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YEAR 12 *Trial Exam Paper*

2017

FURTHER MATHEMATICS

Written examination 1

Worked solutions

This book presents:

➢ worked solutions, giving you a series of points to show you how to work through the questions
➢ mark allocations
➢ tips on how to approach the exam

SECTION A – Core

Data analysis

Question 1

Answer: D

Explanatory notes

$$\frac{58}{261} \times 100 = 22.22\%$$

This is closest to 22%.

Question 2

Answer: A

Explanatory notes

Both variables are categorical variables, as they place students into a category.

Within categorical data we have nominal or ordinal data.

Nominal data does not contain an order or hierarchy of any kind.

Ordinal data can be sorted into an order or hierarchy.

In this case, we are not placing either *Sporting house* or *Lunch choice* into an order or hierarchy; therefore, they are both nominal variables.

Question 3

Answer: E

Explanatory notes

It is not suitable to use a stem plot, histogram, dot plot or scatterplot for categorical data; a segmented bar chart is the most suitable graphical tool as it is the most appropriate display when working with two categorical variables. The other types of graphs that have been suggested are suitable for numerical data.

SECTION A

Question 4

Answer: D

Explanatory notes

The median is found by locating the 'middle' data point, when data values are listed in ascending order.

In this stem plot there are 34 data values; therefore, the 'middle' is between the 17th and 18th data values.

Reading off the stem plot, the middle value is between 5.2 and 5.2. Therefore, the median is 5.2 or $5.20.

Tip

- *When finding the middle of two points, add them together and divide by 2.*

Question 5

Answer: D

Explanatory notes

Describing the weights as 'approximately normally distributed' indicates that we should use a bell curve to calculate the percentage of students who fall within this interval.

As seen below, we can then use the bell curve to determine which percentages we need to include.
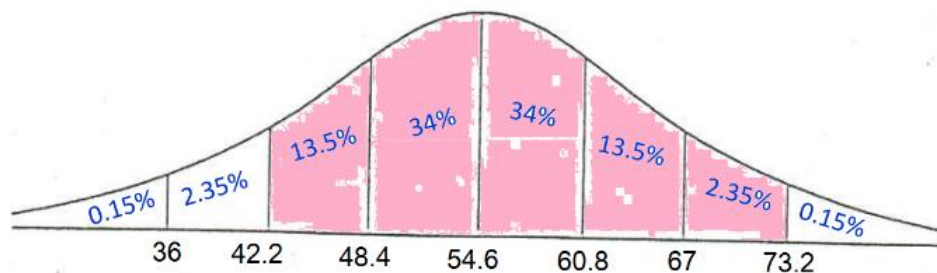
In this case, we require

$$100\% - 2.5\% - 0.15\% = 97.35\%$$

We now need to calculate 97.35% of 186 students:

$$\frac{97.35}{100} \times 186 = 181.071$$

This is closest to 181.

SECTION A

Question 6

Answer: B

Explanatory notes
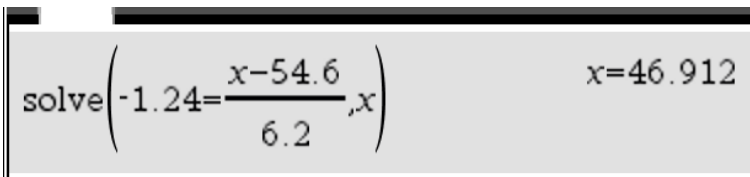
$$z = \frac{x - \bar{x}}{s}$$

$$-1.24 = \frac{x - 54.6}{6.2}$$

$$-7.688 = x - 54.6$$

$$x = 46.912$$

$$\therefore x = 47 \text{ kg}$$

Your CAS calculator can be used to solve this equation directly from the second line of working above.

$$\text{solve}\left(-1.24 = \frac{x - 54.6}{6.2}, x\right) \qquad x = 46.912$$

Question 7

Answer: B

Explanatory notes

$$\log_{10}(0.01) = -2$$

$$0.01 = 10^x \quad \text{or} \quad \frac{1}{100} = 10^x$$

$$\therefore x = -2$$

The frequency (*number*) of data less than –2 on the histogram is equal to $1 + 2 = 3$.

Total frequency is equal to $1 + 2 + 9 + 13 + 4 + 1 = 30$.

Therefore, the percentage of data less than 0.01 is equal to

$$\frac{3}{30} \times 100 = 10\%$$

This is closest to 10%.

Question 8

Answer: D

Explanatory

Option A is true because the 1st quartile for the Year 9 boxplot is approximately equal to the 3rd quartile for the Year 8 boxplot.

Option B is true because the distribution for Year 8 male students does not contain an outlier and is positively skewed.

Option C is true because when we compare the interquartile range for the boxplots, we can see that the interquartile range (IQR) for Year 8 students is approximately 12, whereas the IQR for Year 9 students is approximately 15. From this we can conclude that the boxplots have approximately the same IQR and are therefore similar in variance.

Option D is **not** true because it is not suitable to use the mean as a measure of centre for distributions that are either skewed or contain an outlier, or both. In this case, the Year 8 distribution is skewed. This question asks for the statement that is **not** true, this is therefore our answer.

