2016
VCE
Further Mathematics
Trial Examination 2
Suggested Answers

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Data analysis

Question 1

a.

There were 4 year 12 students who got 8, 9 or 10 pictures correct.

$$\frac{4}{12} \times 100 = 33\frac{1}{3}\%$$

(1 mark)

b.

4, 5, 5, /5, 6, 6 7, 7, 8, / 8, 9, 10

The lower quartile is 5 and the upper quartile is 8

The interquartile range = 8 – 5 = 3.

(1 mark)

c.

Use calculator to find that Mean = 5.25

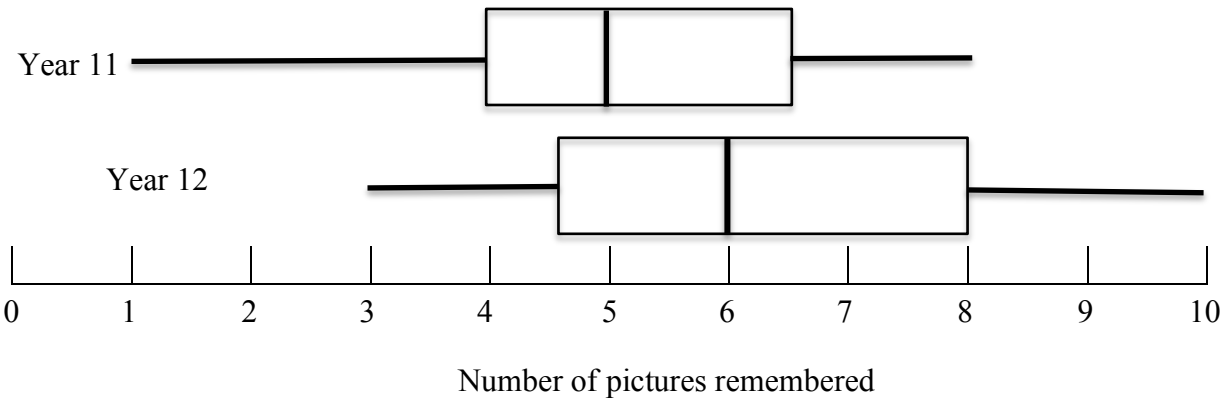
(1 mark)

Standard deviation = 1.96

(1 mark)

d.

Use calculator



1 mark for median and end points correct, and 1 mark for lower and upper quartile correct.

(2 marks)

Data analysis

e.

$IQR = 6.5 - 4 = 2.5$

$2.5 \times 1.5 = 3.75$

$4 - 3.75 = 0.25$

$6.5 + 3.75 = 10.25$

Since there are no values less than 0.25 and no values greater than10.25, there are no outliers.
(1 mark for calculation, and 1 mark for explanation.)

(2 marks)

