The Mathematical Association of Victoria

Trial Examination 2017

FURTHER MATHEMATICS

Trial Written Examination 1 - SOLUTIONS

SECTION A: Core

Question	Answer	Question	Answer
1	В	13	Е
2	А	14	D
3	D	15	D
4	D	16	А
5	С	17	В
6	D	18	E
7	D	19	С
8	С	20	А
9	В	21	С
10	Е	22	D
11	А	23	D
12	С	24	А

SECTION B: Modules Module 1: Matrices

Question	Answer
1	D
2	Е
3	В
4	А
5	С
6	Е
7	Е
8	Е

Module 3: Geometry & measurement

Question	Answer
1	C
2	А
3	В
4	D
5	Е
6	C
7	C
8	Е

Module 2 : Networks & decision mathematics

Question	Answer
1	D
2	E
3	D
4	В
5	С
6	В
7	А
8	D

Module 4 : Graphs & relations

Question	Answer
1	В
2	С
3	В
4	В
5	D
6	С
7	А
8	E

Data Analysis

Question 1 Answer B

There are a total of 25 years in the statistics and there are 10 years where there were values of 8.3 or more, so the following calculation is used:

$$Percentage = \frac{10}{25} \times 100$$
$$= 40\%$$

Question 2 Answer A

The data can be produced in a table as shown below:

Mark out of 10	Frequency
4	1
5	2
6	4
7	3
8	6
9	8
10	1

Using a CAS calculator the mean, median and mode are found to be 7.56, 8 and 9 respectively.

Question 3 Answer D

The traders with an income greater than \$10 000 is required. As this is a log scale graph, the value on the horizontal scale that represents \$10 000 is actually log $(10\ 000) = 4$. There are a total of 6 values in the columns from 4 - < 5 and from 5 - < 6 out of a total of 1 + 8 + 5 + 4 + 2 = 20 traders in the sample.

$$Percentage = \frac{6}{20} \times 100$$
$$= 30\%$$

Question 4 Answer D

The size of a person's house (1 = small, 2 = medium, 3 = large) is an ordinal categorical variable and their income in dollars is a numerical variable. For one categorical and one numerical variable the appropriate displays are back to back stem and leaf plots or parallel box plots. Because there are three categories, a back to back stem and leaf plot is not appropriate (maximum is two categories) so a set of parallel box plots is the only option.

Question 5 Answer C

By adding the life vest each person's weight increases by 2 kg, making the mean weight also increase by 2 kg to 77 kg. The standard deviation does not change because all weights have increased by the same amount and so there is no change in the distances of weight from the mean.

Question 6 Answer D

The sum of the five tests is 79 + 84 + 75 + 81 + 76 = 395.

For six tests to have a mean of 80, the sum of the values will be $6 \times 80 = 480$.

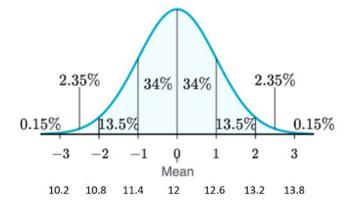
Hence the sixth value is 480 - 395 = 85.

Question 7 Answer D

The value of $10.8 = 12 - 2 \times 0.6$, so it is two standard deviations below the mean.

The value of $11.4 = 12 - 1 \times 0.6$, so it is one standard deviation below the mean.

The data is normally distributed so, using the 68 - 95 - 99.7% rule there are 16% of values below 11.4 seconds and 2.5% of values below 10.8 seconds, so there are 16 - 2.5 = 13.5% between the two values.



Question 8 Answer C

Liam's and Con's times can be calculated as shown below:

Liam	Con
$1.2 = \frac{x - 12.0}{x - 12.0}$	$-0.3 = \frac{x - 12.0}{x - 12.0}$
0.6	0.6
x = 12.72	x = 11.82

Therefore Con runs 12.72 - 11.82 = 0.9 seconds faster.

Question 9 Answer B

Samples drawn from a population are most likely to have similar statistics to population parameters if the sample is collected in a manner which avoids bias, so B is the best answer.

Answer A is incorrect because it is possible, even though less likely, for a biased sample to have similar statistics to population parameters.

Answers C and D are incorrect because choosing an unbiased sample is not a guarantee that the summary statistics always will or will not be the same as population parameters.

