YEAR 12 *Trial Exam Paper*

2016
FURTHER MATHEMATICS

Written examination 1

Worked solutions

This book presents:

- worked solutions, giving you a series of points to show you how to work through the questions
- mark allocations
- tips on how to approach the exam

SECTION A – Core

Data analysis

Question 1

Answer: C

Worked solution

The data is recorded as a numerical variable. Marks are allocated on a scale from 0 to 40, with scores given to the nearest whole mark.

Tips

- *Numerical discrete data is represented by numbers that are counted.*
- *Numerical continuous data is represented by numbers that are measured rather than counted.*

Question 2

Answer: D

Worked solution

A. Histogram – a histogram does not allow us to consider numerical variables that have been split into categories.

B. Segmented bar chart – a segmented bar chart allows us to compare two categorical variables.

C. Scatterplot – a scatterplot is best used for considering the relationship between two numerical variables.

D. Parallel boxplots – parallel boxplots will allow us to display numerical data for two categories, and compare the distribution for these categories.

E. Dot plot – a single dot plot does not accommodate two categorical variables.

Question 3

Answer: A

Worked solution

In order to include an outlier in our description of this data distribution, we need to calculate the lower fence.

Lower fence:

$Q_1 - 1.5 \times IQR$

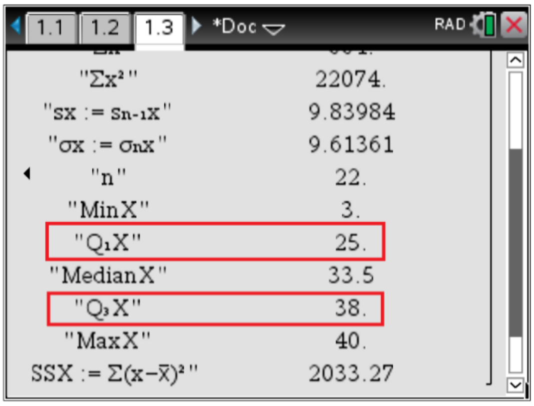
$= 25 - 1.5 \times 13$

$= 5.5$

As the test score 3 is <u>less than</u> the lower fence, it can be considered an outlier.

Tip

- *Often it is easier to use your calculator to get a statistical summary, then calculate the IQR and upper/lower fence.*



Question 4

Answer: D

Worked solution

A. Option A is incorrect because, using quartiles to summarise numerical data, 75% of the data is below the third quartile. For this distribution, Q_3 is equal to 38.

B. Option B is incorrect because the mean value for the distribution is 30.1818. Rounded to the nearest whole number, this is 30

C. Option C is incorrect because, for a negatively skewed distribution, the mean is less than the median. For this distribution, the mean is 30 and the median is 34 (to the nearest whole number).

D. Option D is correct because it is not appropriate to use the median as a measure of centre when a distribution is skewed or contains an outlier. The distribution for *Further Mathematics test scores* is both skewed and contains an outlier.

E. Option E is incorrect because it would also be appropriate to display the data for *Further Mathematics test scores* as a dot plot.

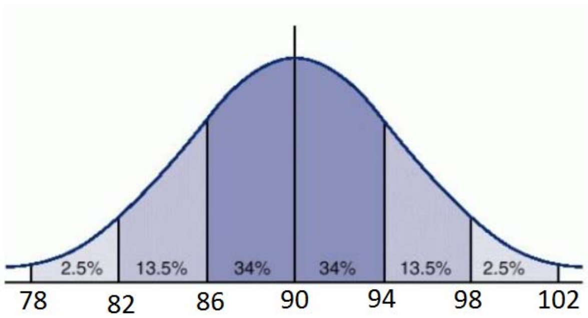
Question 5

Answer: E

Worked solution

The data is approximately normally distributed, we are therefore considering a bell-shaped distribution.

A diagram is very useful for calculating our answer.



In this question we are asked to consider students between 82 and 94.

We first use the bell curve to calculate the percentage.

Percentage between 82 and 94 = 13.5 + 34 + 34 = 81.5%

We now calculate 81.5% of 284 students.

Number of students = $\frac{81.5}{100} \times 284 = 231.46$

Rounded to the nearest whole number, this is 231 students.

Question 6

Answer: A

Worked solution

A. Option A is correct because the range for A is 37, and the range for B is 33.

*Hint: The range includes the outlier.

B. Option B is incorrect because the boxplot for 12A is negatively skewed.

C. Option C is incorrect because the median for 12A is higher than that for 12B.

D. Option D is incorrect because:

Q3 for 12A is it a 38, Q3 for 12B is at 32.

Therefore, the top 25% of scores for 12A are higher than for 12B.

E. Option E is incorrect because the outlier at 3 is the minimum value for 12A, this is lower than the minimum for 12B.

Question 7

Answer: B

Worked solution

In this case, the explanatory variable is *Further Mathematics test score*. The response variable is *Science test score*.

With the information given, we can use the following formulae to calculate the slope, *b*, and *y*-intercept, *a*, for our regression equation:

$$b = \frac{r \times s_y}{s_x}$$

$$b = \frac{0.82 \times 8.71}{9.84}$$

$$b = 0.73$$

$$a = \bar{y} - b\bar{x}$$

$$a = 32.00 - 0.73 \times 30.18$$

$$a = 9.97$$

where, *r* is the correlation coefficient

s_y is the standard deviation for *y*.

s_x is the standard deviation for *x*.

$\bar{x}$ is the mean for *x*.

$\bar{y}$ is the mean for *y*.

Therefore, our regression equation is:

$Science = 9.97 + 0.73 \times Further\ \ Mathematics.$

Question 8

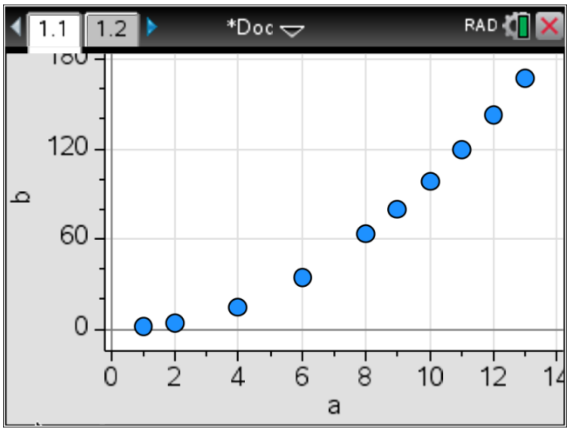
Answer: D

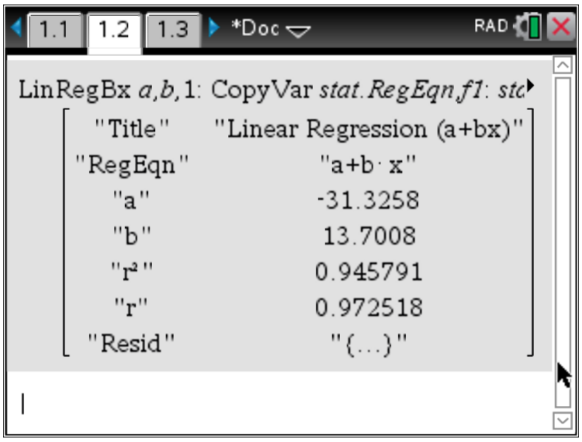
Worked solution

The value of Pearson's correlation coefficient is a positive value, therefore suggesting a positive relationship. Its value is 0.9505, therefore the relationship is strong.

