2017
VCE
Further Mathematics
Trial Examination 2
Suggested Answers

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Data analysis

Question 1

a.

Sydney						Melbourne				
			9	1	**1**	8				
8	7	6	5	5	**2**	0	1			
	5	3	1	1	**3**	1	2	2	4	5
			0	0	**4**	0	1	1	2	3

Key 1/8 = 18^0C

(1 mark)

b. i.
At least 32^0C on 9 days.
$\frac{9}{13} \times 100 = 69.2\%$

(1 mark)

b. ii.
Range for Melbourne = 43 – 18 = 25^0C

(1 mark)

b. iii.
Use calculator
33.08^{0}C

(1 mark)

c. i.
Symmetric

(1 mark)

c. ii.
Negatively skewed.

(1 mark)

d. i.
The median is the middle number.
Median = 28^0C

(1 mark)

d. ii.
Lower Quartile = 25
Upper Quartile = 34
Interquartile range = 34 – 25 = 9^0C

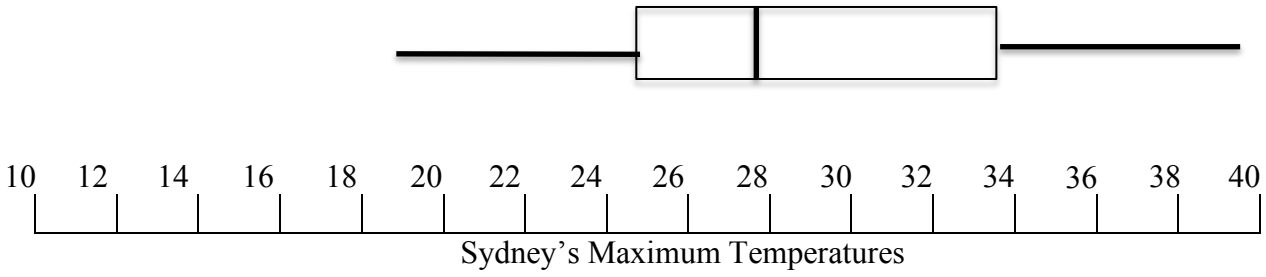
(1 mark)

d. iii.
1.5 × 9 = 13.5
Upper fence = 34 + 13.5 = 47.5
Lower fence = 25 – 13.5 = 11.5
There is one value less than 11.5 so 11 is an outlier. There are no values greater than 47.5
(1 mark for getting 13.5 and one mark for identifying the outlier.

(2 marks)

Data analysis

d. iv.

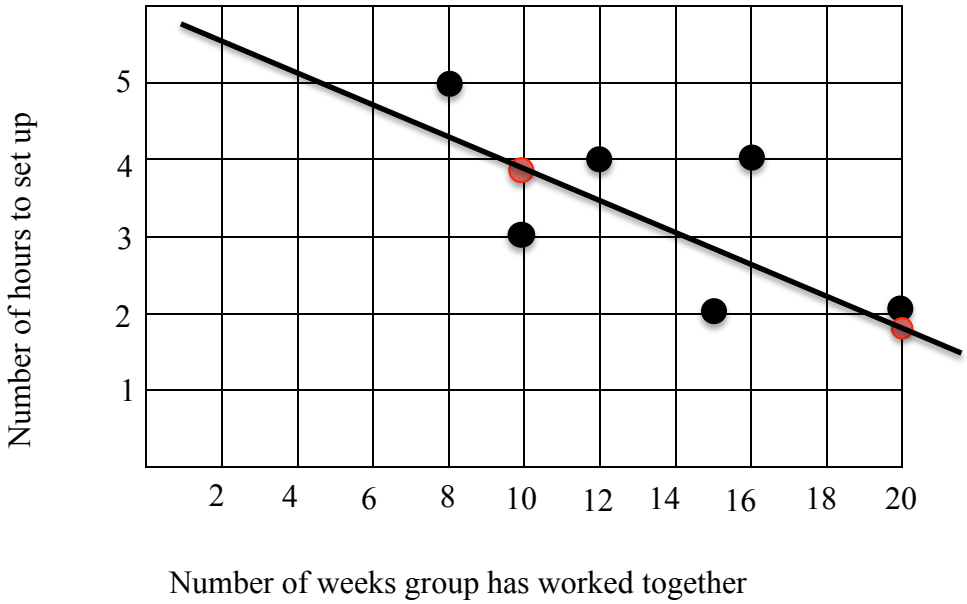


(1 mark for outlier entered correctly and 1 mark for remainder of boxplot correct.)