Option E is true because the distribution for Year 8 male students is positively skewed, with a centre at 58 kg.

Question 9

Answer: C

Explanatory notes

Option A is true. We have been given the correlation coefficient, r, and can square this to find the coefficient of determination, r^2. $(0.7883)^2 = 0.6214$

Option B is true. This is an interpretation of the slope of our regression equation.

Option C is **not** true. This is an interpretation of the coefficient of determination, r^2; however, the correlation coefficient, r, has been used in this interpretation. This question asks for the statement that is **not** true, this is therefore our answer.

Option D is true. An r value of 0.7883 corresponds to a positive, strong relationship. We can observe from the scatterplot that the relationship is linear.

Option E is true. This is an interpretation of the *y*-intercept.

Tip

- *With questions that ask you to determine which answer is **not** true, make sure you test out all five answers to see which are true and which **one** is not true.*

SECTION A

Question 10

Answer: B

Explanatory notes

First, we need to substitute Belinda's study hours into the least squares regression equation:

$exam\ result = 36.8 + 5.7 \times 7.6$
$exam\ result = 80.12$

We now calculate our residual value:

$residual = actual - predicted$
$residual = 75 - 80.12$
$residual = -5.12$

Question 11

Answer: D

Explanatory notes

Pick out some key points and look at their residual values.

Note that a point above the least squares regression line will have a positive residual value, whereas a point below the least squares regression line will have a negative residual value.

Question 12

Answer: E

Explanatory notes

Use your CAS calculator to perform a transformation on the original data.

Tip

- *Ensure that you are performing the transformation to number of hours studied, in this case, the explanatory variable.*

SECTION A

Question 13

Answer: C

Explanatory notes

Note that we can describe this graph as only having 'irregular fluctuations'.
There is no repeating pattern within a year (seasonality) or within a regular, longer period of time (cyclical). The fluctuations are random.

Question 14

Answer: E

Explanatory notes

When interpreting a seasonal index, we consider a seasonal index of 1.0 as being *on average*.

Any value below 1.0 is *below average*. And any value higher than 1.0 is *above average*.

To find the percentage above or below the average, we compare this to 1.0.

For example, for a seasonal index of 0.87, we subtract the index from 1.0:

$1.0 - 0.87 = 0.13$

We can therefore conclude that this is 13% below the average.

With a seasonal index above 1, we can subtract 1.0 from the seasonal index to find the difference.

For example, for a seasonal index of 1.2, we subtract 1.0 from the index:

$1.2 - 1.0 = 0.2$

We can therefore conclude that this is 20% above the average.

Question 15

Answer: B

Explanatory notes

Step 1 – Calculate an average for the week:

$(1\,820 + 1\,700 + 1\,467 + 1\,476 + 1\,765 + 1\,239 + 1\,348) \div 7 = 1\,545$

Step 2 – Divide the sales for Tuesday by the weekly average:

$1\,700 \div 1\,545 = 1.10$

This matches the calculation in option B.

SECTION A

Question 16

Answer: B

Explanatory notes

$$\text{deseasonalised sales} = \frac{\text{actual sales}}{\text{seasonal index}}$$
$$\text{deseasonalised sales} = \frac{1825}{1.14}$$
$$\text{deseasonalised sales} = 1600$$

Recursion and financial modelling

Question 17

Answer: B

Explanatory notes

$A_0 = 3$
$A_1 = -2 \times 3 + 3 = -6 + 3 = -3$
$A_2 = -2 \times -3 + 3 = 6 + 3 = 9$
$A_3 = -2 \times 9 + 3 = -18 + 3 = -15$

Therefore, the first four terms written in a sequence are:

3, –3, 9, –15, …

Question 18

Answer: C

Explanatory notes

A. This represents a combination of geometric decay and arithmetic growth.

B. This represents geometric growth only.

C. This represents geometric decay only.

D. This represents a combination of geometric growth and arithmetic growth.

E. This represents arithmetic growth only.

SECTION A

Question 19

Answer: D

Explanatory notes

Compound interest can be modelled using the following general relation:

$V_0 = $ inital investment, $\qquad V_{n+1} = R \times V_n$

where $R = 1 + \dfrac{r}{100}$

In this case, the interest is calculated monthly, and so we need to convert the interest to a monthly rate.

$3.8 \div 12 = 0.317$

Therefore:

$R = 1 + \dfrac{0.317}{100} = 1.00317$

We must also remember to add the additional payment to the recurrence relation, therefore leading to the answer:

$V_0 = 10\,000, \qquad V_{n+1} = 1.00317 \times V_n + 250$

Tip

- *As this is an investment, the payment that Jessica is making is added to the overall value. If this were a loan, we would subtract this payment. When we have compound interest (or a reducing balance loan) we may need to change the annual interest rate.*

  The following information is useful when doing this:

  compounding bi-yearly   divide by 2

  compounding quarterly   divide by 4

  compounding monthly   divide by 12

  compounding fortnightly   divide by 26

  compounding weekly   divide by 52

Question 20

Answer: E

Explanatory note

$$V_0 = 32\,000, \qquad V_{n+1} = 1.018 \times V_n - 785$$

Using this recurrence relation:

$$V_1 = 1.018 \times V_0 - 785 = 1.018 \times 32\,000 - 785 = 31\,791$$

$$V_2 = 1.018 \times V_1 - 785 = 1.018 \times 31\,791 - 785 = 31\,578.2$$

$$V_3 = 1.018 \times V_2 - 785 = 1.018 \times 31\,578.2 - 785 = 31\,361.6$$

$$V_4 = 1.018 \times V_3 - 785 = 1.018 \times 31\,361.6 - 785 = 31\,141.2$$

Question 21

Answer: C

Explanatory notes

The graph shows an increase in the value, and is therefore an investment. This excludes options A, B and D.