Option E is incorrect because there are no situations where summary statistics will always be the same as population parameters.

Question 10 Answer E

While all options contain true statements only option E makes a comparison of a response variable based on differences in the explanatory variable.

Option A is incorrect as it would not support the contention that there is an association as the values stated are similar.

Options B and D are incorrect as they make no comparisons between each type of voter.

Option C is incorrect because the statement doesn't compare one response variable but instead changes response variables.

Question 11 Answer A

The value of r can be calculated using $\sqrt{0.919} = \pm 0.959$. The relationship between these two variables is negative as the gradient of the given equation is negative, so the correct answer is -0.959.

Question 12 Answer C

The following calculations are used:

Day	Rainfall	4 point moving average	Centred Value
2	12		
3	3		
		$\frac{12+3+8+23}{4} = 11.5$	
4	8		$\frac{11.5+9.5}{2} = 10.5$
		$\frac{3\!+\!8\!+\!23\!+\!4}{4}\!=\!9.5$	
5	23		
6	4		

Question 13 Answer E

There is seasonality as there are regular peaks and troughs every seven (7) days. The major peaks on the sixth day of each week decrease with time and the homework time in the first four days of each week are decreasing with each week, so there is an overall decreasing trend.

Question 14 Answer D

With four seasons, the sum of all the seasonal indices should be 4.000. Quarter 4 value = 4.000 - (0.857 + 0.987 + 1.152) = 1.004

Question 15 Answer D

The quarterly average index is 1.000 or 100%. Quarter 3 index = 1.152 or 115.2%This is GREATER than the quarterly average by 115.2 - 100 = 15.2%

Question 16 Answer A

Quarter 1, 2017 is t = 9 using the time scale given. The deseasonalised cost is given by *deseasonalised cost* = $180.590 + 15.10 \times 9 = 316.40$ The seasonal index for Quarter 1 is 0.857. The following calculation determines the actual data value: Actual value = 316.40×0.857

= \$271.15

Recursion and Financial Modelling

Question 17 Answer B

Reducing balance depreciation represents a multiple of $1 - \frac{18}{100} = 0.82$. The form of the recurrence relation requires that the starting value is given as $V_0 = 32\ 000$.

Question 18 Answer E

The book value has reduced by 3500 - 1795 = \$1705. There are $1705 \div 7.75 = 220$ units of depreciation. The number of copies is $220 \times 500 = 110\ 000$ copies.

Question 19 Answer C

The monthly interest rate is $\frac{52}{8000} \times 100 = 0.65\%$. The yearly rate is given by $0.65 \times 12 = 7.8\%$.

Question 20 Answer A

The three missing entries can be calculated as follows: The interest to be paid is $\frac{0.65}{100} \times 737.57 = 4.79$ correct to the nearest cent. The amount to be paid is the previous balance plus interest or 737.57 + 4.79 = \$742.36. The principal reduction to reach zero must be the same as the previous balance, that is \$737.57.

Question 21 Answer C

The rate of reducing balance depreciation is higher than the rate of flat rate depreciation because there is a greater amount of depreciation in the first year. This is indicated by the book after one year of \$15 300 compared to a book value of \$16 200 for flat rate.

Question 22 Answer D

A perpetuity is an investment where the balance remains the same because only interest is taken from the account. This means that for the recurrence relation to represent a perpetuity the amount taken each time must be equivalent to the interest paid. This is the case in option D because $1.057 \times 125\ 000 = 132\ 125\ and\ 132125 - 7125 = \$125\ 000$, which is the original investment amount.

The following calculations are used:	
The original payment must be determined for	Compound Interest
the loan as originally stated. The required equal	N 24
payments would be \$743.53	1% 4.7
	PV 17000
	PMT -743.5317234
	FV 0
	P/Y 12
	C/Y 12
Amount owing after 18 months must be	Compound Interest
determined. This is \$4400.70 and will be the starting value for the next phase of the loan.	N 18
starting value for the next phase of the toan.	1% 4.7
	PV 17000 PMT -743.53
	PMT -743.53 FV -4400.700082
	P/Y 12
	C/Y 12
The new payment for the last 6 months is	Compound Interest
calculated as \$745.95 per month.	N 6
	1% 5.4
	PV 4400.7
	PMT -745.0450592
	FV 0
	P/Y 12
	C/Y 12
Therefore he needs to pay $745.05 - 743.53 = 1.5	
\$9.12 over the six month period. This is closest to) \$9.10.