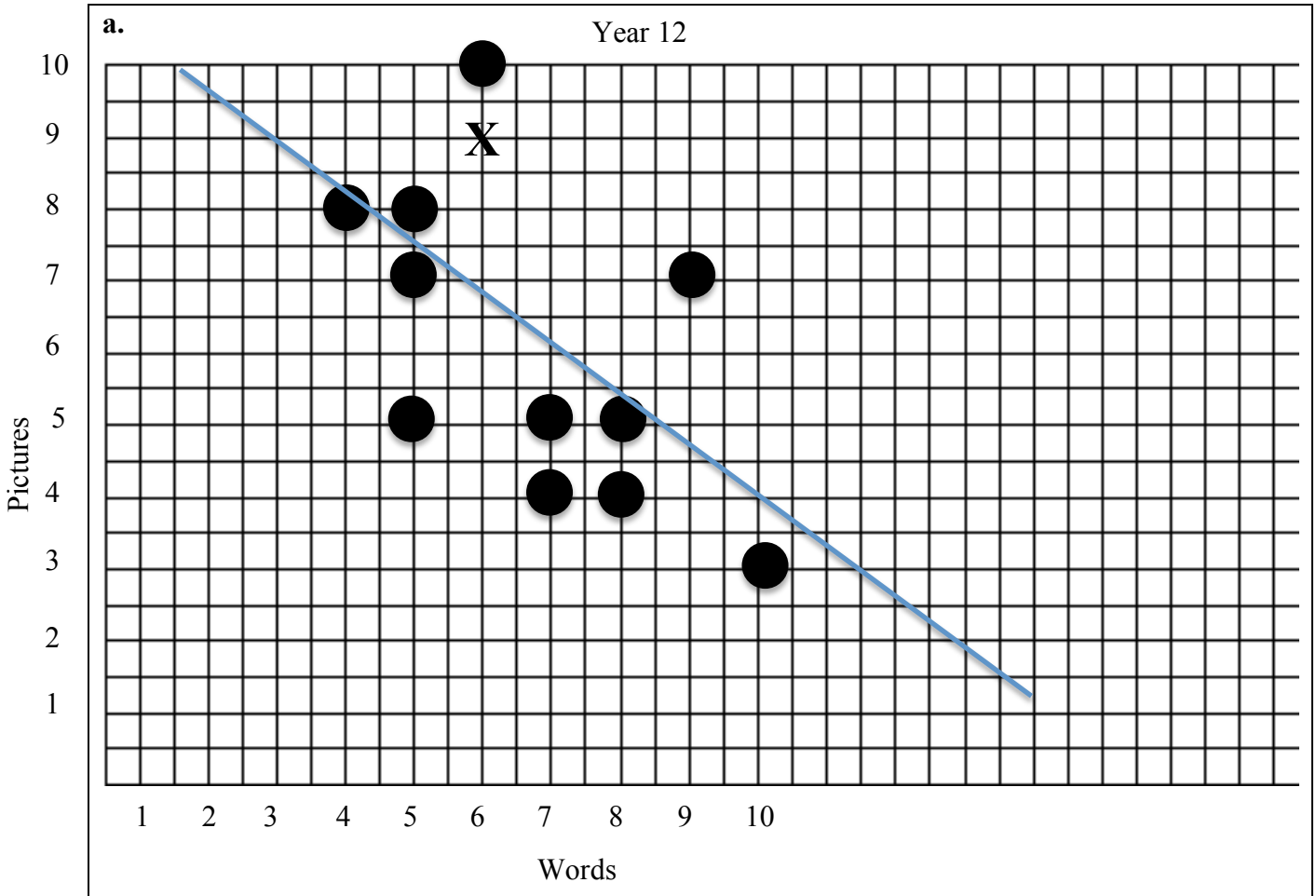
f.

On the whole, the Year 12 students remembered more pictures than the Year 11 students. The year 12 median, lower quartile and upper quartile were all higher than the corresponding values for the Year 11 students. Also the year 12 students had a higher upper and lower value.
(1 mark for recognising that Year 12 remembered more than Year 11 and one mark for linking to the important points on the boxplot.)

(2 marks)

Question 2

a.



The **X** shows the missing 6 words, 9 pictures. (1 mark)

Data analysis

Question 2 (continued)

b.

$$Number\ of\ pictures\ remembered = -0.7 \times number\ words\ remembered + 11.0$$

1 mark for each value.

(2 marks)

c.

Get two points that lie on the line.
When number of words = 2, number of pictures = -1.4 + 11 = 9.6
When number of words = 10, number of pictures = -7+ 11 = 4
Draw a line on the graph passing through these two points.

(1 mark)

d.

$$6 = -0.7 \times number\ words\ remembered + 11$$

Use calculator to solve this equation to get, number of words remembered = 7.1
The number of words would have to be a whole number so round to 7.

(1 mark)

e.

From calculator, $r = -0.58$. This means that the strength of the linear relationship is a moderate negative relationship.

(1 mark)

f.

From the calculator, the coefficient of determination, r^2, = 0.33984
This means that 34 % of the variation in the number of pictures remembered can be explained by the variation in the number of words remembered.