Tips

- *With an r value this high, a non-linear relationship would be obvious in the graph.*
- *For example, it may appear to be a curve, with points close together, as shown below:*





This graph has a high Pearson's correlation coefficient, similar to that in our question. Due to this high value, the strong non-linear relationship is obvious.

Question 9

Answer: B

Worked solution

Ensure that you enter *Test 1* as your explanatory variable, x.

Choose 2 points on the graph:

For example, (2, 12) and (30, 32)

Find the slope using $\frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$.

Slope = $\frac{32-12}{30-2} = \frac{20}{28} = 0.71$

Use the general formula for a linear equation: $y - y_1 = m(x - x_1)$

Equation is:

$$y - 12 = 0.71(x - 2)$$
$$y - 12 = 0.71x - 1.42$$
$$y = 0.71x + 10.58$$

The equation is: $y = 10.58 + 0.71x$

We need to make sure that we write this using our correct variables, however.

$$Test\ 2 = 10.58 + 0.71 \times Test\ 1$$

Question 10

Answer: B

Worked solution

A. Option A is incorrect because the slope for the regression equation is positive; therefore, *Test 2* scores will increase by the slope as *Test 1* scores increase by one.

B. Option B is correct because the value of the coefficient of determination is 0.903. This is therefore a correct interpretation of our r^2 value.

C. Option C is incorrect because we have no information to suggest this outcome. As we do not have a data value below 3 for *Test 1*, this is extrapolating. Also, we would look at the y-intercept to obtain information here.

D. Option D is incorrect because we can use a regression analysis to look at correlation between two variables only. We need to be very careful that we do not use the word 'cause'.

E. Option E is incorrect. Because the data appears to form a linear relationship, there is no need to perform a data transformation.

Question 11

Answer: E

Worked solution

By matching points on our original scatterplot with those predicted by the linear regression line, we can eliminate options, as the residual values do not match.

Question 12

Answer: C

Worked solution

By matching points to the 'circle of transformations', we can see that the most appropriate options are those listed in option C.

Question 13

Answer: C

Worked solution

Year	2010	2011	2012	2013	2014	2015
Average test score	32	18	22	26	35	25

$$\frac{22+26}{2}=24 \qquad \frac{26+35}{2}=30.5$$

$$\frac{24+30.5}{2}=27.25$$

Question 14

Answer: C

Worked solution

As there are 11 months of data recorded, the seasonal indices must add to 11.

We therefore subtract our known seasonal indices from 11:

$$0.65+0.75+0.76+0.83+1.45+1.5+1.45+0.84+0.72+0.73=9.68$$

$$11-9.68=1.32$$

Question 15

Answer: B

Worked solution

Substitute the predicted sales figure for March 2016 into the equation:

$1250+1.37\times 4200=7004$

Multiply by the seasonal index for March, 0.76, to get the actual sales for March:

$7004\times 0.76=5323$

Round to the nearest hundred:

$=5300$

Question 16

Answer: C

Worked solution

We wish to reduce the seasonal index for June from 1.5 to 1.0.

Therefore we need to reduce the index by 1.5 – 1.0 = 0.5

$$\frac{0.5}{1.5} \times 100 = 33.33\%$$

We therefore wish to reduce the index by 33%.

Recursion and financial modelling

Question 17

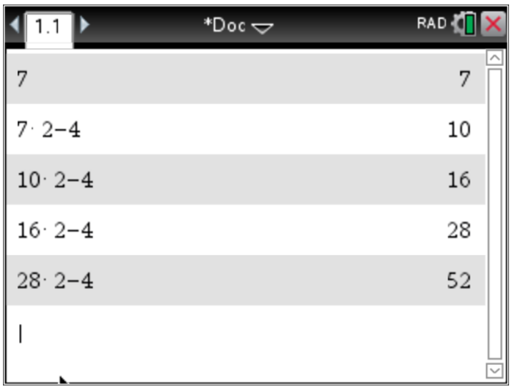
Answer: C

Worked solution

We can calculate iterations by hand, using the recurrence relation.

$$\begin{array}{lllll} & V_1 = 2 \times V_0 - 4 & V_2 = 2 \times V_1 - 4 & V_3 = 2 \times V_2 - 4 & V_4 = 2 \times V_3 - 4 \\ V_0 = 7 & V_1 = 2 \times 7 - 4 & V_2 = 2 \times 10 - 4 & V_3 = 2 \times 16 - 4 & V_4 = 2 \times 28 - 4 \\ & V_1 = 10 & V_2 = 16 & V_3 = 28 & V_4 = 52 \end{array}$$

You can use your calculator to quickly define a recurrence relation, and find iterations.



Tips

- *Remember that for a recurrence relation, you must use the previous term to find your next term.*
- *For example, we use V_2 to find V_3.*
- *Do not keep using V_0 to find each term.*

Question 18

Answer: D

Worked solution

Linear growth or decay will be represented by a straight line graph. This means we could use either option D or E. As we are modelling linear decay, the graph should be displayed as having a negative slope; option D is our answer.

Options A and B are both curved, therefore representations of geometric growth and decay.

Option C is an example of payments made at irregular intervals, and the payment is changing. This is also clearly a growth, rather than decay.

Question 19

Answer: D

Worked solution

The general model for representing flat rate depreciation is:

V_0 = initial value of the asset, $V_{n+1} = V_n - D$, where $D = \frac{r}{100} \times V_0$.

In this scenario, we have the following:

$V_0 = 32000,\ r = 20$

$$D = \frac{20}{100} \times 32000 = 6400$$

Therefore, our recurrence relation is: $V_0 = 32000$, $V_{n+1} = V_n - 6400$

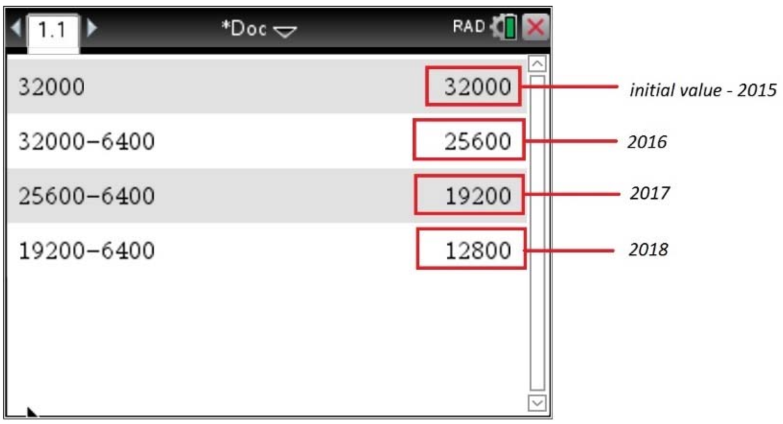
This is option D.