Option E would result in a graph that increased by the same amount each month. This graph is increasing by slightly more each month.

Option C is correct because it shows an investment, increasing by slightly more each month. This shows a compounding interest account.

Question 22

Answer: C

Explanatory notes

Calculate the interest earned on the loan:

$$\frac{(5.4/12)}{100} \times 274\,724.38 = 1\,236.26$$

Calculate the balance of the loan:

Balance + interest – payment

$$274\,724.38 + 1\,236.26 - 1\,375 = 274\,585.64$$

Calculate the reduction in principal:

$$274\,724.38 - 274\,585.64 = 138.74$$

SECTION A

Question 23

Answer: C

Explanatory notes

Using the finance solver in the CAS calculator, find the value of Christine's investment after 2 years.

Use this value to find the interest earned on the account, by subtracting the money that Christine has added to the investment herself:

interest $=$ value $-$ principal $-$ payments

interest $= 8\,916.57 - 4\,500 - 8 \times 500$

interest $= 416.57$

 Tips

- *Remember that if money is being invested in the bank, your value should be entered as a negative into the finance solver.*

- *Carefully read the question – do not assume that interest or payments are added monthly. In this case, we have payments and interest paid quarterly. This means that PpY and CpY are 4.*

SECTION A

Question 24

Answer: C

Explanatory notes

Start by calculating the value of Peter's loan after 2 years of monthly payments at $648.53.

```
Finance Solver
      N:   24
   I(%):   4.25
     PV:   35000
    Pmt:   -648.53
    FV:   -21883.942275924
    PpY:   12
          Finance Solver info stored into
          tvm.n, tvm.i, tvm.pv, tvm.pmt, …
```

Now change the payment to $750, and calculate the time that it will take to repay the loan under these new conditions.

Notice that we will move the FV into PV for these new conditions.

```
Finance Solver
      N:   30.853415414758
   I(%):   4.25
     PV:   21883.9
    Pmt:   -750
    FV:   0
    PpY:   12
          Finance Solver info stored into
          tvm.n, tvm.i, tvm.pv, tvm.pmt, …
```

It will now take 31 months to repay the remaining balance of this loan.

In total, this is $24 + 31 = 55$ months.

Therefore, Peter has saved 5 months on his loan period.

SECTION A

SECTION B – Modules
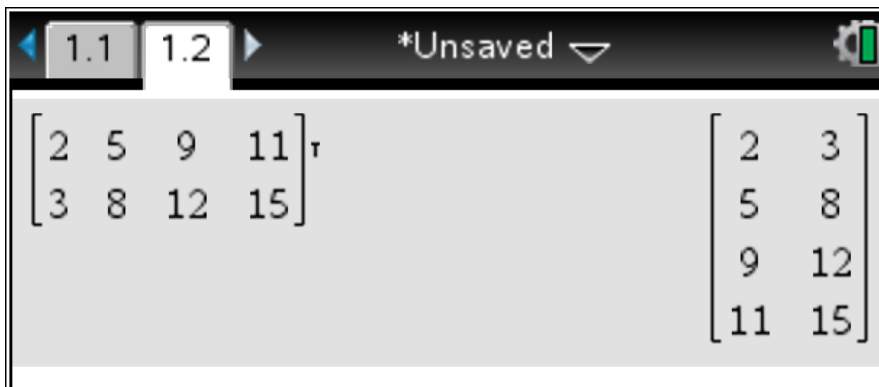
Module 1 – Matrices

Question 1

Answer: C

Explanatory notes

The transpose of a matrix is found by 'swapping' the position of all rows and columns; for example, the element in row 1, column 2 would move to row 2, column 1.

This can also be done using your CAS calculator.

Using the Ti-Nspire™ – this can be found through MENU – 7 – 2.

Question 2

Answer: B

Explanatory notes

A set of simultaneous linear equations have a unique solution only when the determinant does not equal zero.

Using the 2×2 matrix, the determinant is equal to $\det = ad - bc$.

As the determinant cannot equal zero, we can set up an equation to solve.

We find where the determinant equals zero.

$$8\times2 - 4\times s = 0$$
$$16 - 4s = 0$$
$$-4s = -16$$
$$s = 4$$

Therefore, $s \neq 4$ for these equations to have a unique solution.

Question 3

Answer: E

Explanatory notes

In order to find a total, we are looking for a matrix multiplication that will result in a 1×1 matrix.