(2 marks)

Question 2

a.



(1 mark)

b. i.

$$\text{Number of hours to set up} = -0.2 \times \text{Number of weeks group has worked together} + 5.9$$

(1 mark)

Data analysis

Question 2 (continued)

b. ii.
When $x = 10$, $y = 3.9$
When $x = 20$, $y = 1.9$
Mark these two points on the graph and draw the least squares regression line through these two points.

(1 mark)

b. iii.
For every extra week that the group has worked together, there is a decrease of 0.2 hours in the time it takes for the group to set up the concert venue.

(1 mark)

c
From calculator, $r^2 = 0.463$
This means that 46.3% of the variation in the number of hours taken to set up the concert venue can be explained by the variation in the number of weeks that the group has worked together,

(1 mark)

d.
An explanatory variable value of 3 lies outside the values used in the given data. Hence, we would have to extrapolate to find the value of the response variable when the explanatory variable is 3, so the result would not be reliable,

(1 mark)

Question 3

a.

$$z = \frac{3.6 - 4.8}{0.6} = -2$$

(1 mark)

b.

$$\frac{1997}{2000} = 0.9985$$

$$1 - 0.9985 = 0.0015$$

$$= 0.15\%$$

So 0.15% of the students were ranked below Nathan.
Z value for this = -3

$$-3 = \frac{x - 5.2}{0.8}$$

$$x = 2.8\text{m}$$

(1 mark)

Data analysis

Question 4

a.
Quarterly profit ÷ seasonal average = seasonal index for that quarter
40000 ÷ seasonal average = 0.74
Seasonal average for 2014 = \$54054 = \$54000 to the nearest thousand.

(1 mark)

b.
Seasonal index for second quarter of 2015 must be the same as that for the first quarter of that year because their profits are the same.
Sum of seasonal indices = 4
0.82 + 0.82 + 1.24 = 2.88
4 – 2.88 = 1.12

(1 mark)

c.
First mean =
(42000 + 42000 + 58000 + 64000) ÷ 4
= 51500
Second mean =
(42000 + 58000 + 64000 + 46000) ÷ 4
= 52500
Central mean = (51500 + 52500) ÷ 2 = \$52000

1 mark for correct first and second mean
1 mark for correct centring of the mean

(2 marks)

Recursion and financial modelling

Question 5

a.
Interest = 10% of 500 = \$50
Each year 150 + 50 = 200 is added to the account.
Recurrence equation is
$V_{n+1} = V_n + 200, \quad V_0 = 500$

(1 mark)

b.
Use calculator and the recurrence relation to get 700, 900, 1100, 1300, 1500, 1700, 1900, 2100, 2300, 2500.
\$2500

(1 mark)

c.
$A = 500 + 200n$

(1 mark)

d.
$A = 500 + 200 \times 50 = \$10500$

(1 mark)

e. i.
$V_{n+1} = 1.05V_n + 150, \quad V_0 = 500$

(1 mark)

e. ii.
Use calculator and the recurrence relation to get \$2701.13 on day after tenth birthday.
Difference = 2701.13−2500 = \$201.13 = \$201 to the nearest dollar.

(1 mark)

Recursion and financial modelling

Question 6

a.
Monthly interest rate = 6 ÷ 12 = 0.5%

(1 mark)

b.
Use solver on calculator
$N = 144$
$I = 6$
$PV = 200000$
$PMT =$
$FV = 0$
$P/Y = 12$
$C/Y = 12$
This gives $PMT = \$1951.70$

(1 mark)

c.
144 repayments of \$1951.70
$= 144 \times 1951.70 =$ \$281045
Interest = 281045 −200000 = \$81045

(1 mark)

Question 7

a.
Use solver on calculator
$N = 120$
$I = 5$
$PV =$ -150000
$PMT =$ -500
$FV =$
$P/Y = 12$
$C/Y = 12$
This gives $FV =$ \$324,692.56

(1 mark)

b.
Use solver on calculator
$N =$
$I = 6$
$PV =$ -324,692.56
$PMT =$ 30000
$FV = 0$
$P/Y = 1$
$C/Y = 1$
This gives $N = 17.9$
She would only be able to withdraw \$30000 per year for 17 years.

