The Mathematical Association of Victoria

Trial Examination 2016
FURTHER MATHEMATICS

Written Examination 2 - SOLUTIONS

Core Statistics

Q1 a) Continuous numerical **1 mark**

Data is numerical and age can take any value within a range, so it is continuous.

Q1 b) 22 years **1 mark**
The median is represented by the central line in the box of the boxplot. This line is positioned at 22 years.

Q1 c) Positively skewed. **1 mark**
The data is clustered at the lower end of the age range.

Q1 d) From the boxplot it can be seen that the IQR is 3 ($24 - 21 = 3$) and there are outliers at the upper end. The upper fence is $24 + 1.5 \times 3 = 28.5$ years **1 mark**
31, 35 and 54 are all greater than 28.5 and are therefore outliers. **1 mark**

Q2 a) 2.5% **1 mark**

$13\,000 = 24\,000 - 2 \times 5500 = \bar{x} - 2s$ so using the 68-95-99.7% rule there are 2.5% of values below this point.

Q2 b) 13.5% **1 mark**
\$29 500 is one standard deviation above the mean, so there are 16% of values above this value.
\$35 000 is two standard deviations above the mean, so there are 2.5% of values above this value.
Therefore there are $16 - 2.5 = 13.5\%$ of value between these two values.

Q2 c) \$14 500 **1 mark**

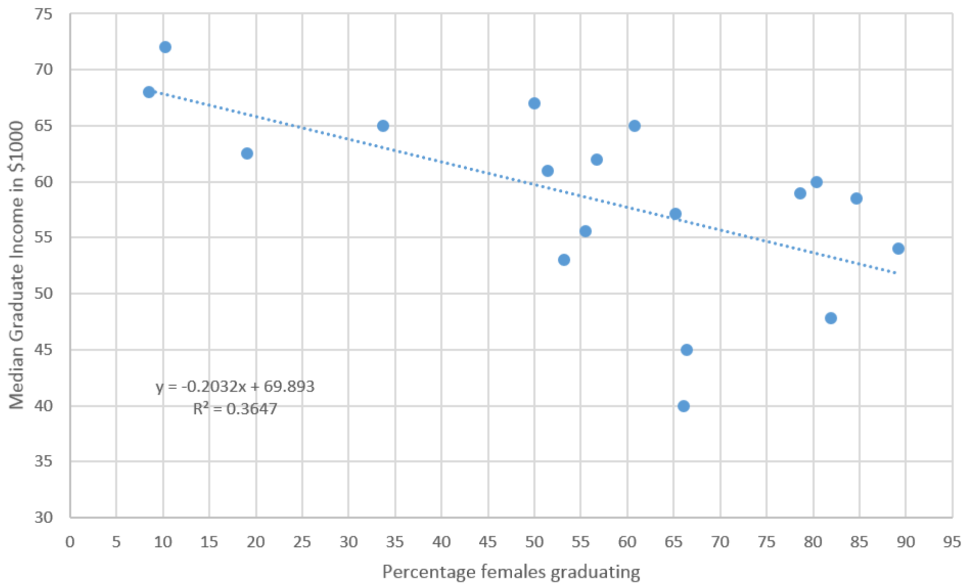
The following calculation is used and solved : $\dfrac{x - 24000}{5500} = -1.73$, $x = 14\,485 \approx 14\,500$ to 3 sig figs

Q3 Yes, as the years have progressed the percentage of female graduates has increased OR Yes, as the years have progressed the percentage of male graduates has decreased **1 mark**

The percentage of female graduates has increased from 45.3% in 1975 to 54.6% in 1995 to 57.4% in 2015
OR
The percentage of male graduates has decreased from 54.7% in 1975 to 45.4% in 1995 to 42.6% in 2015
1 mark

Q4 a) \$59 743 **1 mark**
$MGIT = 69.893 - 0.203 \times 50 = 59.743$
$59.743 \times 1000 = \$59\,743$

Q4 b)



1 mark

Q4 c) On average when the percentage of females graduating is zero, the median graduate income is predicted to be \$69 893. **1 mark**

Q4 d) On average for every extra 1% of female graduates, the median graduate income decreases by \$203. **1 mark**

Q4 e) -0.60 **1 mark**

$r = \pm\sqrt{0.3647} = \pm 0.6039...$ In this case the gradient is negative so the negative root is required.