Question 20

Answer: C

Worked solution

Using our recurrence relation (we will show iterations on the calculator)



Question 21

Answer: B

Worked solution

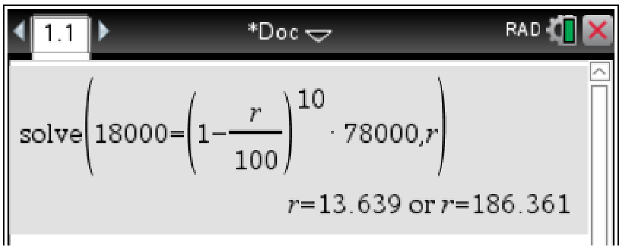
We start by defining the terms in the rule for reducing-balance depreciation.

$$V_0 = 78000$$
$$V_n = 18000$$
$$R = 1 - \frac{r}{100},$$
$$n = 10$$

$$V_n = \left(1 - \frac{r}{100}\right)^n \times V_0$$

$$18000 = \left(1 - \frac{r}{100}\right)^{10} \times 78000$$

We can solve this on our calculator:



Tips

- *The calculator has given us two possible answers.*
- *A percentage yearly rate of 186% would reduce our value by a large amount over the first year:*

$$\left(1 - \frac{186}{100}\right)^{10} \times 78000 = 17261.5$$

- *This is already below the value given to us for 10 years of use.*
- *Therefore, we must discount this answer as being impossible.*

Question 22

Answer: D

Worked solution

The interest is calculated from the balance of the loan after the last payment.

Therefore: Interest = $1224.98 \times \frac{1.5}{100} = \$18.37$

Note: In the initial information given, the interest rate is provided as an annual interest rate. However, we are calculating interest on a monthly basis. We must therefore convert our annual interest rate to a monthly rate $18\% \div 12 = 1.5\%$.

Question 23

Answer: E

Worked solution

The interest is calculated from the balance of the loan after the last payment.

Therefore: $\text{Interest} = 329.69 \times \dfrac{1.5}{100} = \$4.95$

The final payment must cover the balance owing <u>and the interest</u>.

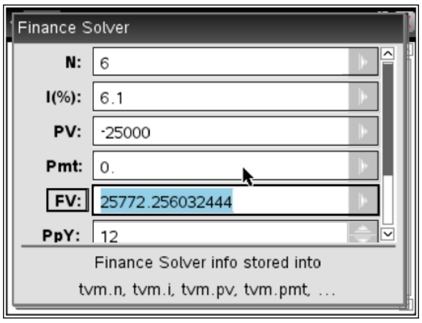
Therefore: $\text{Final payment} = 329.69 + 4.95 = \$334.64$

Question 24

Answer: C

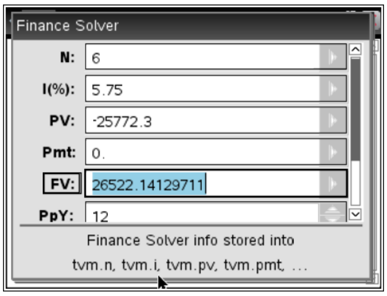
Worked solution

Start by calculating the *future value* for the investment after 6 months, at the interest rate of 6.1%.



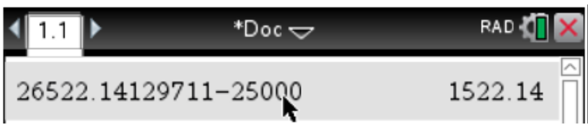
Remember to change your PpY and CpY to 12.

Next, we will change our present value to the value of the investment after 6 months, and calculate the *future value* after another 6 months. We must change our interest rate to 5.75



Remember to change this value to a negative, as you are investing this money in the bank.

Finally, calculate interest earned by subtracting the initial value, 25 000, from the *future value* after 12 months.



SECTION B – Modules

Module 1 – Matrices

Question 1

Answer: B

Worked solution

Incorrect elements in each matrix are highlighted below.

A. $\begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$

B. $\begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 2 \\ 1 & 0 & 0 & 2 & 0 \\ 1 & 1 & 2 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \end{bmatrix}$

C. $\begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$

D. $\begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$

E. $\begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 2 \\ 1 & 0 & 0 & 2 & 0 \\ 1 & 0 & 2 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 \end{bmatrix}$

Question 2

Answer: A

Worked solution

A triangular matrix will have zero elements either above or below the main diagonal.

In this case, we have zero elements above the main diagonal; this is a *lower triangular matrix*.

Question 3

Answer: E

Worked solution

We are looking for a matrix multiplication that will result in a 3 × 1 matrix.

A. $\begin{bmatrix} 25 & 18 & 32 & 8 & 24 \\ 32 & 24 & 27 & 12 & 35 \\ 42 & 19 & 32 & 17 & 39 \end{bmatrix} \times \begin{bmatrix} 13 & 14.5 & 16 & 15.5 & 18 \end{bmatrix}$	This matrix multiplication is undefined: $3 \times 5 \quad 1 \times 5$ The columns in the first matrix do not equal the rows in the second.
B. $\begin{bmatrix} 13 & 14.5 & 16 & 15.5 & 18 \end{bmatrix} \times \begin{bmatrix} 25 & 18 & 32 & 8 & 24 \\ 32 & 24 & 27 & 12 & 35 \\ 42 & 19 & 32 & 17 & 39 \end{bmatrix}$	This matrix multiplication is undefined: $1 \times 5 \quad 3 \times 5$ The columns in the first matrix do not equal the rows in the second.
C. $\begin{bmatrix} 13 \\ 14.5 \\ 16 \\ 15.5 \\ 18 \end{bmatrix} \times \begin{bmatrix} 25 & 32 & 42 \\ 18 & 24 & 19 \\ 32 & 27 & 32 \\ 8 & 12 & 17 \\ 24 & 35 & 39 \end{bmatrix}$	This matrix multiplication is undefined: $5 \times 1 \quad 5 \times 3$ The columns in the first matrix do not equal the rows in the second
D. $\begin{bmatrix} 25 & 32 & 42 \\ 18 & 24 & 19 \\ 32 & 27 & 32 \\ 8 & 12 & 17 \\ 24 & 35 & 39 \end{bmatrix} \times \begin{bmatrix} 13 \\ 14.5 \\ 16 \\ 15.5 \\ 18 \end{bmatrix}$	This matrix multiplication is undefined: $5 \times 3 \quad 5 \times 1$ The columns in the first matrix do not equal the rows in the second.
E. $\begin{bmatrix} 25 & 18 & 32 & 8 & 24 \\ 32 & 24 & 27 & 12 & 35 \\ 42 & 19 & 32 & 17 & 39 \end{bmatrix} \times \begin{bmatrix} 13 \\ 14.5 \\ 16 \\ 15.5 \\ 18 \end{bmatrix}$	This matrix multiplication is defined: $3 \times 5 \quad 5 \times 1$ The columns in the first matrix equal the rows in the second. This matrix multiplication will result in the following: $\begin{bmatrix} 1654 \\ 2012 \\ 2299 \end{bmatrix}$ giving a total for stores A, B and C.