(1 mark)

c.
$3000 = 8\% \times x$
$x =$ \$37500

(1 mark)

Module 1 – Matrices

Question 1

a.

X is a 2×4 matrix because it has 2 rows and 4 columns.

(1 mark)

b.

$$Z = \begin{bmatrix} 200 & 50 & 95 & 180 \\ 126 & 174 & 140 & 108 \end{bmatrix} \begin{bmatrix} 1.8 \\ 2.0 \\ 2.5 \\ 1.6 \end{bmatrix} = \begin{bmatrix} 985.5 \\ 1097.6 \end{bmatrix}$$

(1 mark)

c.

The number of columns in Y is 1. This does not equal the number of rows in X, which is 2.

(1 mark)

d.

985.5 is the number of hours that females in the given town spent watching movies on *Movieflix* on the particular evening and 1097.6 is the number of hours that males in the same town spent watching movies on *Movieflix* on the same evening.

(1 mark)

Question 2

a.

$$P = 1.1 \begin{bmatrix} 500 \\ 480 \\ 220 \\ 540 \end{bmatrix} + \begin{bmatrix} -40 \\ -80 \\ -30 \\ 0 \end{bmatrix}$$

$a = 1.1$

(1 mark)

$b = -40$, $c = -80$, $d = -30$, $e = 0$

(1 mark for b, c, d and e all correct)

b.

2018 =

$$P = 1.1 \begin{bmatrix} 500 \\ 480 \\ 220 \\ 540 \end{bmatrix} + \begin{bmatrix} -40 \\ -80 \\ -30 \\ 0 \end{bmatrix} = \begin{bmatrix} 510 \\ 448 \\ 212 \\ 594 \end{bmatrix}$$

2019 =

$$P = 1.1 \begin{bmatrix} 510 \\ 448 \\ 212 \\ 594 \end{bmatrix} + \begin{bmatrix} -40 \\ -80 \\ -30 \\ 0 \end{bmatrix} = \begin{bmatrix} 521 \\ 412.8 \\ 203.2 \\ 653.4 \end{bmatrix}$$

2020 =

$$P = 1.1 \begin{bmatrix} 521 \\ 412.8 \\ 203.2 \\ 653.4 \end{bmatrix} + \begin{bmatrix} -40 \\ -80 \\ -30 \\ 0 \end{bmatrix} = \begin{bmatrix} 533.1 \\ 374.08 \\ 193.52 \\ 718.74 \end{bmatrix}$$

There will be 374 Westerns available in 2020.

(1 mark)

Module 1 – Matrices

Question 2 (continued)

c.

Use calculator to multiply previous answer by 1.1 and add $\begin{bmatrix} -40 \\ -80 \\ -30 \\ 0 \end{bmatrix}$

This gives 374.08 after 3 years, 331.4 after 4 years, 284.6 after 5 years, 233.1 after 6 years and 176.4 after 7 years. So less than 200 after 7 years.

(1 mark)

Question 3

a.

$$\begin{array}{cc} & \begin{matrix} T & W & SF & C \end{matrix} \\ \begin{matrix} To \\ \\ \\ \\ \end{matrix} & \begin{matrix} T \\ W \\ SF \\ C \end{matrix} \begin{bmatrix} 0.85 & 0.08 & 0.00 & 0.03 \\ 0.10 & 0.80 & 0.00 & 0.00 \\ 0.00 & 0.09 & 1.00 & 0.27 \\ 0.05 & 0.03 & 0.00 & 0.70 \end{bmatrix} \end{array}$$

From

(1 mark)

b.

It tells us that originally 1000 people watched Thrillers, 2000 people watched Westerns, 500 people watched Science Fiction and 3000 people watched Comedies.