Q4 f) $-\$16\,480$ **1 mark**

When there are 66.1% females, the predicted income is

$MGIT = 69.893 - 0.203 \times 66.1 = 56.4797, \quad 56.4797 \times 1000 = \$56\,479.70$

Residual is $actual - predicted = 40\,000 - 56\,479.70 = -\$16\,479.70 \approx -\$16\,480$

Q4 g) \$8320 **1 mark**

Using $b = \dfrac{r \times S_y}{S_x}$, $-0.203 = \dfrac{-0.60 \times S_y}{24.6}$, $S_y = 8.323$, i.e. $8.323 \times 1000 = \$8323 \approx \$8320$

Q5 a) $NFG(in\ thousands) = -51\,829.31 + 26.26 \times year$ **1 mark**

Q5 b) 1 478 000 female graduates **1 mark**

$NFG(in\ thousands) = -51\,829.31 + 26.26 \times 2030 = 1478.49, \quad 1478.49 \times 1000 = 1\,478\,490 \approx 1\,478\,000$

Q5c) The prediction may not be reliable as it is extrapolated beyond the range of the available data. **1 mark**

Q6 a) 1.1 **1 mark**

The sum of the seasonal indices should add up to the number of points in a cycle, here 4.

$SI = 4 - 1.65 - 0.8 - 0.45 = 1.1$

Q6 b) Quarter 1 has sales 65% above the seasonal average. **1 mark**

Q6 c) $29 100

Quarter 2, 2015 is t = 18.The predicted deseasonalised sales is ds sales = 32 500 + 215 × 18 = 36 370 .
The actual sales are the deseasonalised sales × seasonal index for the quarter.

Actual = 36 370 × 0.8 = $29 096 ≈ $29 100

Core: Recursion and financial mathematics.

Q1 a) 0.009 **1 mark**

$$\text{Quarterly Interest rate : } = \frac{3.6\%}{4} = 0.9\ \% = 0.009$$

Q1 b) $B_0 = \$850 \quad B_{n+1} = 1.009\, B_n$ **1 mark**

Q1 c) $913.16 **1 mark**

Number of compounding periods = 2 × 4 = 8

$B_8 = 1.009^8 \times 850 = 913.1628\ldots \quad \$913.16$

Q2 a) $B_0 = \$1100,\ B_{n+1} = 1.0075\, B_n - 190$ **1 mark**

Q2 b) $178.85

Interest = $177.52 × 0.0075 = $1.33 **1 mark**

Final payment = $177.52 + $1.33 = $178.85 **1 mark**

Q2 c) 9.38% **1 mark**

$$r_{effective} = \left(\left(1+\frac{9.0}{100\times 12}\right)^{12} - 1\right)\times 100\% = 9.3806\ldots \approx 9.38\ \%$$

Q3 a) $553.62

N	I(%)	PV	Pmt	FV	PpY, CpY
120	4.80	– 350 000	0	750 000	12

*Solve for **Pmt** : 1203.6249… He will have to pay $1203.62.* **1 mark**

Extra to pay = $1203.62 – $650.00 = $553.62 **1 mark**

Q3 b) $750 000 **1 mark**

For a perpetuity, the full $750 000 since he will only be paid the interest.