Question 4

Answer: B

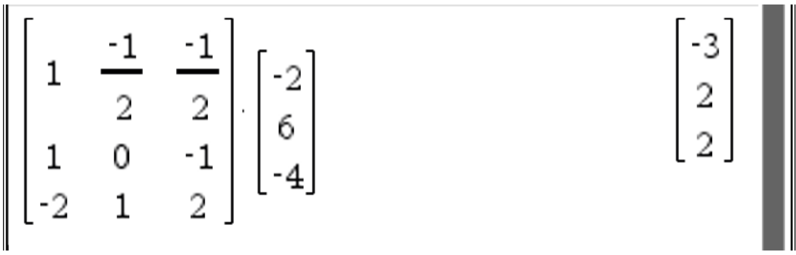
Worked solution

To solve the simultaneous equations: $AX = B$

Step 1: Find the inverse, A^{-1}.

Use your calculator for this.

Step 2: Multiply matrix *B* by the inverse of matrix *A*.



Question 5

Answer: D

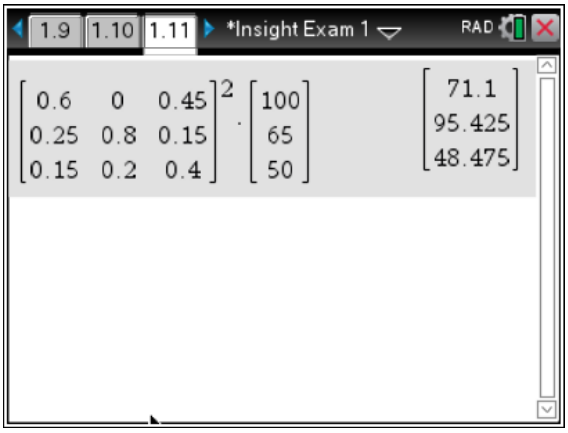
Worked solution

To find S_3, we use the transition matrix, T.

As our initial matrix is S_1, this is information for our first week. Therefore, we only require 2 transitions to find the third week.

$S_3 = T^2 \times S_1$

On the calculator:



Tips

- *Your numbers here must add up to the initial number, 215. This will require you to think carefully about your rounding.*
- *Beware of questions where the initial state matrix is S_1. Your transitions will need to be* ***one less*** *than the state matrix you require: $S_n = T^{n-1} \times S_1$.*

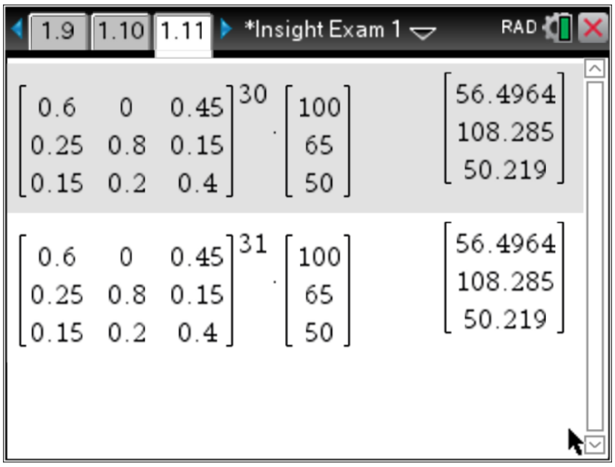
Question 6

Answer: D

Worked solution

We are looking for two consecutive matrices between which there is no change in state.

This can be seen below for S_{30} and S_{31}.



Question 7

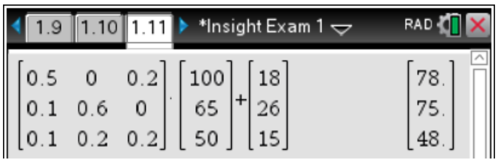
Answer: C

Worked solution

For a recurrence relation of this form, we must work out S_2 first, then use this in the equation to find S_3.

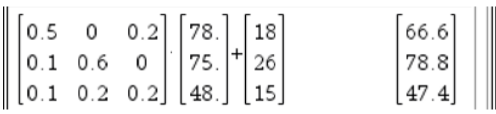
Step 1: Find S_2.

$S_2 = S_1 \times T + B$



Step 2: Find S_3.

$S_3 = S_2 \times T + B$



Question 8

Answer: C

Worked solution

The two-step dominance matrix can be found by *squaring* the one-step dominance matrix.

One-step dominance is found when a student has a direct link to another, with the arrow facing towards their opponent. For example, student A defeated student B. Therefore a ‘1’ is placed in the matrix.

The one-step dominance matrix is shown below:

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} \text{A} & \text{B} & \text{C} & \text{D} & \text{E} \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

We can square this to find the two-step dominances, as shown on the calculator page below.

$$\begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}^2 \qquad \begin{bmatrix} 0 & 1 & 0 & 2 & 2 \\ 2 & 0 & 0 & 1 & 0 \\ 2 & 0 & 0 & 2 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

Module 2 – Networks and decision mathematics

Question 1

Answer: E

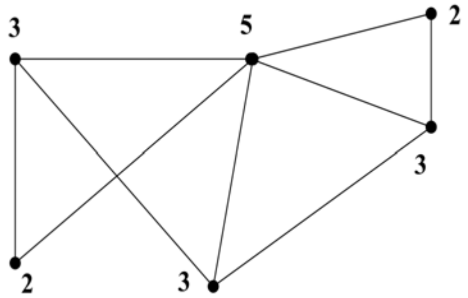
Question 2

Answer: D

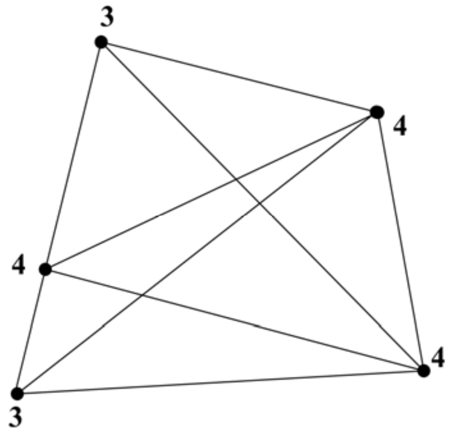
Worked solution

For a network to contain a Euler trail, the network must contain exactly 2 vertices with odd degree, with the rest of the vertices having an even degree.