The calculation that will give us this is $\begin{bmatrix} 3.50 & 2.50 & 4.00 & 1.75 \end{bmatrix} \begin{bmatrix} 65 \\ 156 \\ 105 \\ 256 \end{bmatrix}$

If we consider the matrix multiplication being performed,

$3.50\times65+2.50\times156+4.00\times105+1.75\times256=1485.50$

we obtain the total spent on drinks.

Question 4

Answer: D

Explanatory notes

$\begin{bmatrix} \text{row 1, column 1} & \text{row 1, column 2} \\ \text{row 2, column 1} & \text{row 2, column 2} \\ \text{row 3, column 1} & \text{row 3, column 2} \end{bmatrix} = \begin{bmatrix} i=1, j=1 & i=1, j=2 \\ i=2, j=1 & i=2, j=2 \\ i=3, j=1 & i=3, j=2 \end{bmatrix} = \begin{bmatrix} 2\times1+1 & 2\times2+1 \\ 2\times1+2 & 2\times2+2 \\ 2\times1+3 & 2\times2+3 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 4 & 6 \\ 5 & 7 \end{bmatrix}$

Question 5

Answer: D

Explanatory notes

Each of the columns of a transition matrix, where the total number does not change, should add to 1.

Option D is the only matrix in which each column adds to 1.

Tip

- *In this matrix, 1 means that when a family has chosen to shop at Ultramarket, they will not change their selection. They will continue to return to Ultramarket.*

Question 6

Answer: C

Explanatory notes

$$\begin{bmatrix} 0.65 & 0.24 & 0.39 \\ 0.15 & 0.54 & 0.33 \\ 0.20 & 0.22 & 0.28 \end{bmatrix}^2 \times \begin{bmatrix} 215 \\ 175 \\ 60 \end{bmatrix} = \begin{bmatrix} 206.857 \\ 142.349 \\ 100.795 \end{bmatrix}$$

The number of students selecting juice, rounded to the nearest whole number, is equal to 142.

Question 7

Answer: A

Explanatory notes

To find the long-term prediction, we need to find the steady state matrix.

$$\begin{bmatrix} 0.65 & 0.24 & 0.39 \\ 0.15 & 0.54 & 0.33 \\ 0.20 & 0.22 & 0.28 \end{bmatrix}^{30} \times \begin{bmatrix} 215 \\ 175 \\ 60 \end{bmatrix} = \begin{bmatrix} 208.698 \\ 140.423 \\ 100.879 \end{bmatrix}$$

To ensure this is the steady state, we find a consecutive matrix to confirm that our state is unchanging.

$$\begin{bmatrix} 0.65 & 0.24 & 0.39 \\ 0.15 & 0.54 & 0.33 \\ 0.20 & 0.22 & 0.28 \end{bmatrix}^{31} \times \begin{bmatrix} 215 \\ 175 \\ 60 \end{bmatrix} = \begin{bmatrix} 208.698 \\ 140.423 \\ 100.879 \end{bmatrix}$$

The number of students selecting iced tea, in the long term, is closest to 209.

SECTION B – Module 1

Question 8

Answer: B

Explanatory notes

We must first consider one-step dominance. A matrix is the best way to record this.

$$D_1 = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{ccccc} A & B & C & D & E \\ \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

Next, we consider the two-step dominance.

This can be found by squaring the one-step dominance matrix:

$$D_2 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix}^2 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 2 & 2 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

We can rank the players based on their one- and two-step dominances:

One-step
$A = 1$
$B = 3$
$C = 1$
$D = 2$
$E = 3$

Two-step
$A = 1$
$B = 5$
$C = 2$
$D = 4$
$E = 4$

Rankings can be made by adding the dominance scores together:

$A = 1 + 1 = 2$
$B = 3 + 5 = 8$
$C = 1 + 2 = 3$
$D = 2 + 4 = 6$
$E = 3 + 4 = 7$

Therefore, the winner of the tournament is Benjamin.

SECTION B – Module 1

Module 2 – Networks and decision mathematics

Question 1

Answer: A

Explanatory notes

A trail is a walk that contains no repeated edges.

The walk shown repeats vertices, but not edges.

Additional information for students

A path is a walk with no repeated edges or vertices.

A cycle is a path (no repeated edges or vertices) that starts and ends at the same vertex.

A circuit is a trail (no repeated edges) that starts and ends at the same vertex.

Question 2

Answer: C

Explanatory notes

We can use trial and error to find the shortest distance between *A* and *D*.

Alternatively, we can use Dijkstra's algorithm.

