

Victorian Certificate of Education

Year

SUPERVISOR TO ATTACH PROCESSING LABEL HERE



FURTHER MATHEMATICS Written examination 2

Day Date

Reading time: *.** to *.** (15 minutes) Writing time: *.** to *.** (1 hour 30 minutes)

QUESTION AND ANSWER BOOK

Structure of book

Section A – Core	Number of questions	Number of questions to be answered	Number of marks
	9	9	36
Section B – Modules	Number of modules	Number of modules to be answered	Number of marks
	4	2	24
			Total 60

- Students are to write in blue or black pen.
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 30 pages.
- Formula sheet.
- Working space is provided throughout the book.

Instructions

- Write your student number in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Version 4 - September 2016

SECTION A – Core

Instructions for Section A

Answer all questions in the spaces provided. Write using blue or black pen.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π , surds or fractions.

In 'Recursion and financial modelling', all answers should be rounded to the nearest cent unless otherwise instructed.

Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Data analysis

Question 1 (3 marks)

The segmented bar chart below shows the age distribution of people in three countries, Australia, India and Japan, for the year 2010.



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ersion 4	- September 2016 3	FURMATH EXAM 2 (SAMPL
. V	Vrite down the percentage of people in Australia who were aged 0–14 years in 2010.	1 n	nark
	n 2010, the population of Japan was 128000000.		
Н	low many people in Japan were aged 65 years and over in 2010?	1 r	nark
F tł	rom the graph on page 2, it appears that there is no association between the percentage one 15–64 age group and the country in which they live.	of people in	
E	xplain why, quoting appropriate percentages to support your explanation.	1 n	nark



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Question 3 (6 marks)

The scatterplot below shows the *population* and *area* (in square kilometres) of a sample of inner suburbs of a large city.



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SECTION A – Question 3 – continued TURN OVER

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i.	Calculate the residual when the least squares regression line is used to predict the population of Wiston from its area.	1 r
		11
ii.	What percentage of the variation in the population of the suburbs is explained by the variation in area?	
	Round your answer to one decimal place.	1 n

Question 4 (3 marks)

The scatterplot and table below show the *population*, in thousands, and the *area*, in square kilometres, for a sample of 21 outer suburbs of the same city.



Area (km²)	Population
	(thousands)
1.6	5.2
4.4	14.3
4.6	7.5
5.6	11.0
6.3	17.1
7.0	19.4
7.3	15.5
8.0	11.3
8.8	17.1
11.1	19.7
13.0	17.9
18.5	18.7
21.3	24.6
24.2	15.2
27.0	13.6
62.1	26.1
66.5	16.4
101.4	26.2
119.2	16.5
130.7	18.9
135.4	31.3

In the outer suburbs, the relationship between *population* and *area* is non-linear. A **log** transformation can be applied to the variable *area* to linearise the scatterplot.

Apply the log transformation to the data and determine the equation of the least squares regression line that allows the population of an outer suburb to be predicted from the logarithm of its area.
 Write the slope and intercept of this least squares regression line in the boxes provided below.
 Round your answers to two significant figures.



b. Use the equation of the least squares regression line in **part a.** to predict the population of an outer suburb with an area of 90 km².

Round your answer to the nearest one thousand people.

2 marks

1 mark

SECTION A – continued TURN OVER

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1 mark

1 mark

1 mark

1 mark

Question 5 (4 marks)

There is a negative association between the variables *population density*, in people per square kilometre, and area, in square kilometres, of 38 inner suburbs of the same city.

For this association, $r^2 = 0.141$

Write down the value of the correlation coefficient for this association between the variables a. population density and area.

Round your answer to three decimal places.

The mean and standard deviation of the variables population density and area for these b. 38 inner suburbs are shown in the table below.

	<i>Population density</i> (people per km ²)	Area (km²)
Mean	4370	3.4
Standard deviation	1560	1.6

One of these suburbs has a population density of 3082 people per square kilometre.

- i. Determine the standard *z*-score of this suburb's population density. Round your answer to one decimal place.
- ii. Interpret the z-score of this suburb's population density with reference to the mean population density.
- iii. Assume the areas of these inner suburbs are approximately normally distributed. How many of these 38 suburbs are expected to have an area that is two standard deviations or more above the mean? Round your answer to the nearest whole number.

SECTION A – continued

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Question 6 (5 marks)

Table 1 shows the Australian gross domestic product (*GDP*) per person, in dollars, at five yearly intervals (*year*) for the period 1980 to 2005.

Table 1

Year	1980	1985	1990	1995	2000	2005
GDP	20900	22300	25 000	26400	30900	33 800



a. Complete the **time series plot above** by plotting the *GDP* for the years 2000 and 2005.