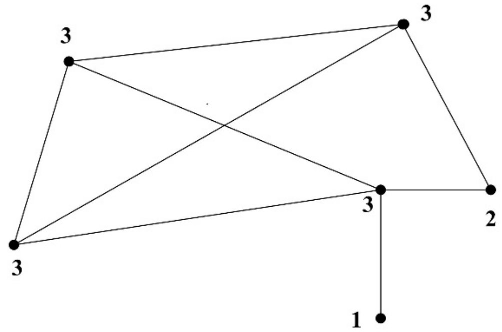
Network A has 2 vertices with even degree, and the rest are odd – this does not satisfy the conditions for an Eulerian trail.



Network B has 2 vertices with odd degree, and the rest are even – this satisfies the conditions for an Eulerian trail.



Network C: has 5 vertices with odd degree, and only one even – this does not satisfy conditions for Euler trail.

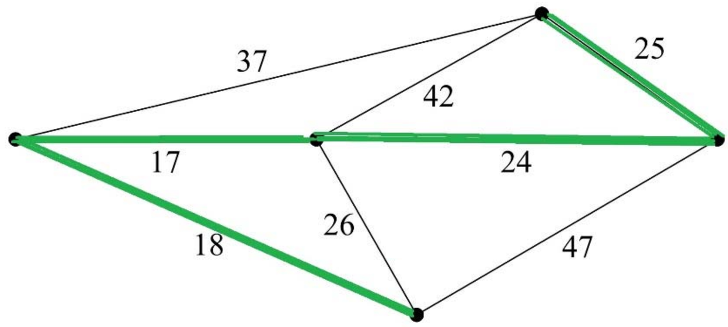


Question 3

Answer: A

Worked solution

The minimum spanning tree is shown below:



The length of this spanning tree is 17 + 18 + 24 + 25 = 84

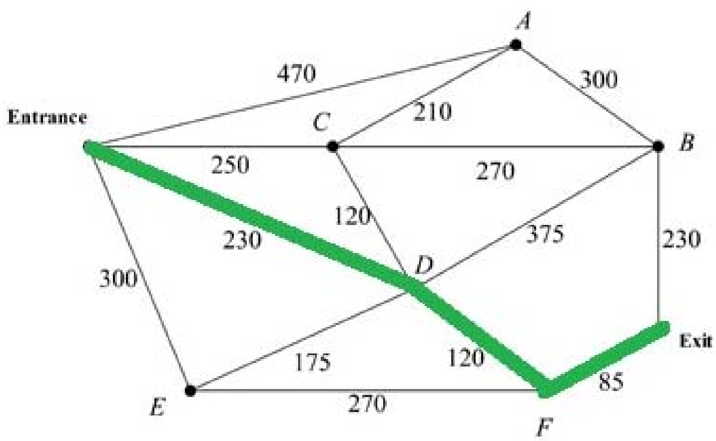
Question 4

Answer: A

Worked solution

The shortest path can be found using one of two methods:

Method 1: By inspection



Method 2: Using Dijkstra's algorithm

	A	B	C	D	E	F	Exit
Ent	470	×	250	$\boxed{230}$	300	×	×
		230 + 375 = 605	230 + 120 = 450		230 + 175 = 405	230 + 120 = 350	
D	$\boxed{230}$	605	$\boxed{250}$	$\boxed{230}$	300	350	×
C	$\boxed{230}$	$\boxed{250}$	$\boxed{250}$	$\boxed{230}$	$\boxed{300}$	350	×
						300 + 270 = 570	
E	$\boxed{230}$	$\boxed{250}$	$\boxed{250}$	$\boxed{230}$	$\boxed{300}$	$\boxed{350}$	×
							350 + 28 = 435
F	$\boxed{230}$	$\boxed{250}$	$\boxed{250}$	$\boxed{230}$	$\boxed{300}$	$\boxed{350}$	$\boxed{435}$

Explanatory notes

1. Create a table with the starting vertex (entrance) as the first row vertex.
2. Write the other vertices as column vertices.
3. Write down the distance from entrance to all other vertices.
4. Put a box around the smallest number in the row.
5. Look at the column vertex for this box number; this becomes the next row vertex.
6. Continue process until the destination is reached.
7. Backtrack from the smallest number in row F, to the destination.

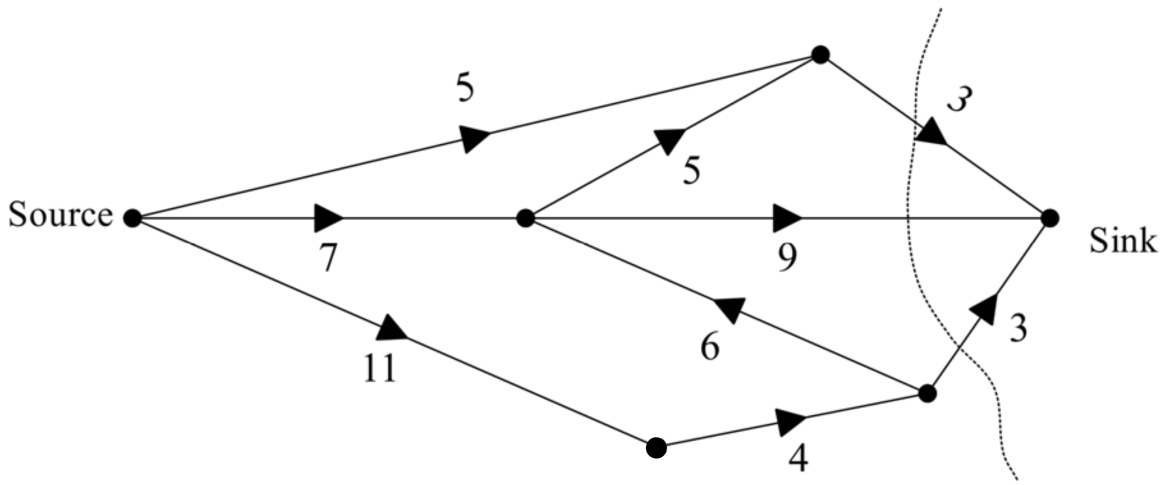
Question 5

Answer: D

Worked solution

Maximum flow = minimum cut

= 3 + 9 + 3 = 15



Question 6

Answer: E

Worked solution

Use the Hungarian algorithm.

Step 1: row reduction

Subtract the smallest number from each row, from every number in the same row.