(1 mark)

c.

$$\begin{bmatrix} 0.85 & 0.08 & 0.00 & 0.03 \\ 0.10 & 0.80 & 0.00 & 0.00 \\ 0.00 & 0.09 & 1.00 & 0.27 \\ 0.05 & 0.03 & 0.00 & 0.70 \end{bmatrix}^3 \begin{bmatrix} 1000 \\ 2000 \\ 500 \\ 3000 \end{bmatrix}$$

$$= \begin{bmatrix} 1133.9 \\ 1289.7 \\ 2818.3 \\ 1258.1 \end{bmatrix}$$

1290 people will watch a western for their fourth movie.

(1 mark)

d.

$$\begin{bmatrix} 0.85 & 0.08 & 0.00 & 0.03 \\ 0.10 & 0.80 & 0.00 & 0.00 \\ 0.00 & 0.09 & 1.00 & 0.27 \\ 0.05 & 0.03 & 0.00 & 0.70 \end{bmatrix}^{100} \begin{bmatrix} 1000 \\ 2000 \\ 500 \\ 3000 \end{bmatrix}$$

$$= \begin{bmatrix} 0.7 \\ 0.5 \\ 6498.6 \\ 0.2 \end{bmatrix}$$

None will watch comedy in the long run. In fact everyone will watch Science fiction.

(1 mark)

Module 2: Networks and decision mathematics

Question 1

a.
A path has no repeated edges or vertices.
Can go R – V – A = 4 + 5 = 9
R – V – T – A = 4 + 2 + 1 = 7
R– V – T – S – A = 4 + 2 + 3 + 4 = 13
R– V – T – S – U – A = 4 + 2 + 3 +1 + 3 = 13
R– V – U – A = 4 + 1 + 3 = 8
R– V – U – S – A = 4 + 1 + 1 + 4 = 10
R– V – U – S – T – A = 4 + 1 + 1 + 3 + 1 = 10
R– U – A = 7 + 3 = 10
R– U – V – A = 7 + 1 + 5 = 13
R– U – V – T – A = 7 + 1 + 2 + 1 = 11
R– U– V – T – S – A = 7 + 1 + 2 +3 + 4 = 17
R– U – S –A = 7 + 1 + 4 = 12
R– U – S –T– A = 7 + 1 + 3 + 1 = 12
R– U – S – T – V – A = 7 + 1 + 3 + 2 + 5 = 18
There are 14 different paths from R to A.

(1 mark)

b.
The shortest path is
R – V – T – A
4 + 2 + 1 = 7 km

(1 mark)

c.
An Euler circuit exists only when all the vertices are even. Here T and S are not even.
An Euler path exists if only 2 of the vertices are odd, which is the case here.

(1 mark)

d.(i)
For an Euler path you begin and end on an odd vertex. Begin on T and end on S or begin on S and end on T.

(1 mark)

Question 2

a.
The numbers going down the column are
1 – 1, 3 – 1, 8 – 1, 2 – 1, 7 – 1
= 0, 2, 7, 1, 6

(1 mark)

b.
The minimum number of lines required to cover all the zeros is 4, which is less than 5.

(1 mark

c.
Sue should do the wet areas.
Tom should move the paintings.
Val should do the vacuuming.
Umberto should polish.
Riza should dust.

Umberto should be allocated the task of polishing.

(1 mark

d.
Sue should do the wet areas = 9 hours.
Tom should move the paintings = 6 hours.
Val should do the vacuuming = 7 hours.
Umberto should polish = 10 hours.
Riza should dust = 6 hours.
Total number of hours
= 9 + 6 + 7 + 10 + 6 = 38 hours.

(1 mark)

Module 2: Networks and decision mathematics

Question 3

a. The critical path is the longest path. Here, the longest path is C – H – I = 3 + 15 + 8 = 26 weeks. This is the minimum time for the extension to be completed. (1 mark)	**b.** There is a new critical path now The longest path is now C – G – J = 3 + 12 + 12 = 27 weeks. This is now the minimum time for the extension to be completed. So the extension will take one week longer to complete. (1 mark)
c. Extra workers for activity A will not reduce the overall time for the project, so it is a waste of money to bring in extra workers for activity A. The project will still take 27 weeks to complete, at \$6000 per week = 27 × 6000 = \$162000 plus 3 × 800 = 2400 for the extra workers. The whole project would cost \$2400 more. (1 mark)	**d.** Extra workers for activity G would produce a new critical path C – H – I, which would take 26 weeks. This would reduce the overall time for the project by 1 week and thus save \$6000. The project will now take 26 weeks to complete, at \$6000 per week = 26 × 6000 = \$156000 plus 3 × 800 + 1 × 1600 = \$4000 for the extra workers. So they save \$6000 - \$4000 = \$2000 (1 mark)