Question 23 Answer D The following calculations are used:

Question 24 Answer A The following calculations are used:

The following calculations are used:	
The amount invested under the annuity to receive \$4200 each month for 20 years is \$663 874.83.	N 240 1% 4.5 PV -663874.8346 PMT 4200 FV 0 P/Y 12 C/Y 12
The amount required under a perpetuity would be given by $4200 = \frac{4.5}{100} \times x$ $x = \$1120000$	This means an increase of 1 120 000 – 663 874.83 = \$456 125.17. This is \$456 000 correct to three sig figs.

SECTION B – Modules

Module 1 - Matrices

Question 1 Answer D

The first column becomes the first row (eliminate option A). The second column becomes the second row (eliminate options B and E). The third column becomes the third row (eliminate option C)

Question 2 Answer E

For a unique solution, the determinant does NOT equal zero, i.e. $ad - bc \neq 0$. Checking the options :

А	$6 \times 4 - 8 \times 3 = 24 - 24 = 0$	NO
В	$2 \times 6 - 4 \times 3 = 12 - 12 = 0$	NO
С	$9 \times 4 - 12 \times 3 = 36 - 36 = 0$	NO
D	$2 \times 9 - 6 \times 3 = 18 - 18 = 0$	NO
E	$4 \times 2 - 3 \times 3 = 8 - 9 = -1$	YES

Question 3	Answer B		
Matrix assembly	y is $\begin{bmatrix} 1-1+2\\ 2-1+2\\ 3-1+2\\ 4-1+2 \end{bmatrix}$	$\begin{bmatrix} 1-2+2\\ 2-2+2\\ 3-2+2\\ 4-2+2 \end{bmatrix} =$	$ \left[\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$

Question 4 Answer A

Raising the permutation matrix to the same power as the number of moves taking place is essentially the same as multiplying the data matrix by the identity matrix.

In this case, there are only two moves occurring in the permutation matrix, as the third and fourth letters always remains the third and fourth letters, and the power required is two, so the word formed is the same as that give initially (OPTS).

This can be seen by using the calculator as shown below:

 $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^2$

Alternatively change matrix
$$K = \begin{bmatrix} O \\ P \\ T \\ S \end{bmatrix}$$
 to $K = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$.
Then $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^2 \times \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$, so it can be seen that the matrix does not change.

Question 5 Answer C

For a 25% discount, the final price will be 100 - 25 = 75% of original, a multiple of 0.75 for A. For a 30% discount, the final price will be 100 - 30 = 70% of original, a multiple of 0.70 for B. For a 25% mark-up, the final price will be 100 + 25 = 125% of original, a multiple of 1.25 for C. For a 30% mark-up, the final price will be 100 + 30 = 130% of original, a multiple of 1.30 for D. Only one matrix, C, contains these in correct order down lead diagonal.

Question 6 Answer E

Given that March is S_3 the January is S_1 . Because $S_3 = T^2 \times S_1$ then $S_1 = T^{-2} \times S_3$.

 $\begin{bmatrix} 0.75 & 0.30 \\ 0.25 & 0.70 \end{bmatrix}^{-2} \times \begin{bmatrix} 258 \\ 242 \end{bmatrix} = \begin{bmatrix} 200 \\ 300 \end{bmatrix}$

Question 7 Answer E

To find the long term values, calculate either T^n or T^nS_0 for large values of n.

$$T^{20} = T^{21} = \begin{bmatrix} 0.583... & 0.583... \\ 0.416... & 0.416... \end{bmatrix}$$
 or $T^{20}S_0 = T^{21}S_0 = \begin{bmatrix} 58.3... \\ 41.6... \end{bmatrix}$

Boulevard therefore has a market share of approximately 58 % which is approximately 7 % DOWN on 65% where it started.