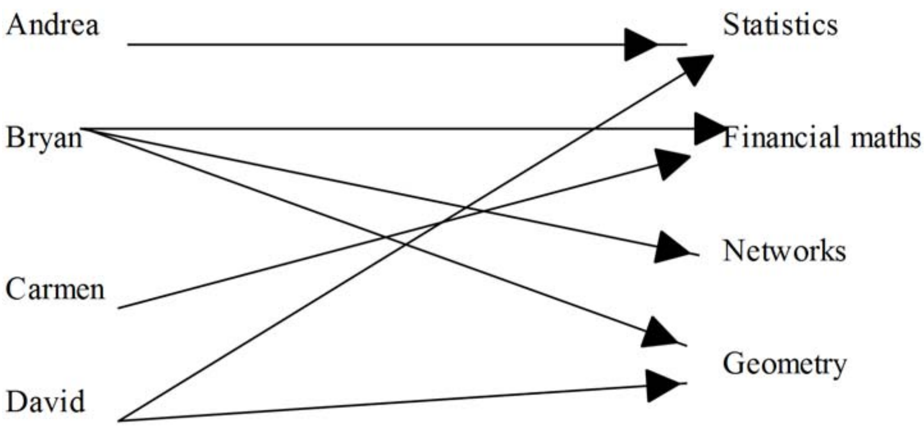
Student	Mathematics task to complete			
	Statistics	Financial maths	Networks	Geometry
Andrea	0	2	9	6
Bryan	1	0	3	2
Carmen	4	0	7	6
David	0	2	7	2

Step 2: column reduction

Subtract the smallest number from any column that does not already contain a 0 from each number in the column.

Student	Mathematics task to complete			
	Statistics	Financial maths	Networks	Geometry
Andrea	0	2	6	4
Bryan	1	0	0	0
Carmen	4	0	4	4
David	0	2	4	0

Step 3: If the minimum number of lines required to cover all the zeros in the table is equal to the number of allocation to be made, draw a **directed bipartite graph**, with an edge for every zero value in the table



Step 4: Use the bipartite graph to make allocations.

Andrea only has one possible task – **Statistics**.

David now only has **Geometry** remaining.

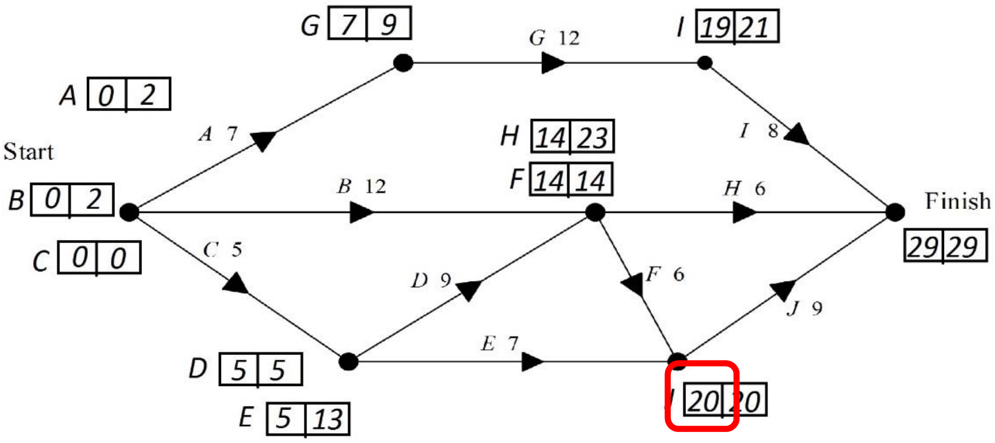
Carmen only had one possible task – **Financial maths**.

Therefore, **Bryan** will do the remaining task – **Networks**.

Question 7

Answer: D

Worked solution

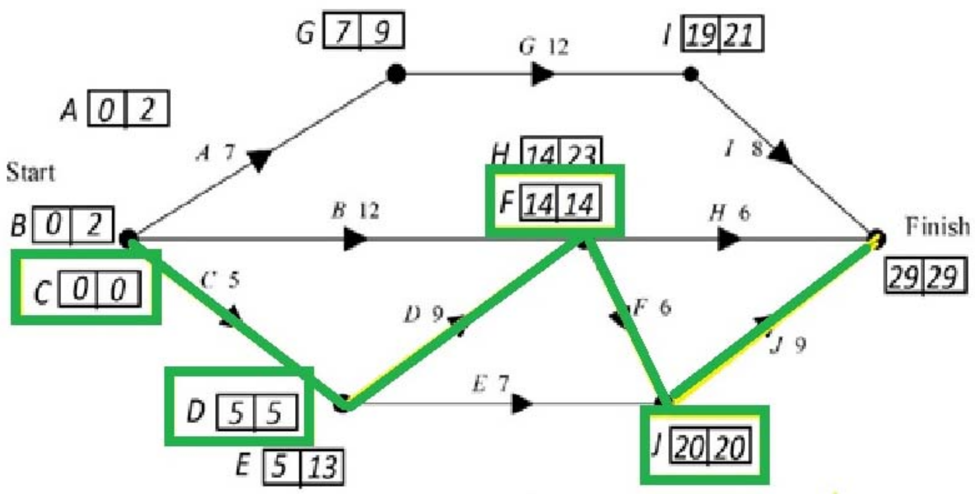


Question 8

Answer: B

Worked solution

The critical path is made up of those activities that have no float/slack time. These are our critical activities, which must be completed on time, in order for the project to be completed in the minimum time.



Module 3 – Geometry and measurement

Question 1

Answer: C

Worked solution

As the sails are similar triangles, we can calculate the scale ratio for the triangles.

$$\frac{2}{1.5} = 1.333333$$

The area of the smaller triangle is equal to 1.65 m^2. We can multiply this by square of the scale ratio in order to calculate the area of the larger triangle.

$$\text{Area larger triangle} = \left(\frac{2}{1.5}\right)^2 \times 1.65 = 2.93\ \text{m}^2$$

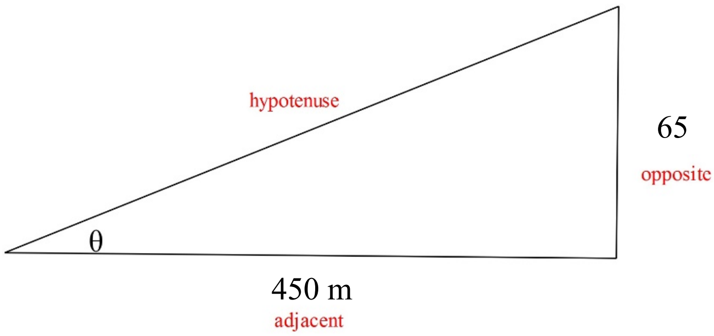
We must remember to square the scale ratio as we are calculating area, which is in units squared.