(1 mark)

Data analysis

Question 3

a.
9 is 3 standard deviations below the mean.
99.7% lie within ± 3 standard deviations of the mean. This leaves 0.3% outside. 0.15% would be less than 9 and 0.15% would be greater than 15 + 6.
Probability > 9 = Everything except 0.15%
=100 – 0.15 = 99.85%

(1 mark)

b.
$z = \frac{21-15}{2} = 3$
$z = \frac{11-15}{2} = -2$
Probability Z < -2 = (100 – 95) ÷ 2 = 2.5%
Probability Z > 3 = (100 – 99.7) ÷ 2 = 0.15%
We want everything except these two percentages.
Required region = 100 – (2.5 + 0.15) = 97.35%

(1 mark)

c.
$z = \frac{17-15}{2} = 1$

Probability Z >1 = 16%

Pr Z < 1 = 100 – 16 = 84%

84% of 2000 = 1,680

(1 mark)

Question 4

a.
It is seasonal.

(1 mark)

b.
This value and the two points either side when placed in ascending order are 1000, 2000, 3000, 6000, 9000.
The median of these 5 values is 3000.

(1 mark)

c.
Average for 2014
= (9 + 2 + 4 + 8) × 1000 ÷ 4 = 5750
SI for June 2014 = 2000 ÷ 5750
= 0.347826087
Seasonally adjusted value for June 2014
= 2000 ÷ 0.347826087 = 5750

1 mark for correct SI value
1 mark for seasonally adjusted value

(2 marks)

Recursion and financial modelling

Question 5

a. $0.94^2 \times 1400 = 1237$ (1 mark)	**b.** 100 – 94 = 6% (1 mark)
c. Use calculator to find $K_n < 700$ when $n = 12$. (1 mark)	**d.** 0.94 × 1400 = 1316 This means the population is reduced by 1400 – 1316 = 84 in the first year, so $d = 84$ to keep the population at 1400. (1 mark)

Question 6

a. a = 100 + 4 = 104% = 1.04 b is a loss in numbers so b = -12 1 mark for each. (2 marks)	**b.** 1.04 × 86 = 89.44. This means that the trees increase by about 3 from the planting but decrease by 12 because of natural disasters. Hence, this model would not give an increase in the number of trees. (1 mark)
c. $T_{n+1} = 1.2^n \times 86 - 12 > 200$ Use calculator to find this occurs when $n = 5$. After 5 years. (1 mark)	

Question 7

a. 5200 is the flat rate reduction, so 8% × original value = 5200 Solve on calculator to get $V_0 = \$65{,}000$ (1 mark)	**b.** $V_1 = 65000 - 5200 = 59800$ $V_2 = 59800 - 5200 = 54600$ $V_3 = 54600 - 5200 = 49400$ $V_4 = 49400 - 5200 = 44200$ $V_5 = 44200 - 5200 = \$39000$ (1 mark)
c. $0.88^{10} \times 65000 = \$18{,}102.56$ (1 mark)	

Recursion and financial modelling

Question 7 (continued)

d. Use calculator to find the following

Year	Flat Rate	Reducing Balance
1	59800	57200
2	54600	50336
3	49400	44295.68
4	44200	38980.20
5	39000	34302.57
6	33800	30186.27
7	28600	26563.91
8	23400	23376.24
9	18200	20571.09

So after 9 years.

(1 mark)

Module 1 – Matrices

Question 1

a.

B is a 3×1 matrix because it has 3 rows and 1 column.

(1 mark)

b.

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 2 \\ 5 & 1 & 3 \end{bmatrix}$$

The columns represent the size of the cones and the rows represent the different families

(1 mark)

c.