Q4

Sam will repay 12 × $2600 = $31 200 each year through monthly repayments, whereas he will repay 26 × $1300 = $33 800 each year through fortnightly repayments. **1 mark**

He will pay an extra $2600 each year through fortnightly repayments which is equivalent to an extra month's repayments. **1 mark**

Module 1 :Matrices

Q1 a) $S_0 = \begin{bmatrix} 125 \\ 100 \end{bmatrix}$ **1 mark**

Q1 b)

$$T = \begin{array}{c} \\ Grove \\ Kates \end{array} \begin{array}{c} \begin{array}{cc} Grove & Kates \end{array} \\ \begin{bmatrix} 0.75 & 0.30 \\ 0.25 & 0.70 \end{bmatrix} \end{array}$$

1 mark

Q1 c) $S_4 = T^4 S_0 = \begin{bmatrix} 0.75 & 0.3 \\ 0.25 & 0.7 \end{bmatrix}^4 \begin{bmatrix} 125 \\ 100 \end{bmatrix} = \begin{bmatrix} 122.820... \\ 102.179... \end{bmatrix}$ **1 mark**

Q1 d) $S_1 = \begin{bmatrix} 0.77 & 0.27 \\ 0.23 & 0.73 \end{bmatrix} \begin{bmatrix} 123 \\ 102 \end{bmatrix} + \begin{bmatrix} 19 \\ 13 \end{bmatrix} = \begin{bmatrix} 141.25 \\ 115.75 \end{bmatrix}$ **1 mark**

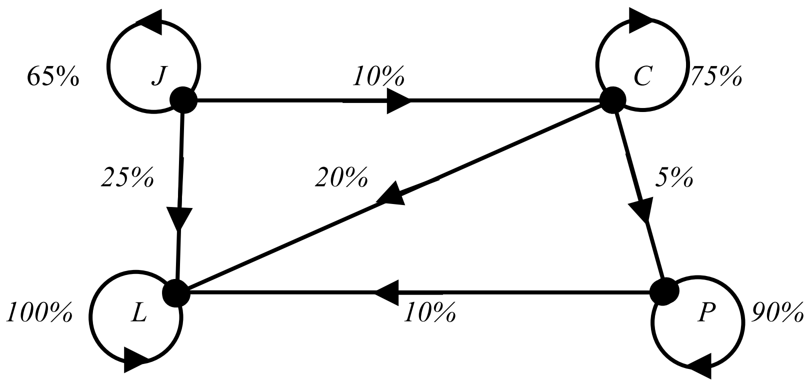
$$S_2 = \begin{bmatrix} 0.77 & 0.27 \\ 0.23 & 0.73 \end{bmatrix} \begin{bmatrix} 141.25 \\ 115.75 \end{bmatrix} + \begin{bmatrix} 19 \\ 13 \end{bmatrix} = \begin{bmatrix} 159.015 \\ 129.985 \end{bmatrix}$$

$$S_2 = \begin{bmatrix} 159 \\ 130 \end{bmatrix}$$ **1 mark**

Note : Using $S_1 = \begin{bmatrix} 141 \\ 116 \end{bmatrix}$ *will also give the same rounded answer, from* $\begin{bmatrix} 158.89 \\ 130.11 \end{bmatrix}$.

Q2 a) Once a worker leaves the supermarket, they never return. **1 mark**

Q2 b)



2 marks

Q2 c) She will need to find seven (7) junior casual staff and five (5) senior casual staff.

Junior casual staff : 25% of 20 leave, so 5 replacements needed.
10% of 20 become senior casuals, so 2 replacements needed
Senior casual staff : 20% of 20 leave, so 4 replacements needed.
Senior casual staff : 5% of 20 are promoted, so 1 replacement needed. **2 marks**

Q3

$$D = \begin{array}{c} \\ \\ \end{array}\begin{array}{cc} \begin{array}{cccc} & & \textit{Losers} & \\ K & L & M & N \end{array} & \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} & \begin{array}{c} K \\ L \\ M \\ N \end{array} \textit{Winners} \end{array}$$

Leanne won one game and as a result picked up one two-step dominance.
The only way to do this was that Leanne defeated Neil (who had won one game).

Ken won two games and as a result picked up two two-step dominances.
The only way to do this was that Ken defeated Leanne and Neil (who had won one game each). **1 mark**
Neil won one game and as a result picked up two two-step dominances.
The only way to do this was that Neil defeated Maggie, since we have already determined that Neil has lost to Ken and Leanne.