Question 2

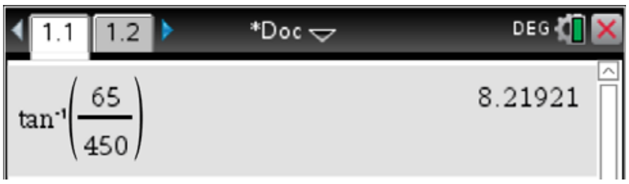
Answer: B

Worked solution

We start by labelling the sides of the triangle, in relation to the angle we are trying to find:



We have been given the numerical value of the 'opposite' side and the 'adjacent' side, and we therefore use $\tan\theta = \dfrac{\text{opp}}{\text{adj}}$



Tips

- *Your calculator settings must be in **Degrees** for this calculation.*
- *If your calculator is in **Radians**, you have 2 possible options:*
  - *Change your settings to degrees*
  - *Convert your answer to degrees, using the appropriate function in your CAS calculator.*

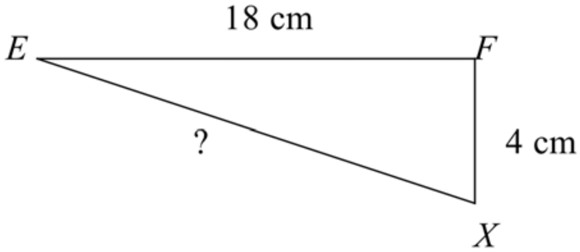
Question 3

Answer: C

Worked solution

Step 1: Calculate the length of the base of the triangle created by the rod.

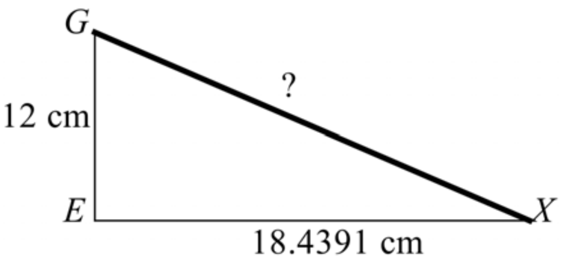
Find the length of the diagonal within the rectangular base:



Pythagoras' theorem: $c^2 = a^2 + b^2$

We are trying to find *c*: $c = \sqrt{a^2 + b^2} = \sqrt{18^2 + 4^2} = 18.4391$

Step 2: Find the length of the rod, by creating a vertical triangle within the box.



We are trying to find *c*: $c = \sqrt{a^2 + b^2} = \sqrt{12^2 + 18.4391^2} = 22\text{ cm}$

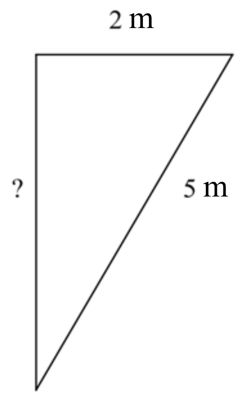
Question 4

Answer: D

Worked solution

Step 1: Calculate the height of the trapezium, using Pythagoras' theorem.

The base of the triangle is: $\dfrac{8-4}{2}=2$



We are trying to find *b*: $b=\sqrt{c^2-a^2}=\sqrt{5^2-2^2}=4.58$

Step 2: Calculate the TSA by calculating area of individual shapes.

The trapezium

$$\text{area}=\frac{1}{2}\times(a+b)\times h$$
$$\text{area}=\frac{1}{2}\times(8+4)\times 4.58$$
$$\text{area}=27.48$$

We have 2 trapeziums: $\text{area}=2\times 27.48=54.96$

The rectangular sides

$$\text{area}=\text{length}\times\text{width}$$
$$\text{area}=12\times 5$$
$$\text{area}=60$$

Again, we have 2 of these: $\text{area}=2\times 60=120$

The rectangular base

$$\text{area}=\text{length}\times\text{width}$$
$$\text{area}=4\times 12$$
$$\text{area}=48$$

Total surface area

$$54.96+120+48=222.96\approx 223$$

Question 5

Answer: B

Worked solution

Step 1: Calculate the volume of the cake.

$V = l \times w \times h$

$V = 12 \times 18 \times 10$

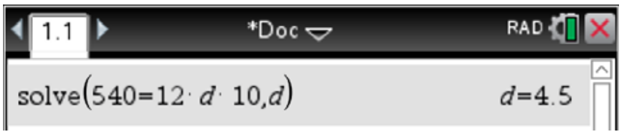
$V = 2160\ \text{cm}^3$

Step 2: Calculate one quarter of this volume.

$2160 \div 4 = 540$

Step 3: Find the value of *d*, using the formula for the volume of a rectangular prism.

$V = l \times w \times h$



Question 6

Answer: B

Worked solution

The two cities have the same longitude, therefore they are on the same great circle.

The total angle of both points representing the cities is $(38+23)^\circ = 61^\circ$

We now find the arc length between these two points.

The formula for arc length = $s = \dfrac{\pi r \theta}{180}$

When looking at the Earth as our sphere, the radius is 6400 km.

The angle, θ, is equal to 61.

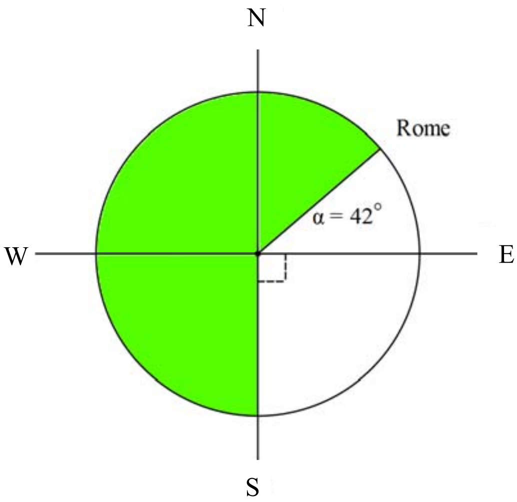
Therefore, the length of the arc $s = \dfrac{\pi \times 6400 \times 61}{180} = 6813.77\ \text{km} \approx 6814\ \text{km}$

Question 7

Answer: E

Worked solution

The diagram shows the plane of the meridian 13°E.



First, we calculate the angle from the point at Rome to the South Pole.

$$\theta = (42 + 90)^\circ = 132^\circ$$

The length of the arc $s = \dfrac{\pi r \theta}{180} = \dfrac{\pi \times 6400 \times 132}{180} = 14\,744.5 \approx 14\,745 \text{ km}$

This distance from Rome to the South Pole along the great circle is 14 745 km.

Question 8

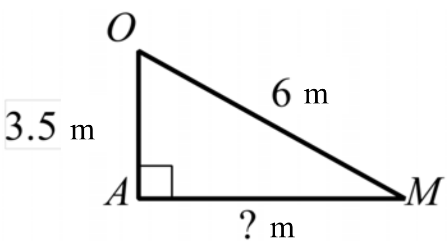
Answer: A

Worked solution

The side length *OM*, is equal to the radius of the hemisphere. It is therefore 6 m.