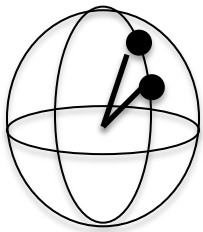
Module 3: Geometry and measurement

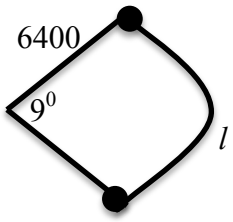
Question 1

a.
Towns on the same longitude have the same time.

(1 mark)

b.





Difference between 66^0N and 57^0N
$= 66 - 57 = 9^0$

$$l = \frac{2\pi \times 6400 \times 9}{360} = 1005\,\text{km}$$

(1 mark)

Question 2

a.

$AC^2 = 12^2 + 28^2$

$AC = 30.5$ cm

(1 mark)

b.

Height of triangular base = 8 cm. (Use triad of right angle triangle 6:8:10)

Area of 2 triangles = $\frac{1}{2} \times 12 \times 8 \times 2 = 96$

Area of 2 slant rectangles = 28 × 10 × 2 = 560

Area of flat rectangle = 12 × 28 = 336

Total surface area = 96 + 560 + 336 = 992 cm^2

(1 mark)

c.

Volume of box = Area of triangular base × height

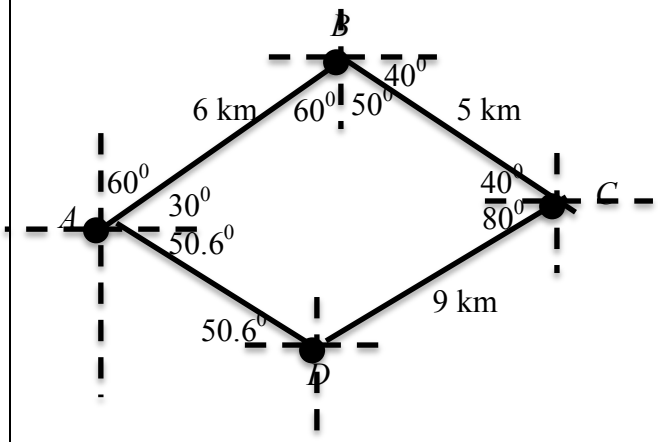
V = ½ × 12 × 8 × 28 = 1344 cm^3

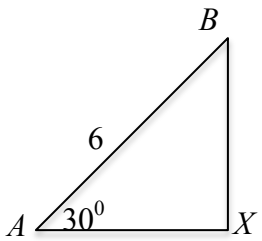
(1 mark)

Module 3: Geometry and measurement

Question 3

a.





Let X be a point east of A and south of B.

$\angle BAX = 30^0$

$AB = 6$

$\cos 30^0 = \dfrac{BX}{6}$

$BX = 5.2 \text{ km}$

(1 mark)

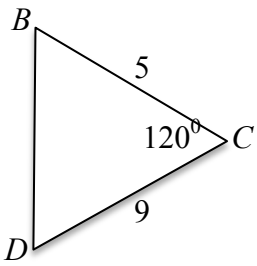
b.

$\angle XAD = 50.6^0$ (alternate angles)

Bearing of D from A = 60 + 30 + 50.6 = 140.6^0T

(1 mark)

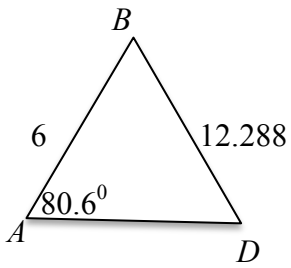
c.