Use your calculator to find the inverse of A

$$A^{-1} = \begin{bmatrix} \frac{5}{3} & -\frac{4}{3} & \frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} & -\frac{1}{6} \\ -\frac{17}{6} & \frac{13}{6} & -\frac{1}{6} \end{bmatrix}$$

(1 mark)

d.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{5}{3} & -\frac{4}{3} & \frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} & -\frac{1}{6} \\ -\frac{17}{6} & \frac{13}{6} & -\frac{1}{6} \end{bmatrix} \times \begin{bmatrix} 44 \\ 67 \\ 63 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 10 \end{bmatrix}$$

The cost of a large cone is \$10

(1 mark)

Module 1 – Matrices

Question 2

a.

$$W_1 = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

(1 mark)

b.

$$W_2 = \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 3 & 0 & 0 & 1 & 2 \\ 2 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

(1 mark)

c.

$$W_3 = 0.6 \begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 3 & 0 & 0 & 1 & 2 \\ 2 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$W_3 = \begin{bmatrix} 0 & 0 & 0.6 & 0.6 & 0.6 \\ 1.8 & 0 & 0 & 0.6 & 1.2 \\ 1.2 & 0.6 & 0 & 0 & 0.6 \\ 0.6 & 0.6 & 0 & 0 & 0 \\ 0 & 0.6 & 0 & 0 & 0 \end{bmatrix}$$

(1 mark)

d.

From first dominance
A = 1
B = 3
C = 3
D = 2
E = 1
From weighted second dominance
A = 1.8
B = 3.6
C = 2.4
D = 1.2
E = 0.6
Adding these values
A = 2.8
B = 6.6
C = 5.4
D = 3.2
E = 1.6
B is first, C is second and D is third.

(1 mark)

Module 1 – Matrices

Question 3

a.

$$\begin{bmatrix} 0.25 & 0.30 & 0.40 \\ 0.40 & 0.10 & 0.10 \\ 0.35 & 0.60 & 0.50 \end{bmatrix}$$

(1 mark)

b.

$$S_0 = \begin{bmatrix} 750 \\ 1250 \\ 2000 \end{bmatrix}$$

(1 mark)

c.

Wednesday night is in 2 night's time

$$\begin{bmatrix} 0.25 & 0.30 & 0.40 \\ 0.40 & 0.10 & 0.10 \\ 0.35 & 0.60 & 0.50 \end{bmatrix}^2 \begin{bmatrix} 750 \\ 1250 \\ 2000 \end{bmatrix} = \begin{bmatrix} 1333 \\ 809 \\ 1858 \end{bmatrix}$$

Would expect 809 to eat at the Chinese restaurant on Wednesday night.

(1 mark)

d.

$$\begin{bmatrix} 0.25 & 0.30 & 0.40 \\ 0.40 & 0.10 & 0.10 \\ 0.35 & 0.60 & 0.50 \end{bmatrix}^{100} \begin{bmatrix} 750 \\ 1250 \\ 2000 \end{bmatrix} = \begin{bmatrix} 1322 \\ 797 \\ 1881 \end{bmatrix}$$

Number expected to eat at a restaurant in the long term is 1322 + 797 = 2119

(1 mark)

Module 2: Networks and decision mathematics

Question 1

a. Can go C – D – E = 70 + 60 = 130 or C – B – D – E = 50 + 40 + 60 = 150 or C – B – F – E = 50 + 30 + 40 = 120 Shortest distance = 120 km (1 mark)	**b.** A – F – E – D – C – B – F – A 20 + 40 + 60 + 70 + 50 + 30 + 20 = 290 km (1 mark)
c. A Hamiltonian path is passing through each town once only and starting and finishing at a different town. Leave out the longest distance of 80 km and use the shorter distances of 20, 30 and or 40 km. To do this Jerome could go C – B – D – E – F – A = 50 + 40 + 60 + 40 + 20 = 210 km. (1 mark)	**d.(i)** Bo would have to start and end at an odd vertex, i.e. at Fairview and Darebin. (1 mark)
d.(ii) An Eulerian path means travelling along each road once only and starting and finishing at a different point. Bo could go F – A – B – F – E – D – B – C – D = 20 + 80 + 30 + 40 + 60 + 40 + 50 + 70 = 390 km (1 mark)	**e.** An Eulerian circuit exists when all the vertices are even. When the road between B and D is removed, the odd vertices will be B and F, so an extra road between these two towns would make all the vertices even. Between Fairview and Brentwood. (1 mark)