	B	C	F	G	E	D
A	72	47	45	X	X	X
F	67	47	45	X	X	83
C	67	47	45	159	X	83
B	67	47	45	159	X	83

Question 3
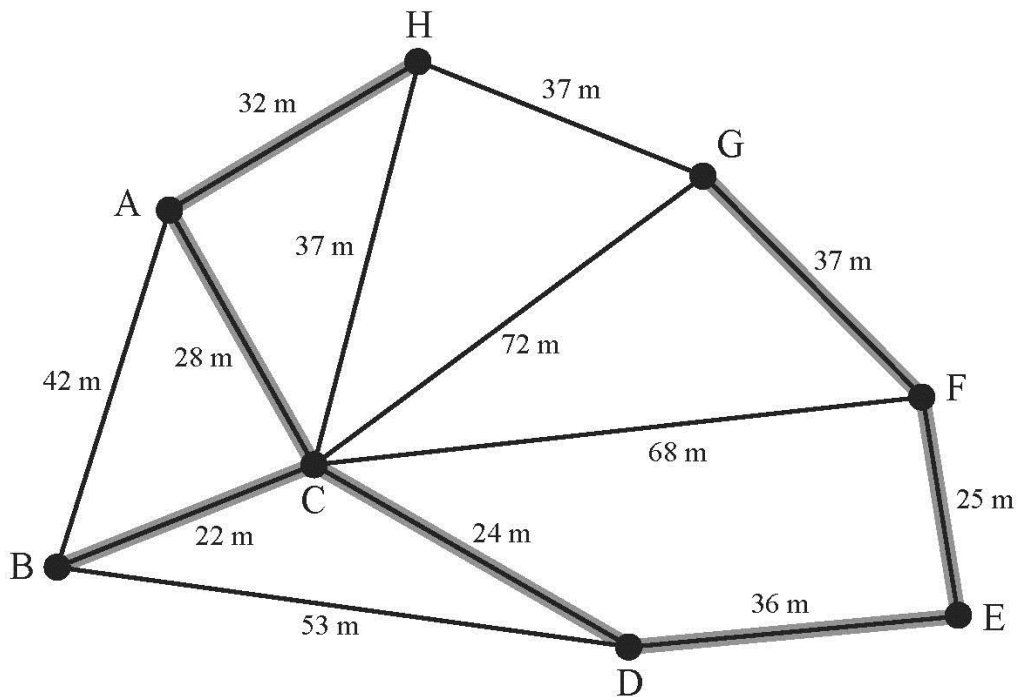
Answer: E

Explanatory notes

A Eulerian circuit is possible when all vertices in the network have a degree that is an even number.

In this network, vertex *A* has a degree of 3 and vertex *E* also has a degree of 3. Therefore, they have an 'odd degree'.

If we were to add an edge between these vertices, they would both have an even degree, therefore making a Eulerian circuit possible.

Question 4

Answer: D

Explanatory notes

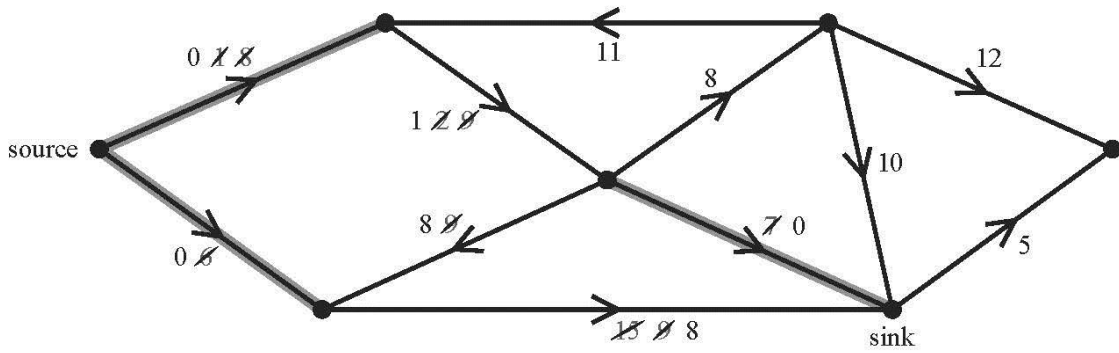The minimum spanning tree is shown on the network below.

As we can see, the edge between *G* and *C* is not used in the minimum spanning tree.
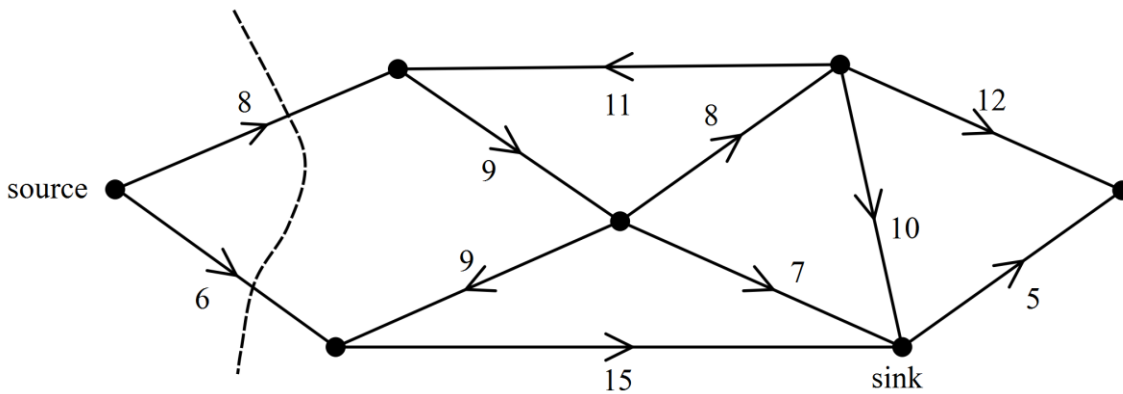
SECTION B – Module 2

Question 5

Answer: B

Explanatory notes

Using tracking

Alternatively, this can be found using the minimum cut = maximum flow method.