$BD = \sqrt{5^2 + 9^2 - 2 \times 5 \times 9 \cos 120^0}$

$BD = 12.288$

(1 mark)



$\dfrac{12.288}{\sin 80.6^0} = \dfrac{6}{\sin D}$

$D = 28.8^0$

$B = 180 - (28.8 + 80.6) = 70.6^0$

$\dfrac{AD}{\sin 70.6^0} = \dfrac{6}{\sin 28.8^0}$

$AD = 11.747 = 12 \text{ km}$

(1 mark)

Module 3: Geometry and measurement

Question 4

a.

$$x = \frac{360}{6} = 60^0$$

(1 mark)

b.

$$\text{Area of one triangle} = \frac{1}{2} \times 8 \times 8 \sin 60^0$$

$$= 27.71$$

$$\text{Area of 6 triangles} = \times 27.71 = 166.3 \text{ cm}^2$$

(1 mark)

c.

$V = \text{Area of base} \times \text{height}$

$832 = 166.3 \times d$

$d = 5 \text{ cm}$

$V_1 : V_2 = 832 : 13 = 64 : 1$

$l_1 : l_2 = \sqrt[3]{64} : \sqrt[3]{1} = 4 : 1$

$d_1 : d_2 = 4 : 1$

$d_1 = 5$

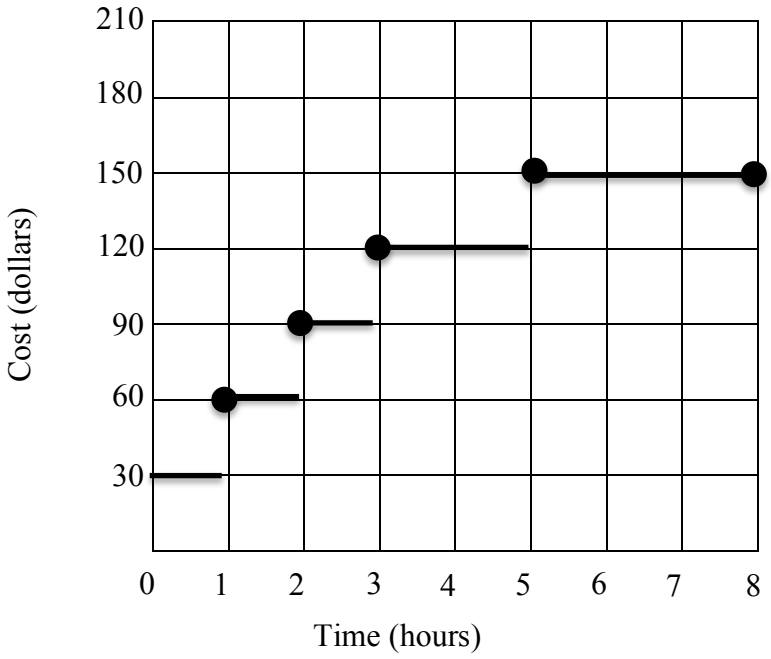
$d_2 = \frac{5}{4} = 1.25 \text{ cm}$

(1 mark)

Module 4: Graphs and relations

Question 1

a.



(1 mark)

b.

$120

(1 mark)

c.

$$C = \begin{cases} 30 & 0 < t < 1 \\ 60 & 1 \le t < 2 \\ 90 & 2 \le t < 3 \\ 120 & 3 \le t < 5 \\ 150 & 5 \le t \le 8 \end{cases}$$