Module 2: Networks and decision mathematics

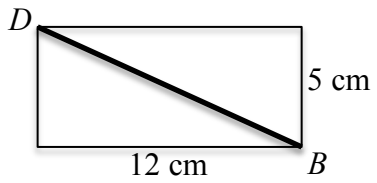
Question 2

a. A, D, F, B, E, G and H (1 mark)	**b.** The critical path is the longest path. B – E – F – J - M (1 mark
c. 4 + 10 + 6 + 5 + 1 = 26 days (1 mark)	**d.** HJM will take longer than KM, all of which require G to be completed before they start. 26 – (1 + 5 + 6 + 8) = 6. The latest starting time for G is the 6^{th} day. (1 mark)
e. i. I is not on the critical path and will not create a new critical path, so only B will affect the time taken. If B is reduced by 1 day to 3 days, then F can start on day 13 instead of day 14 and 1 day will be saved. Project will now take 25 days. (1 mark)	**e. ii.** Darren will now be 1 day over so instead of paying \$2000 for being 2 days over he will pay \$1000 + \$150. He will save 2000 – 1150 = \$850 (1 mark)

Module 3: Geometry and measurement

Question 1

a. i.

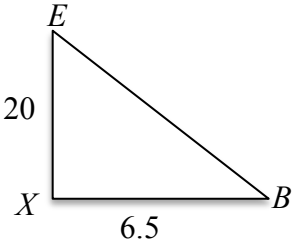


Know your triads, 5 : 12 : 13 or use Pythagoras.

$BD = \sqrt{12^2 + 5^2} = 13$ cm.

(1 mark)

a. ii.



X is the centre of the base of the pyramid.

$BX = \frac{1}{2}BD = 6.5$

$BE = \sqrt{20^2 + 6.5^2} = 21.03$ cm.

(1 mark)

a. iii.

$\angle EBD = \angle EBX$

$\tan \angle EBX = \frac{20}{6.5}$

$\angle EBX = \tan^{-1}\left(\frac{20}{6.5}\right) = 72^0$

(1 mark)

b. i.

The figures are similar so the ratio of the lengths of the sides is 5 : 20 = 1 : 4

$\frac{PR}{13} = \frac{1}{4}$

$PR = 3.25$ cm

(1 mark)

b. ii.

V of large pyramid $= \frac{1}{3} \times 12 \times 5 \times 20 = 400$

Ratio of volumes $= 1^3 : 4^3 = 1 : 64$

V of small pyramid $= \frac{1}{64} \times 400 = 6.25$ cm^3

(1 mark)

Module 3: Geometry and measurement

Question 2

a.

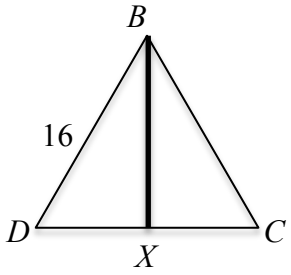
$$\frac{35}{\sin 30^0} = \frac{16}{\sin \angle BAC}$$

$$\angle BAC = 13.21^0$$

$$\angle BDC = 30 + 13.21 = 43.2^0$$

(1 mark)

b.



$$\cos 43.2^0 = \frac{DX}{16}$$

$$DX = 11.66$$

$$DC = 2 \times 11.66 = 23.3$$

$$AC = 23.3 + 35 = 58.3 \text{ cm.}$$

(2 marks)

Question 3

a.

8 p.m. Sunday

(1 mark)

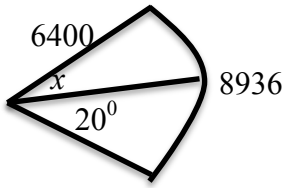
b.

$$l = \frac{20 + x}{180} \times \pi \times 6400 = 8936$$

$$20 + x = 80$$

$$x = 60^0$$

B is 60^0 N



(1 mark)

Module 4: Graphs and relations

Question 1

a.
$C = 320n + 12000$

(1 mark)

b.
$R = 480n$

(1 mark)

c.
$320n + 12000 = 480n$

$n = 75$

Bill would have to sell 75 phones a month to break even.