(Answer on the time series plot above.)

b. Briefly describe the general trend in the data.

1 mark

1 mark

c. In Table 2, the variable year has been rescaled using 1980 = 0, 1985 = 5, and so on. The new variable is *time*.

Table 2

Year	1980	1985	1990	1995	2000	2005
Time	0	5	10	15	20	25
GDP	20900	22300	25 000	26400	30900	33 800

i. Use the variables *time* and *GDP* to write down the equation of the least squares regression line that can be used to predict *GDP* from *time*. Take *time* as the explanatory variable.

2 marks

ii. The least squares regression line in part c.i. above has been used to predict the *GDP* in 2010.Explain why this prediction is unreliable.

1 mark

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FURMATH EXAM 2 (SAMPLE)

Question 7 (4 marks) Hugo is a professional bike rider. The value of his bike will be depreciated over time using the flat rate method of depreciation. The value of Hugo's bike, in dollars, after n years, V_n can be modelled using the recurrence relation below $V_0 = 8400$, $V_{n+1} = V_n - 1200$ a. Using the recurrence relation, write down calculations to show that the value of Hugo's bike after two years is \$6000.	
$V_0 = 8400,$ $V_{n+1} = V_n - 1200$ a. Using the recurrence relation, write down calculations to show that the value of Hugo's bike after two years is \$6000. Hugo will sell his bike when its value reduces to \$3600. b. After how many years will Hugo sell his bike? The unit cost method can also be used to depreciate the value of Hugo's bike. A rule for the value of the bike, in dollars, after travelling <i>n</i> kilometres is $V_n = 8400 - 0.25n$ c. What is the depreciation of the bike per kilometre?	
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c. What is the depreciation of the bike per kilometre?	
	1 mark
After two years, the value of the bike when depreciated by the unit cost method will be the same as the value of the bike when depreciated by the flat rate method.	
d. How many kilometres has the bike travelled after two years?	1 mark
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		$S_0 = 5000,$ $S_{n+1} = 1.004 S_n$	
V	Wha	at is the annual interest rate (compounding monthly) for Hugo's savings account?	1 ma
_			
V	Wha	at would be the value of Hugo's savings after 12 months?	1 ma
_			
sing the ponth onth of T_n	a d rate n.	different investment strategy, Hugo could deposit \$3000 into an account earning compound integration of 4.2% per annum, compounding monthly, and make additional payments of \$200 after even the value of Hugo's investment after <i>n</i> months using this strategy.	terest ery
sing the n onth et T_n ne m	a d rate n. , be nont i.	different investment strategy, Hugo could deposit \$3000 into an account earning compound inte of 4.2% per annum, compounding monthly, and make additional payments of \$200 after eve e the value of Hugo's investment after n months using this strategy. thly interest rate for this account is 0.35%. Write down a recurrence relation, in terms of T_{n+1} and T_n , that models the value of Hugo's investment using this strategy.	terest ery 1 ma
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SECTION A – continued

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	Rou	w many repayments are required to fully repay this loan? nd your answer to the nearest whole number.	1 mar
fte	r the ner se	fifth repayment, Hugo increased his monthly repayment so that the loan was fully repaid with a even repayments (that is, 12 repayments in total).	-
	i.	What is the minimum value of Hugo's new monthly repayment?	1 mar
			-
			-
	ii.	What is the value of the final repayment required to ensure the loan is fully repaid after 12 repayments?	1 mar
			-
			-

SECTION B – Modules

Instructions for Section B

Select **two** modules and answer **all** questions within the selected modules. Write using blue or black pen. You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π , surds or fractions.

Unless otherwise indicated, the diagrams in this book are not drawn to scale.

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Module 1 – Matrices

Question 1 (2 marks)

Five trout-breeding ponds, P, Q, R, X and V, are connected by pipes, as shown in the diagram below.



The matrix W is used to represent the information in this diagram.

 $W = \begin{bmatrix} P & Q & R & X & V \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} V$

In matrix *W*:

- the 1 in row 2, column 1, for example, indicates that pond P is directly connected by a pipe to pond Q
- the 0 in row 5, column 1, for example, indicates that pond *P* is not directly connected by a pipe to pond *V*.
- **a.** In terms of the breeding ponds described, what does the sum of the elements in row 3 of matrix *W* represent?

1 mark

The matrix W^2 is shown below.

	Р	Q	R	Х	V	
	2	0	0	2	1	P
	0	3	2	1	1	Q
$W^2 =$	0	2	2	0	1	R
	2	1	0	3	1	X
	1	1	1	1	2	V

b. Matrix W^2 has a 2 in row 2 (Q), column 3 (R).

Explain what this number tells us about the pipe connections between Q and R.