Maggie won two games and as a result picked up three two-step dominances.
Since we have already determined that Maggie has lost to Neil, she must have beaten Ken and Leanne (picking up two two-step dominance and one two-step dominances respectively). **1 mark**

So the one-step dominance matrix is :

$$D = \begin{array}{cc} \begin{array}{cccc} & & \textit{Losers} & \\ K & L & M & N \end{array} & \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} & \begin{array}{c} K \\ L \\ M \\ N \end{array} \textit{Winners} \end{array}$$

Module 2: Networks and decision maths

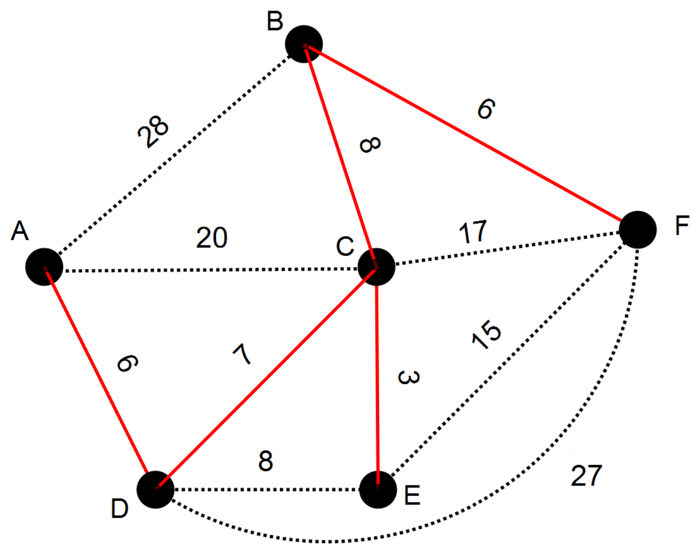
Q1 a) 27 kilometres **1 mark**
Shortest path is ADCBF with a distance of $6 + 7 + 8 + 6 = 27\ kilometres$.

Q1 b) Hamiltonian cycle **1 mark**

Q1 c) The required route is an eulerian circuit. This is not possible because the degree of every vertex is not even and this is the only circumstance that allows for an eulerian circuit. Vertices A, B, C and E all have odd degree vertices. **1 mark**

Q1 d) Two roads **1 mark**
Repeating two roads between A, B, C and E, the circuit can be transversed using every other road once.

Q1 e)



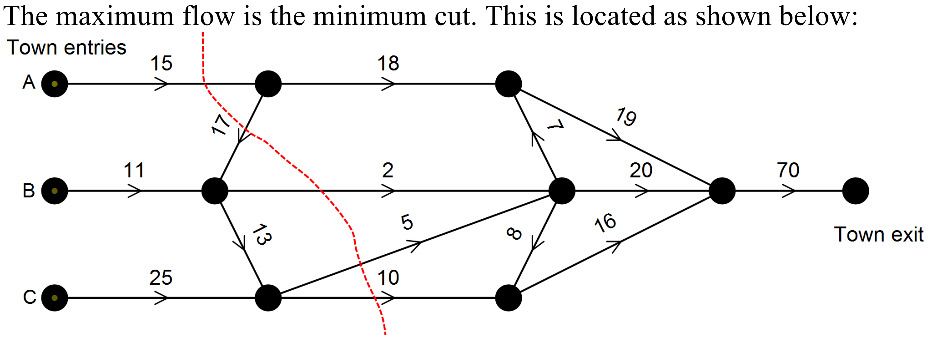
1 mark

Q1 f) 30 kilometres

$6 + 7 + 3 + 8 + 6 = 30$ kilometres

Q2 a) 36 vehicles. **1 mark**
The edge labelled 7 vehicles is not counted as it is reduced to zero when the other edges are cut, so the capacity is $10 + 5 + 2 + 0 + 19 = 36$ vehicles.