Cuts can be found using trial and error. The minimum cut is shown below.

SECTION B – Module 2

Question 6

Answer: A

Explanatory notes

Using the Hungarian algorithm to find the minimum allocation:

Step 1 – Row reduction

(Subtract the smallest number in each row from the other numbers in the row.)

	Adam	Bryce	Carey	Donald
Job 1	0	7	3	4
Job 2	4	0	6	3
Job 3	4	5	0	2
Job 4	2	0	1	3

Step 2 – Column reduction

(Subtract the smallest number in each column from the other numbers in the column.)

	Adam	Bryce	Carey	Donald
Job 1	0	7	3	2
Job 2	4	0	6	1
Job 3	4	5	0	0
Job 4	2	0	1	1

Step 3 – Determine the minimum number of lines required to cover all zeros in the table.

	Adam	Bryce	Carey	Donald
Job 1	0		3	2
Job 2	4	0	6	1
Job 3	4		0	0
Job 4	2	0	1	1

Only three lines are required; however, 4 have jobs. We therefore require an additional step.

SECTION B – Module 2

Step 4 – Find the smallest uncovered number.

Subtract this from each uncovered number.

Add this to any number underneath a point where the lines cross.

	Adam	Bryce	Carey	Donald
Job 1	0	7	3	2
Job 2	3	0	5	0
Job 3	4	0	0	0
Job 4	1	0	0	0

Step 5 – Repeat Step 3, finding the minimum number of lines required to cover the zeros.

	Adam	Bryce	Carey	Donald
Job 1	0	a	1	2
Job 2	3	0	3	0
Job 3	4	0	0	0
Job 4	1	0	0	0

SECTION B – Module 2

The minimum number of lines is now equal to the number of jobs.

We can do our allocations.

Step 6 – Allocate jobs to students, using zeros as a guide.

Option 1:

	Adam	Bryce	Carey	Donald
Job 1	0	8	3	2
Job 2	3	0	5	0
Job 3	4	6	0	0
Job 4	1	0	0	0

Option 2:

	Adam	Bryce	Carey	Donald
Job 1	0	8	3	2
Job 2	3	0	5	0
Job 3	4	6	0	0
Job 4	1	0	0	0

In both allocations, Adam must do Job 1.

SECTION B – Module 2

Question 7

Answer: B

Explanatory notes

A dummy activity is required to indicate that both activity *E* and activity *F* are predecessors of activity *J*; however, only activity *E* is required for activity *H*.

The statement that is correct from above is option B: activity *J* cannot start until both activities *F* and *E* have been completed.

Question 8

Answer: D

Explanatory notes

SECTION B – Module 2
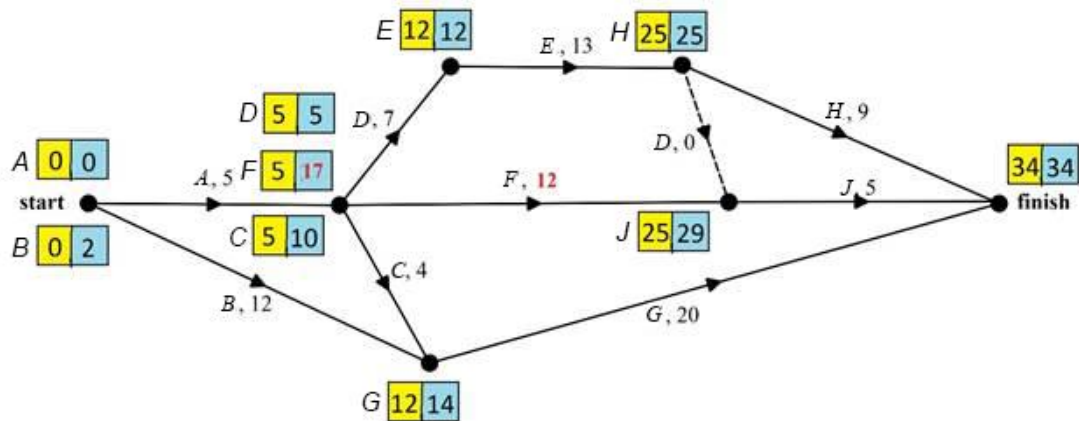
Module 3 – Geometry and measurement

Question 1

Answer: B

Explanatory notes

Area of large circle: $\pi r^2 = \pi \times 6^2 = 113.0973$

Area of smaller circle: $\pi r^2 = \pi \times 2^2 = 12.5669$

Area of triangle: $\dfrac{1}{2} \times b \times h = \dfrac{1}{2} \times 2 \times 5 = 5$

Area of shaded region = area of large circle – area of smaller circles – area of triangle

$$= 113.0973 - 2 \times 12.5668 - 5$$
$$= 82.9643$$
$$\approx 83$$

Question 2

Answer: C

Explanatory notes

Step 1: Find the ratio for these similar triangles.

$$\dfrac{6.3}{4.2} = 1.5$$

Step 2: Set up an equation to find the unknown side, *x*.

$$\dfrac{x + 2.5}{x} = 1.5$$

Step 3: Solve this equation for *x*, using either algebra or your CAS calculator.