1 mark

FURMATH EXAM 2 (SAMPLE)

Question 2 (10 marks)

10000 trout eggs, 1000 baby trout and 800 adult trout are placed in a pond to establish a trout population. In establishing this population:

- eggs (*E*) may die (*D*) or they may live and eventually become baby trout (*B*)
- baby trout (B) may die (D) or they may live and eventually become adult trout (A)
- adult trout (A) may die (D) or they may live for a period of time but will eventually die.

From year to year, this situation can be represented by the transition matrix T, where

$$T = \begin{bmatrix} E & B & A & D \\ 0 & 0 & 0 & 0 \\ 0.4 & 0 & 0 & 0 \\ 0 & 0.25 & 0.5 & 0 \\ 0.6 & 0.75 & 0.5 & 1 \end{bmatrix} D^{E}$$

- **a.** Use the information in the transition matrix *T* to
 - i. determine the number of eggs in this population that die in the first year
 - ii. complete the transition diagram below, showing the relevant percentages.



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1 mark

2 marks

SECTION B – Module 1 – Question 2 – continued

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The initial state matrix for this trout population, S_0 , can be written as				
$\begin{bmatrix} 10000 \end{bmatrix} E$				
		$S_0 = \begin{bmatrix} 1000 \end{bmatrix} B$		
Let	S _n rep	present the state matrix describing the trout population after <i>n</i> years.		
b.	Usir	ng the rule $S_{n+1} = T S_n$, determine		
	i.	S_1	1 mark	
	ii.	the number of adult trout predicted to be in the population after four years.	1 morts	
		Round your answer to the nearest whole number of trout.	Ппатк	
c.	The die.	transition matrix T predicts that, in the long term, all of the eggs, baby trout and adult trout will		
	i.	How many years will it take for all of the adult trout to die (that is, when the number of adult		
		trout in the population is first predicted to be less than one)?	1 mark	
	ii.	What is the largest number of adult trout that is predicted to be in the pond in any one year?	1 mark	
d.	Dete	ermine the number of eggs, baby trout and adult trout that, if added to or removed from the pond at		
	rema	ains constant from year to year.	2 marks	
		SECTION B – Module 1 – Question 2	– continued	

To take breeding into account, assume that every year 50% of the adult trout each lay 500 eggs.

The matrix describing the population after n years, S_n , is now given by the new rule

$$S_{n+1} = T S_n + 500 M S_n$$

where

e. Use this new rule to determine S_2 .

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1 mark

End of Module 1 – SECTION B – continued

Module 2 – Networks and decision mathematics

Question 1 (6 marks)

Water will be pumped from a dam to eight locations on a farm.

The pump and the eight locations (including the house) are shown as vertices in the network diagram below. The numbers on the edges joining the vertices give the shortest distances, in metres, between locations.



The total length of pipe that supplies water from the pump to the eight locations on the farm is a minimum. This minimum length of pipe is laid along some of the edges in the network.

b. i. On the diagram below, draw the minimum length of pipe that is needed to supply water to all locations on the farm.



ii. What is the mathematical term that is used to describe this minimum length of pipe in **part b.i**? 1 mark

1 mark

Question 2 (6 marks)

A project will be undertaken on the farm. This project involves the 13 activities shown in the table below. The duration, in hours, and predecessor(s) of each activity are also included in the table.

Activity	Duration (hours)	Predecessor(s)
A	5	-
В	7	_
С	4	-
D	2	С
Е	3	С
F	15	A
G	4	B, D, H
Н	8	E
Ι	9	<i>F</i> , <i>G</i>
J	9	B, D, H
K	3	J
L	11	J
М	8	I, K

Activity G is missing from the network diagram for this project, which is shown below.



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Givo i.	en that activity <i>G</i> is not on the critical path write down the activities that are on the critical path in the order that they are completed	1 ma
ii.	find the latest starting time for activity <i>D</i> .	1 ma
Con 'If j the	usider the following statement: ust one of the activities in this project is crashed by one hour, then the minimum time to complete entire project will be reduced by one hour.'	1 ma
		1 IIIa
Ass	ume activity F is crashed by two hours.	
Wha	at will be the minimum completion time for the project?	1 ma

End of Module 2 – SECTION B – continued

Module 3 – Geometry and measurement

Question 1 (3 marks)

One of the field events at athletics competitions is the discus.

The field markings for the discus event consist of a circular throwing ring, foul lines and the boundary line of the field, as shown in the diagram below. The shaded area on the diagram is the landing region for a discus throw.



The foul lines meet the boundary line at points *A* and *B*, 65 m from the centre of the throwing ring. The angle θ is 34.92°.

- **a.** What is the length of the boundary line from point *A* to point *B*? Write your answer in metres, rounded to two decimal places.
- b. Calculate the area of the landing region.Round your answer to the nearest square metre.