(1 mark)

Question 2

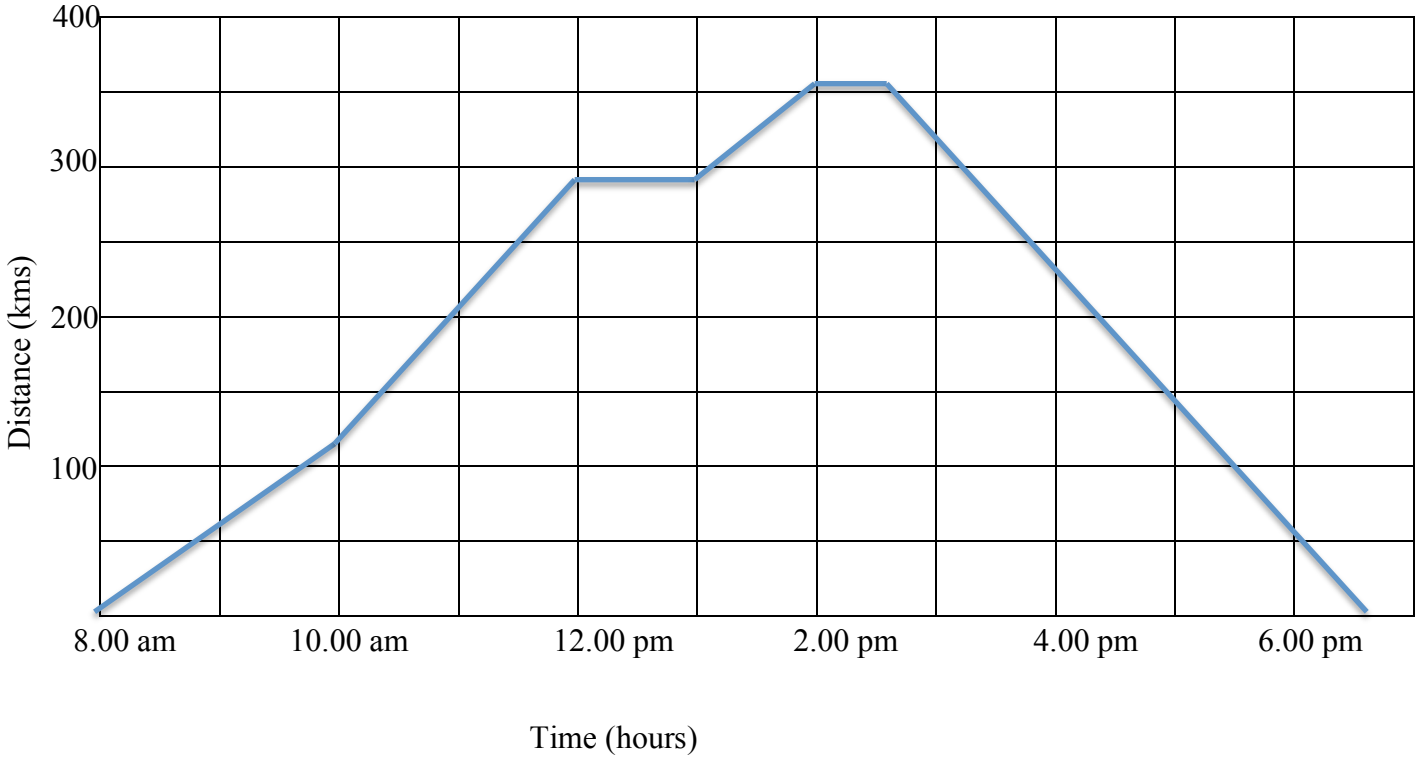
a.
She leaves at 8. She drives for 2 hours at 60 km/hr
This means when time is 10, she is 120 km from home.
She continues at 85 km/hr for 2 hours which means she is at120 + 170 = 290 km from home when the time is 12 pm.
Lunch takes 1 hour so still 290 km rom home when time is 1 pm,
She goes another 70 km after lunch
So is 290 + 70 = 360 km from home then time is
2 pm.
She unloads so is still at 360 km from home at 2:30 pm.
She travels 360 km home at 90 km per hour. This will take 4 hours.
2:30 + 4 = 6:30 pm
Alice will arrive home at 6:30 pm.