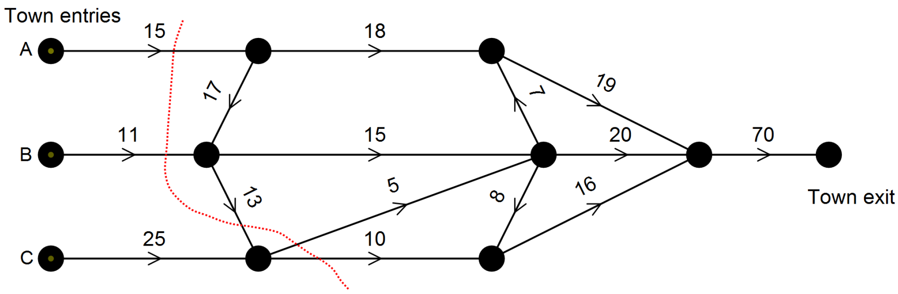
Q2 b) Maximum flow is 32 vehicles. **1 mark**
The maximum flow is the minimum cut. This is located as shown below:



This has a capacity of $15 + 0 + 2 + 5 + 10 = 32$ vehicles.

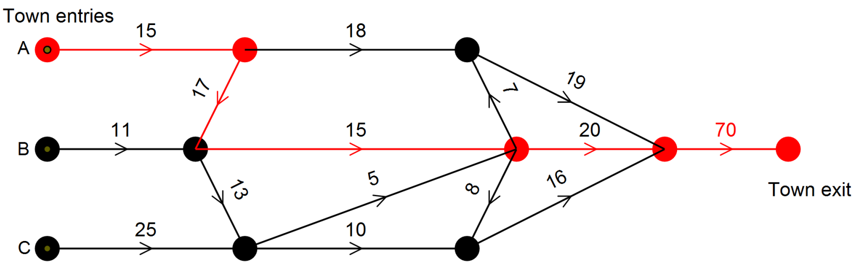
Q2 c) The flow is increased to 41 vehicles. **1 mark**

The next minimum cut (now the new minimum cut) is as shown below:



This has a capacity of 15 + 11 + 0 + 5 + 10 = 41 vehicles. **1 mark**

Q2 d)



1 mark

Q3

Worker	**Task**
Thomas	Fitting
Ursula	Cutting
Victoria	Ironing
Xavier	Sewing

$$\begin{array}{c} \\ T \\ U \\ V \\ X \end{array}\begin{array}{c} \begin{array}{cccc} C & F & S & I \end{array} \\ \begin{bmatrix} 0 & 3 & 4 & 0 \\ 0 & 4 & 5 & 2 \\ 0 & 7 & 4 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \rightarrow \begin{array}{c} \\ T \\ U \\ V \\ X \end{array}\begin{array}{c} \begin{array}{cccc} C & F & S & I \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 4 & 1 & 0 \\ 3 & 0 & 0 & 3 \end{bmatrix} \end{array}$$

1 mark

Module 3: Geometry and measurement

Q1 a) 50 metres **1 mark**

Using the cosine rule, AB $= \sqrt{100^2 + 100^2 - 2 \times 100 \times 100 \times \cos 28.96^o} = 50\ m$

Q1 b) 4.04 metres **1 mark**

CD $= \dfrac{28.96}{360} \times 2 \times \pi \times 8.0 = 4.0357\ldots \approx 4.04$ m

Q1 c) 1065 square metres
Each foul line will now be 120 m long.
If foul lines of 100 m give a field end of 50 m from part (a), then foul lines of 120 will give field end of 60 m.
Distance between field ends = 120 × cos 14.48º − 100 × cos 14.48º
= 116.1881… − 96.8234…
= 19.36 m **1 mark**
Extra area is a trapezium with parallel sides of 50 and 60 m, and a distance between of 19.36 m

Area $= \dfrac{50+60}{2} \times 19.36 = 1064.8 \approx 1065$ square metres **1 mark**