$x + 2.5 = 1.5x$

$\quad 2.5 = 0.5x$

$\quad\quad x = 5$

SECTION B – Module 3

Question 3

Answer: C

Explanatory notes

Area = area of minor sector OAB − area of triangle AOB

Area = $\dfrac{\pi r^2 \theta}{360} - \dfrac{1}{2} r^2 \sin \theta$

Area = $\dfrac{\pi \times (7.5)^2 \times 66}{360} - \dfrac{1}{2} \times (7.5)^2 \times \sin(66°)$

Area = 6.7 cm^2

Question 4

Answer: E

Explanatory notes

The two cities have the same longitude; therefore, they are on the same great circle.

The total angle of both points representing the cities is: $(28+56)° = 84°$.

We now find the arc length between these two points.

The formula for arc length is $s = \dfrac{\pi r \theta}{180}$.

When looking at the Earth as a sphere, the radius is 6400 km.

The angle, θ, is equal to 84.

Therefore, the length of the arc is:
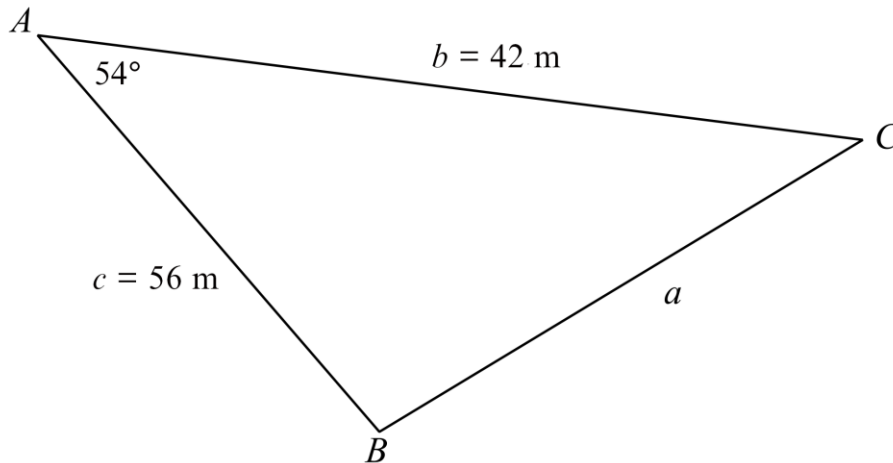
$s = \dfrac{\pi \times 6400 \times 84}{180}$

$s = 9382.89$ km

$s \approx 9383$ km

SECTION B − Module 3

Question 5

Answer: B

Explanatory notes

We have been given a non-right-angled triangle, with two known sides and the included angle. This means that we can use the cosine rule.

We start by labelling the sides and angles.

The cosine rule, in general, is:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Using the values given:

$$a^2 = (42)^2 + (56)^2 - 2 \times 42 \times 56 \cos(54°)$$

$$a^2 = 2\,135.058\,17\,...$$

$$a = \sqrt{2\,135.058\,17\,...}$$

$$a = 46.206\,68\,...$$

$$a = 46.21$$

SECTION B – Module 3

Question 6

Answer: C

Explanatory notes

Find the difference in longitude:

$144 - 142 = 2$

$2 \div 15 = 0.1333\ldots$

Find the time difference:

$0.1333 \times 60 = 8$ minutes

Find the time in Terang:

9.30 a.m. $+\ 8$ min $= 9.38$ a.m.

Georgia will receive her results at 9.38 a.m.

Question 7

Answer: C

Explanatory

Calculate the volume of the cylindrical cup:

$V = \pi r^2 h$
$V = \pi \times (2.5)^2 \times 22$
$V = 431.9689\ldots$

Calculate the volume of the hemispherical lid:

$V = \dfrac{1}{2} \times \dfrac{4}{3}\pi r^3$
$V = \dfrac{1}{2} \times \dfrac{4}{3} \times \pi \times (2.5)^3$
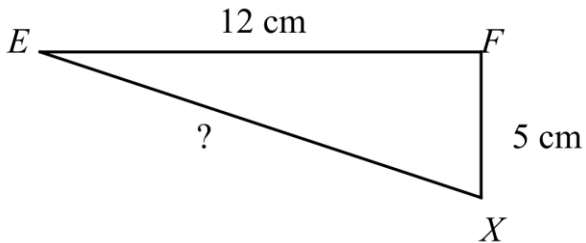$V = 32.7249\ldots$

Total volume is the total of the two sections:

$431.9689 + 32.7249 = 464.6938$
$\therefore V = 465 \, \text{cm}^3$

Question 8

Answer: B

Explanatory notes

Step 1: Calculate the length of the base of the triangle created by the rod.
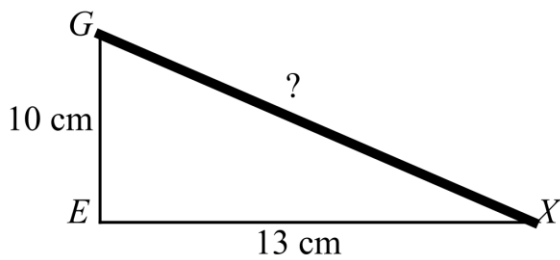
Find the length of the diagonal within the rectangular base.