(1 mark)

Module 4: Graphs and relations

Question 2 (continued)

b.



(1 mark)

c.

Average speed = Distance ÷ time
= 2 × 360 ÷ 10.5 = 68.6 km/hr.

(1 mark)

Question 3

a.

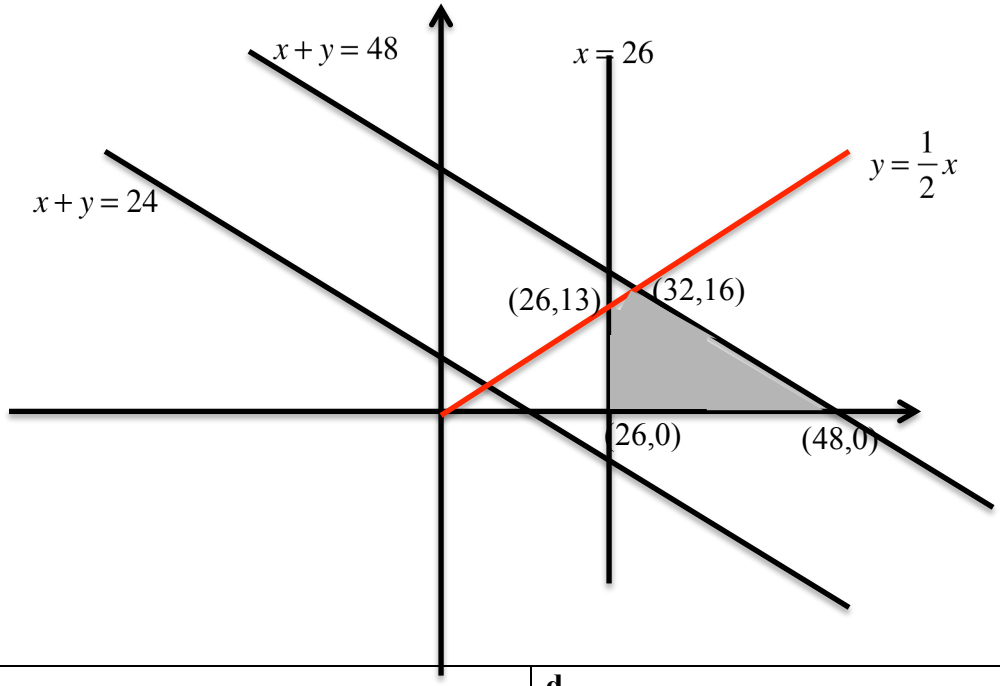
$x \geq 2y \Rightarrow y \leq \frac{1}{2}x$

(1 mark)

Module 4: Graphs and relations

Question 3 (continued)

b.



(1 mark)

c.

Corner points.

Solve simultaneously

$y = \frac{1}{2}x$ and $y = -x + 48$

$x = 32, y = 16$

$y = \frac{1}{2}x$ and $x = 26$

$x = 26, y = 13$

jugs = x and teapots = y

Profit = $20x + 50y$

At (26,13) $P = 520 + 650 = 1170$

At (32,16) $P = 640 + 800 = 1440$

At (26,0) $P = 520$

At (48,0) $P = 960$

Maximum profit is \$1440

(1 mark)

d.

From part c, the number of teapots that give the maximum profit is 16.

(1 mark)

End of Suggested Solutions 2016 VCE Further Mathematics Trial Examination 2

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