OR

Area = Area extended field – area normal field **1 mark**

$$= \left(\frac{1}{2} \times 120 \times 120 \times \sin(28.96) - \pi \times 8^2 \times \frac{28.96}{360}\right) - \left(\frac{1}{2} \times 100 \times 100 \times \sin(28.96) - \pi \times 8^2 \times \frac{28.96}{360}\right)$$

= 3770.057… – 2404.820…
= 1065.237…
≈ 1065 square metres **1 mark**

Q2 a) 1900 metres **1 mark**
Angle between latitudes = 34º – 17º = 17º

Distance along meridian= $\dfrac{6400\pi}{180^o} \times 17^o = 1898.918 \approx 1900$ km

Q2 b) i) 5305.84 km **1 mark**

Radius small circle = 6400 × cos 34º = 5305.8404… ≈ 5305.84 km

Q2 b) ii) 2500 km **1 mark**
Difference in longitudes = 142º – 115º = 27º

Distance $= \dfrac{5305.84\pi}{180^o} \times 27^o = 2500.3181\ldots \approx 2500$ km

Q2 c) 1.30 am **2 marks**
Difference in longitude = 142º – 37º = 105º
Time difference = 105º ÷ 15º = 7 hours
Since Mildura is to the east of Nairobi, it will ahead in time by 7 hours
Report time = 6.30 p.m. + 7 hours = 1.30 a.m. the following morning

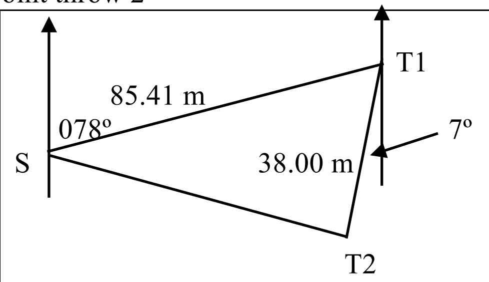
Q3 a) S is start of throw, T1 landing point throw 1, T2 landing point throw 2

$\angle ST1T2 = 78^\circ - 7^\circ = 71^\circ$ 1 mark

From Cosine Rule,

$$ST2 = \sqrt{85.41^2 + 38.00^2 - 2 \times 85.41 \times 38.00 \times \cos 71^\circ}$$

$= 81.397 \approx 81.40$ m 1 mark



Q3 b) From Cosine Rule, or Sine Rule, $\angle T1ST2 = 26.193\ldots^\circ$

Bearing $= 78^\circ + 26^\circ$

$= 104^\circ$T **1 mark**

Module 4: Graphs and relations

Q1a) $132.30 **1 mark**

Value in \$AUS $= 12.60 \times \dfrac{10500}{1000} = \$132.30.$

Q1 b) $0.26 per year **1 mark**

Average increase $= \dfrac{14.4 - 12.6}{8 - 1} = 0.2571 \approx \$0.26$ per year

Q1 c) $18.90 **1 mark**

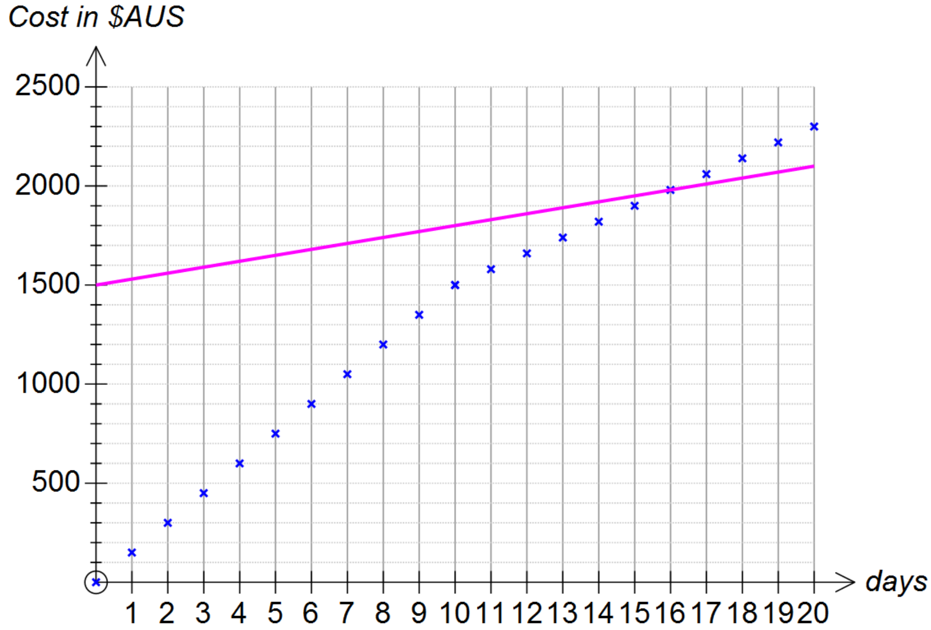
Value in \$AUS $= 14.40 \times \dfrac{10500}{1000} = \$151.20$