Pythagoras' theorem: $c^2 = a^2 + b^2$

We are trying to find c: $c = \sqrt{a^2 + b^2} = \sqrt{12^2 + 5^2} = 13$

Step 2: Find the length of the rod by creating a vertical triangle within the box.

We are trying to find c: $c = \sqrt{a^2 + b^2} = \sqrt{10^2 + 13^2} = 16.4012 \text{ cm} = 16 \text{ cm}$

SECTION B – Module 3

Module 4 – Graphs and relations

Question 1

Answer: E

Explanatory notes

Firstly, we must find the equation for the straight line.

Step 1: Select two points on the graph as an example – (0, 2) and (8, 6).

Use these points to find the gradient of the straight line.

$$\text{gradient} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 2}{8 - 0} = \frac{4}{8} = \frac{1}{2}$$

Step 2: Use the gradient and one point to find the equation for the straight line graph.

$$y - y_1 = m(x - x_1)$$
$$y - 2 = \frac{1}{2}(x - 0)$$
$$y = \frac{1}{2}x + 2$$

To find the *x*-intercept, we must find the point where $y = 0$.

We can therefore solve the equation, when $y = 0$:

$$0 = \frac{1}{2}x + 2$$
$$-2 = \frac{1}{2}x$$
$$x = -4$$

The coordinate of the *x*-intercept is (–4, 0).

Question 2

Answer: D

Explanatory notes

The $29 monthly line rental fee is the constant. The cost of the internet bill alters based on the number of megabytes used.

Question 3

Answer: D

Explanatory notes

Testing each set of coordinates will show that the only set that satisfies the inequality is option D.

$$\left(1, \frac{2}{5}\right), \quad \frac{2}{5} \geq \frac{2}{5} \times 1, \quad \text{true}$$

$$\left(2, \frac{4}{5}\right), \quad \frac{4}{5} \geq \frac{2}{5} \times 2, \quad \text{true}$$

Question 4

Answer: D

Explanatory notes

This is an interpretation of the inequality, in its most literal sense.

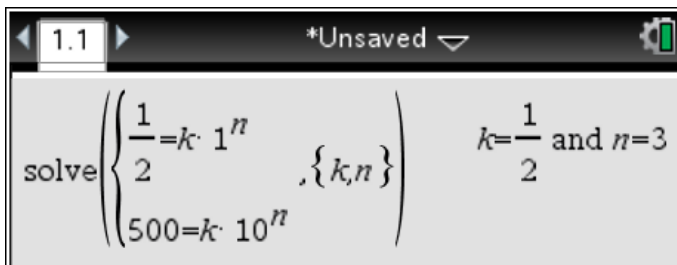We know that the symbol $\geq$ means greater than or equal to.

Question 5

Answer: D

Explanatory notes

Create two simultaneous equations, using the general form $y = kx^n$ in order to solve for k and n.

Equation 1: $\qquad \frac{1}{2} = k \times 1^n$

Equation 2: $\qquad 500 = k \times 10^n$

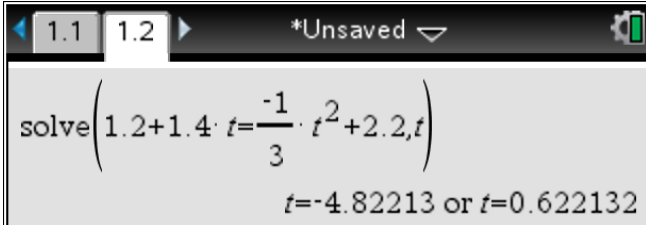Solving simultaneously (this can be done using your CAS calculator) gives

$k = \frac{1}{2}$ and $n = 3$.

SECTION B – Module 4

Question 6

Answer: D

Explanatory

We must first find the point of intersection for the two balls.

We can use our CAS device to solve when the two equations are equal for *t*.

We can ignore the negative answer. Therefore, the intersection occurs when $t = 0.6221$.

We then convert this to seconds by multiplying by 60.

$t = 0.6221 \times 60$

$t = 37.32$

$t \approx 37\,\text{s}$

Question 7

Answer: E

Explanatory notes

$k \times 11 = 600$

$\therefore\ k = 60$

Question 8

Answer: D

Explanatory notes

We must find the equation of the straight line.

We can identify that the *y*-intercept is 4 by substituting $x = 0$ into the equation.

From this, we can identify the line on the graph, recognising that the *x*-intercept is 6.

Substituting (6, 0) into the equation gives:

$a \times 6 + 3 \times 0 = 12$

$6a = 12$

$a = 2$

END OF WORKED SOLUTIONS

SECTION B – Module 4