The length of *OA* is the difference between the radius of the hemisphere, 6, and the height of the water, 2.5:

$$OA = 6 - 2.5 = 3.5$$



To find the length of side *AM*, we use Pythagoras' theorem:

$$b = \sqrt{c^2 - a^2} = \sqrt{6^2 - 3.5^2} = 4.87$$

To find the area of the surface of the water, we use $A = \pi r^2$:

Area = $\pi \times 4.87^2 = 74.51 \text{ m}^2$

Module 4 – Graphs and relations

Question 1

Answer: C

Worked solution

We start by selecting two coordinates on our graph:

(0, 4   and   6, 8)

Use these points to find the gradient of the straight line:

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-4}{6-0} = \frac{4}{6} = \frac{2}{3}$$

We can see that the *y*-intercept for this straight line is 4, from the graph.

Therefore, our equation in gradient–intercept form is : $y = \frac{2}{3}x + 4$

The equations given in the multiple choice options are in intercept form.

Write the equation using a common denominator: $y = \frac{2x+12}{3}$

Multiply both sides by 3: $3y = 2x + 12$

Subtract 2x from both sides: $3y - 2x = 12$

Hence, we have option C.

Question 2

Answer: A

Worked solution

A. $(1, -3)$

$4 \times (1) - 2 \times (-3) \le 24$

$4 + 6 \le 24$

True

B. $(2, -9)$

$4 \times 2 - 2 \times (-9) \le 24$

$8 + 18 \le 24$

False

C. $(3, -7)$

$4 \times 3 - 2 \times (-7) \le 24$

$12 + 14 \le 24$

False

D. $(4, -5)$

$4 \times 4 - 2 \times (-5) \le 24$

$16 + 10 \le 24$

False

E. $(5, -6)$

$4 \times 5 - 2 \times (-6) \le 24$

$20 + 12 \le 24$

False

Question 3

Answer: D

Worked solution

A 35 minute appointment will fall into the third interval, which sits at $40, for $25 < t \le 35$.

Tips

- *Pay attention to the open and closed circles in the step graphs.*
- *An open circle corresponds to a greater than or less than, but not equal to.*
- *A closed circle corresponds to a greater than, less than or equal to.*

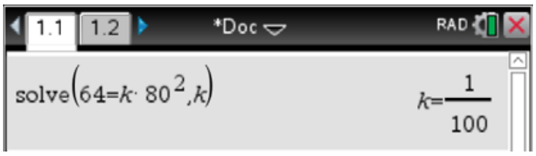
Question 4

Answer: C

Worked solution

We first need to find the value of *k*, in the equation.

We can use the point $(80, 64)$ to solve for *k*.



Now, we can use the equation braking distance $= \frac{1}{100} \times (\text{speed})^2$ to find the braking distance when the speed is equal to 85.

Braking distance = $\frac{1}{100} \times 85^2 = 72.25 \approx 72$ m

Question 5

Answer: B

Worked solution

Average speed = total distance travelled ÷ total time

Average speed = $\frac{850}{9} = 94.4$ km/h

Question 6

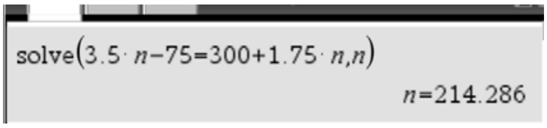
Answer: C

Worked solution

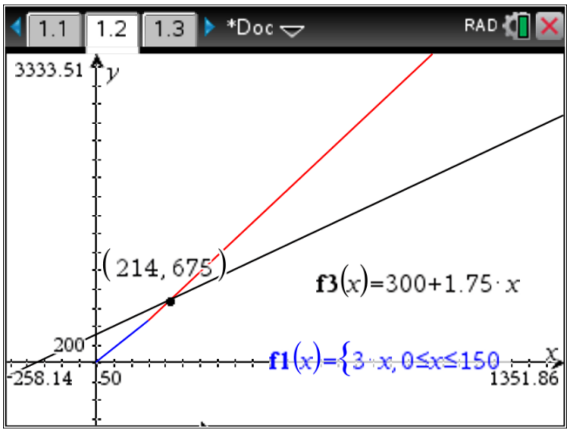
This can be done in one of two ways: <u>algebraically and graphically.</u>

<u>Algebraically:</u> We can solve for the value of *n*, where our second equation equals the Cost equation.

Note: This requires you to check where the first equation ends within the Revenue function, and realise that the Cost function will intersect the second equation after $n = 150$.



<u>Graphically:</u> We can graph the three functions, and find the intersection between the Cost function, and the Revenue function.



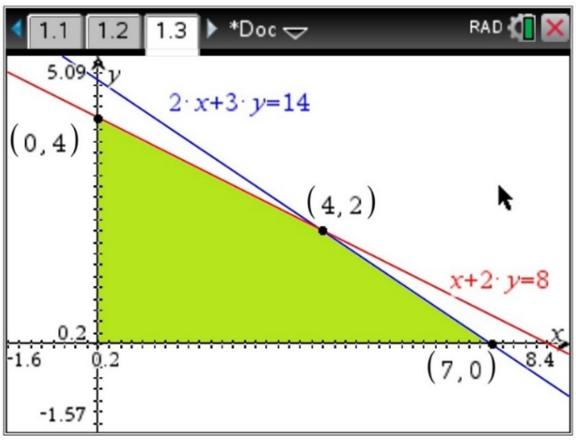
Question 7

Answer: B

Worked solution

This is best done by looking at a graph of the linear equations specified by the programming problem.

The feasible region can be seen below, as shown by the shaded region.



Question 8

Answer: D

Worked solution

A. Option A is incorrect because both the revenue and the cost equation have their highest power of x as 1. They are equations for a straight line graph, and therefore linear.

B. Option B is incorrect because the revenue equation tells us that we will sell each copy for \$75.

C. Option C is incorrect because:

$$C = 18\,000 + 12x = 18\,000 + 12 \times 1000 = 30\,000$$

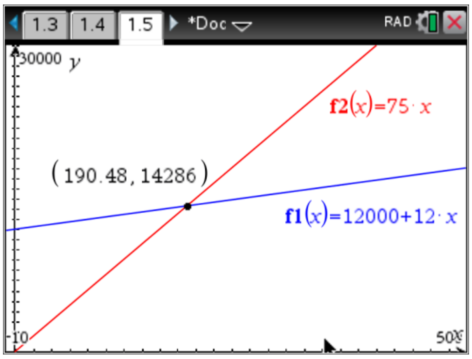
D. Option D is correct because:

Profit = Revenue – Cost

$$P = 75x - 12\,000 + 12x$$
$$P = 63x - 12\,000$$
$$P = 63 \times 1000 - 12\,000$$
$$P = 51\,000$$

E. Option E is incorrect because the revenue is equal to the cost when 190 copies are sold (using rounding to the nearest textbook), and after that, the revenue is more.

We can see this by graphing the Revenue and Cost functions.



END OF WORKED SOLUTIONS