$\$151.20 - \$132.30 = \$18.90$

Q2 a) $a = 10, b = 700, c = 80$ **2 marks**

$$C = \begin{cases} 150n & 0 < n \le 10 \\ 700 + 80n & 10 < n \le 20 \end{cases}$$

Q2 b)



1 mark

Q2 c) 17 days **1 mark**

Break-even is 16 days, so cheaper at 17 days.

Q3 a) 0.02 or $\dfrac{1}{500}$ **1 mark**

$$k = \frac{p}{m^2} = \frac{2}{10^2} = \frac{8}{20^2} = \frac{18}{30^2} = 0.02 \quad or \quad \frac{1}{50}$$

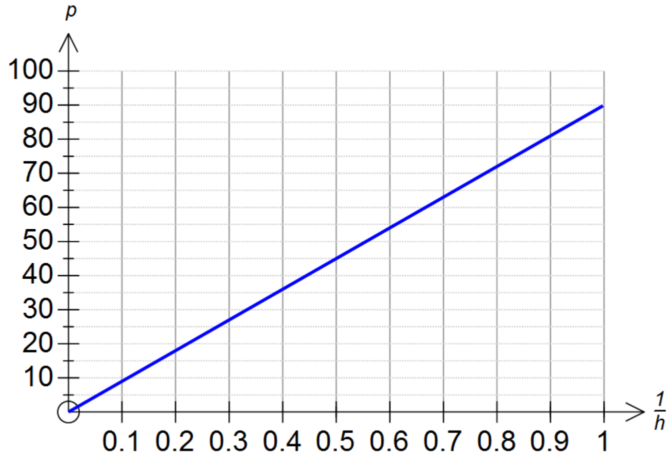
Q3 b) 70.7 minutes **1 mark**

$100 = 0.02 \times m^2$

$m^2 = \dfrac{100}{0.02} = 5000$

$m = \sqrt{5000} = 70.71\ldots \approx 70.7$ minutes

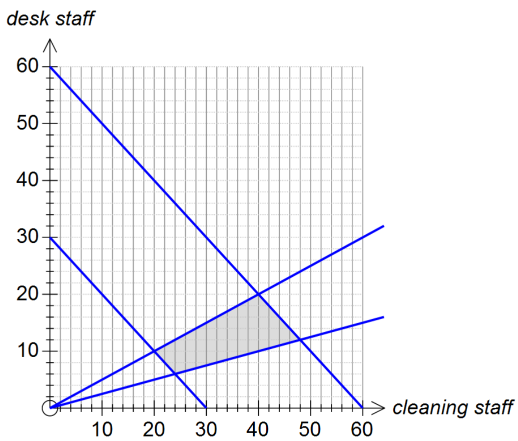
Q3 c)



1 mark

Note that it should be an open circle at origin and pass through (1, 90)

Q4 a)



1 mark

Q4 b)

The number of desk staff must be at least one quarter of the number of cleaning staff and not more than half of the number of cleaning staff OR the number of cleaning staff must be at least twice but not more than 4 times as many as the desk staff. **1 mark**