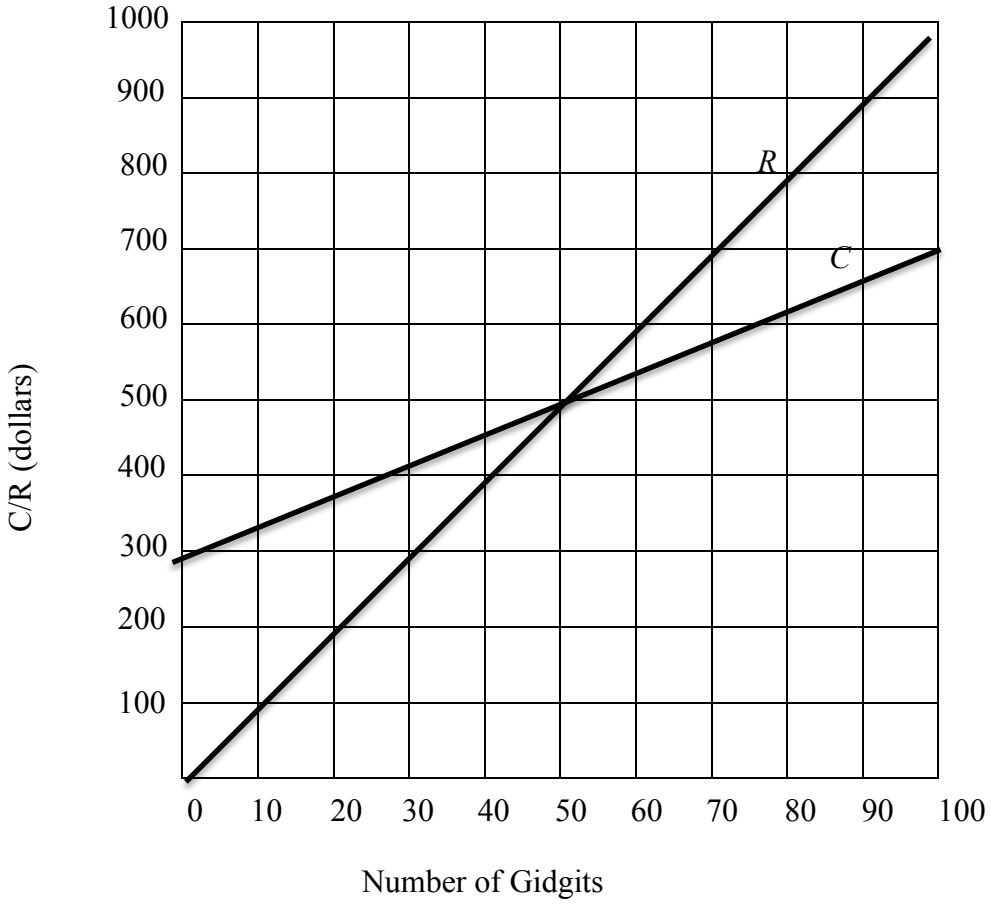
(1 mark)

Module 4: Graphs and relations

Question 2

a. $C = 300 + 4x$ (1 mark)	**b.** $2300 = 300 + 4x$ $x = 500$ (1 mark)
c. $10x = 300 + 4x$ $x = 50$ (1 mark)	**d.** $R = 10x$ contains the points (0,0) and (100, 1000) $C = 300 + 4x$ contains the points (0,300) and (100, 700)

d. (continued)



(2 marks)

e.
To make a profit he must sell more than 50. Since packs of 40 he would have to sell 80 as the minimum number to make a profit. This equals 80 ÷ 40 = 2 packets.

(1 mark)

Module 4: Graphs and relations

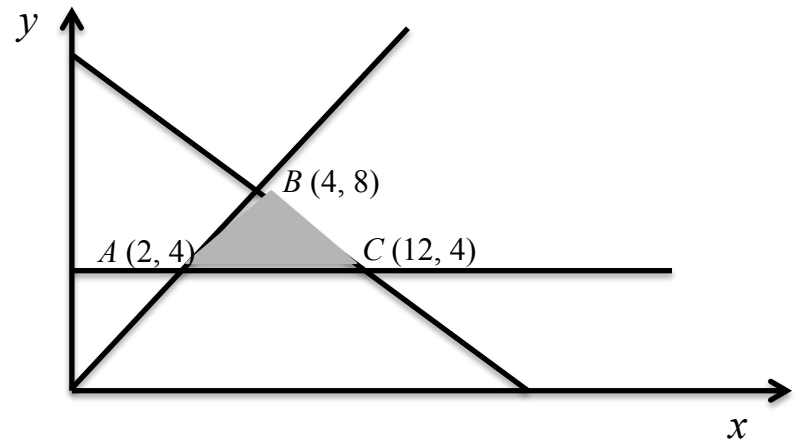
Question 3

a.

$y \leq 2x$

(1 mark)

b.



(1 mark)

c.

$P = 2x + 3y$
At A, $P = 2 \times 2 + 3 \times 4 = 16$
At B, $P = 2 \times 4 + 3 \times 8 = 32$
At C, $P = 2 \times 12 + 3 \times 4 = 36$
He should produce 12 blicks and 4 blocks

(1 mark)

End of Suggested Solutions 2017 VCE Further Mathematics Trial Examination 2

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