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1 mark

2 marks

Finc	the shortest great circle distance to the South Pole from Mildura (34° S, 142° E).	
Rou	nd your answer to the nearest kilometre.	1 n
The i	flight from Mildura (34° S, 142° E) to Sydney (34° S, 151° E) travels along a small circle. Find the radius of this small circle	
1.	Round your answer to two decimal places.	1 n
ii.	Find the distance the plane travels between Mildura (34° S, 142° E) and Sydney (34° S, 151° E). Round your answer to the nearest kilometre.	1 m

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How long after the sun rises in Sydney (34° S, 151° E) will the sun rise in Rome (42° N, 12° E)? Round your answer to the nearest minute.	1 marł
	_
Daniel's flight to Rome leaves Sydney airport on Sunday, 6 March at 10.20 am, local time. The flight arrives in Rome on Monday, 7 March at 2.30 am. Assume the time difference between Sydney and Rome is 10 hours.	-
How long does the flight take to travel from Sydney to Rome? Round your answer to the nearest minute.	1 mar
	_
	_
	_
	_
SECTION B – Module	3 - conti

Question 3 (2 marks)

Daniel will compete in the intermediate division of the discus competition. Competitors in the intermediate division use a smaller discus than the one used in the senior division, but of a similar shape. The total surface area of each discus is given below.



By what value can the volume of the intermediate discus be multiplied to give the volume of the senior discus?

Question 4 (2 marks)

Daniel has qualified for the finals of the discus competition.

On his first throw, Daniel threw the discus to point A, a distance of 53.32 m on a bearing of 057° .

On his second throw from the same point, Daniel threw the discus a distance of 57.51 m.

The second throw landed at point B, on a bearing of 125° , measured from point A.

Determine the distance, in metres, between points *A* and *B*. Round your answer to one decimal place.

End of Module 3 – SECTION B – continued TURN OVER

Module 4 – Graphs and relations

Question 1 (8 marks)

Fastgrow and Booster are two tomato fertilisers that contain the nutrients nitrogen and phosphorus. The amount of nitrogen and phosphorus in each kilogram of Fastgrow and Booster is shown in the table below.

	1 kg of Booster	1 kg of Fastgrow
Nitrogen	0.05 kg	0.05 kg
Phosphorus	0.02 kg	0.06 kg

- **a.** How many kilograms of phosphorus are in 2 kg of Booster?
- **b.** If 100 kg of Booster and 400 kg of Fastgrow are mixed, how many kilograms of nitrogen would be in the mixture?

Arthur is a farmer who grows tomatoes.

He mixes quantities of Booster and Fastgrow to make his own fertiliser.

Let *x* be the number of kilograms of Booster in Arthur's fertiliser.

Let *y* be the number of kilograms of Fastgrow in Arthur's fertiliser.

Inequalities 1 to 4 represent the nitrogen and phosphorus requirements of Arthur's tomato field.

Inequality 1	$x \ge 0$
Inequality 2	$y \ge 0$
Inequality 3 (nitrogen)	$0.05x + 0.05y \ge 200$
Inequality 4 (phosphorus)	$0.02x + 0.06y \ge 120$

Arthur's tomato field also requires at least 180 kg of the nutrient potassium.

Each kilogram of Booster contains 0.06 kg of potassium.

Each kilogram of Fastgrow contains 0.04 kg of potassium.

c. Inequality 5 represents the potassium requirements of Arthur's tomato field.

Write down Inequality 5 in terms of *x* and *y*.

Inequality 5 (potassium) ____

1 mark

1 mark

1 mark

The lines that represent the boundaries of Inequalities 3, 4 and 5 are shown in the graph below.



e.

Question 2 (4 marks)

A shop owner bought 100 kg of Arthur's tomatoes to sell in her shop.

She bought the tomatoes for \$3.50 per kilogram.

The shop owner will offer a discount to her customers based on the number of kilograms of tomatoes they buy in one bag.

The revenue, in dollars, that the shop owner receives from selling the tomatoes is given by the piecewise defined relation below

	5.4 <i>n</i>	$0 < n \leq 2$
revenue = <	10.8 + 4(n-2)	$2 < n \le 10$
	a + 2(n-10)	10 < <i>n</i> < 100

where n is the number of kilograms of tomatoes that a customer buys in one bag.

a. What is the revenue that the shop owner receives from selling 8 kg of tomatoes in one bag?

b. A revenue of \$46.80 is received from selling 12 kg of tomatoes in one bag.

Show that a has the value 42.8 in the revenue equation above.

c. Find the maximum number of kilograms of tomatoes that a customer can buy in one bag, so that the shop owner never makes a loss.

2 marks

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1 mark

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END OF QUESTION AND ANSWER BOOK