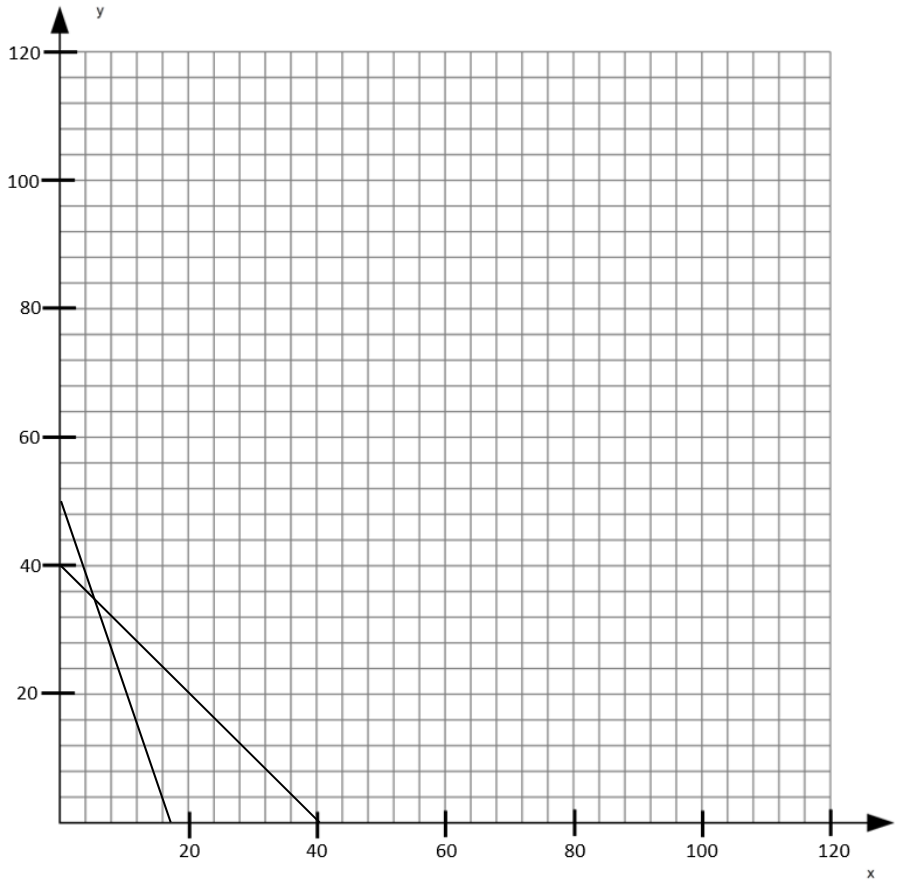
Each millilitre of A contains 9 mg of chemical X.

Each millilitre of B contains 3 mg of chemical X.

c. Inequality 5 represents the requirement of chemical X for Graham's mice.
Write down Inequality 5 in terms of x and y.

1 mark

The lines that represent the boundaries of **some** of the inequalities, including Inequalities 1 and 2, are shown in the graph below



d. Using the information given from the graph above
i. Draw the boundary for the missing Inequality. And hence, write down the line's equation.

1 mark

ii. On the **graph above**, shade the region that satisfies inequalities 1 to 5.

1 mark

Graham would like to use the least amount of his medicine to treat the mice effectively.

e.

i. How many millilitres of his own medicine will Graham need to make?

2 marks

ii. On the **graph above**, show the point(s) where this solution occurs?

1 mark

Total: 8 marks

Question 2

Alex is considering signing up for an internet data plan. He is thinking between two companies, DataNet and DeltaCom.

DataNet charges him \$1 per gigabyte (GB) used.

DeltaCom has a more complicated plan, in which they charge \$30 upfront for a 40 GB allowance. As usage exceeds 40 GB and goes up to 80, they charge \$1.50 per GB in excess of 40. As usage exceeds 80 GB, a charge of \$0.5 per GB in excess of 80 GB is applied.

a. If Alex uses 20 GB in a month, what is the cost difference between DataNet and DeltaCom. Which company is cheaper?

1 mark

b. The cost, in dollars, that DeltaCom charges for their data x GB can be given by

$$\$D = \begin{cases} 30 & 0 < x \leq 40 \\ 1.5x - 30 & 40 < x \leq 80 \\ 0.5x + b & 80 < x \end{cases}$$

Show that b has the value of 50 in the equation above.

1 mark

c. Find the range(s) of data usage, for which it would be more expensive to go with DeltaCom than DataNet.

2 marks

Total: 4 marks

Formula Sheet

Core – Data analysis

standardised score	$z = \frac{x - \bar{x}}{s_x}$
lower and upper fence in a boxplot	lower $Q_1 - 1.5 \times IQR$ upper $Q_3 + 1.5 \times IQR$
least squares line of best fit	$y = a + bx$, where $b = r\frac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$
residual value	residual value = actual value – predicted value
seasonal index	seasonal index $= \frac{\text{actual figure}}{\text{deseasonalised figure}}$

Core – Recursion and financial modelling

first-order linear recurrence relation	$u_0 = a, \quad u_{n+1} = bu_n + c$
effective rate of interest for a compound interest loan or investment	$r_{effective} = \left[\left(1 + \frac{r}{100n}\right)^n - 1\right] \times 100\%$

Module 1 – Matrices

determinant of a 2 × 2 matrix	$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
inverse of a 2 × 2 matrix	$A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $\det A \neq 0$
recurrence relation	$S_0 =$ initial state, $S_{n+1} = TS_n + B$

Module 2 – Networks and decision mathematics

Euler's formula	$v + f = e + 2$

Module 3 – Geometry and measurement

area of a triangle	$A=\frac{1}{2}bc\sin(\theta^\circ)$
Heron's formula	$A=\sqrt{s(s-a)(s-b)(s-c)}$, where $s=\frac{1}{2}(a+b+c)$
sine rule	$\frac{a}{\sin(A)}=\frac{b}{\sin(B)}=\frac{c}{\sin(C)}$
cosine rule	$a^2=b^2+c^2-2bc\cos(A)$
circumference of a circle	$2\pi r$
length of an arc	$r\times\frac{\pi}{180}\times\theta^\circ$
area of a circle	πr^2
area of a sector	$\pi r^2\times\frac{\theta^\circ}{360}$
volume of a sphere	$\frac{4}{3}\pi r^3$
surface area of a sphere	$4\pi r^2$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a prism	area of base × height
volume of a pyramid	$\frac{1}{3}\times$ area of base × height

Module 4 – Graphs and relations

gradient (slope) of a straight line	$m=\frac{y_2-y_1}{x_2-x_1}$
equation of a straight line	$y=mx+c$

End of Booklet