Question 8 Answer E

Two (2) calculations will be required with this matrix recurrence relation.

$$S_{1} = \begin{bmatrix} 0.92 & 0.01 & 0.01 & 0.01 \\ 0.02 & 0.95 & 0.01 & 0.02 \\ 0.04 & 0.02 & 0.97 & 0.03 \\ 0.02 & 0.02 & 0.01 & 0.94 \end{bmatrix} \times \begin{bmatrix} 35500 \\ 43500 \\ 51000 \\ 42000 \end{bmatrix} + \begin{bmatrix} 750 \\ 875 \\ 1025 \\ 825 \end{bmatrix} = \begin{bmatrix} 34775 \\ 44260 \\ 54045 \\ 42395 \end{bmatrix}$$
$$S_{2} = \begin{bmatrix} 0.92 & 0.01 & 0.01 & 0.01 \\ 0.02 & 0.95 & 0.01 & 0.02 \\ 0.04 & 0.02 & 0.97 & 0.03 \\ 0.02 & 0.02 & 0.01 & 0.94 \end{bmatrix} \times \begin{bmatrix} 34775 \\ 44260 \\ 54045 \\ 42395 \end{bmatrix} + \begin{bmatrix} 750 \\ 875 \\ 1025 \\ 825 \end{bmatrix} = \begin{bmatrix} 34150 \\ 45006 \\ 56997 \\ 42797 \end{bmatrix}$$

The required element is 56 997.

Module 2: Networks and decision mathematics

Question 1 Answer D

Option A can be eliminated as the boundary between M and K has been missed. Option B can be eliminated as the boundary between K and G has been missed. Option C can be eliminated as the boundary between B and M has been missed. Option E can be eliminated as a boundary between B and Y that does not exist is in the diagram.

Question 2 Answer E

Using Euler's rule V + R = E + 2, the rule can be rewritten using x as both V and E. x can then be eliminated from both sides:

$$x + R = x + 2$$
$$R = 2$$

Question 3 Answer D

The original diagram has 6 vertices, 9 edges and 5 regions. All options have the same number of vertices edges and regions.

The degrees of each of the vertices in the original diagram are 5, 3, 3, 3, 2 and 2.

Option D has a central vertex of degree 5 and all other vertices have degrees of 3, so it is not the same as the original network.

Question 4 Answer B

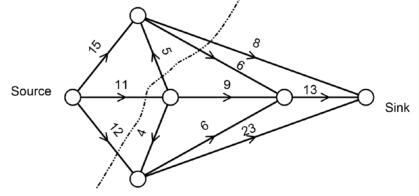
Using Prim's algorithm and starting at the vertex at the top left hand corner, the order of selection is the edge of weight 2, followed by each of the three edges of weight 4, then the edges of weight 6 and 10.

Question 5 Answer C

The capacity of the cut is given by 15 + 11 + 0 + 6 + 23 = 55. The edge labelled 4 is not counted as its capacity is reduced to 0 as the flow into it is already cut.

Question 6 Answer B

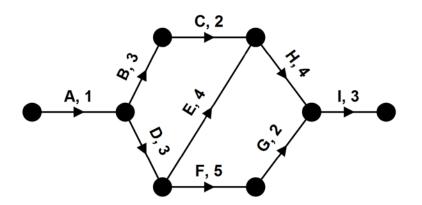
The maximum flow through this network from source to sink is given by the cut shown below:



This cut has a capacity of 8 + 6 + 0 + 11 + 12 = 37.

Question 7 Answer A

The network below can be drawn from the provided table.



The minimum number of days taken for the project is the longest path through this network. The paths through this network are ABCHI (13 days), ADEHI (15 days) and ADFGI (14 days). The longest path therefore is 15 days.

Question 8 Answer D

The paths through the original network and their durations are shown in the table below:

Path	Duration in weeks
ADH	15
ADGI	11
AEI	12
BCDH	26
BCEI	23
BCDGI	22
BFI	18

The critical path is therefore BCDH with a time of 26 weeks. Reducing each of these activities means that the path is reduced to 18 weeks. Other paths would also be reduced by this action, with BCEI reduced by 4 weeks to 19 weeks (B and C on same path), BCDGI reduced by 6 weeks to 16 weeks (B, C and D on same path).

The new critical path is therefore 19 weeks, a reduction of 7 weeks overall.

Module 3: Geometry and Measurement

Question 1 Answer C

The angle directly below θ will be 112° as it is corresponding to the given angle.

 $\theta + 112^{\circ} = 180^{\circ}$ (straight line), so $\theta = 180^{\circ} - 112^{\circ} = 68^{\circ}$.

Question 2 Answer A

Kolkata (88° E) is further east than Ahmedabad (72° E), so sunset will occur in Kolkata BEFORE Ahmedabad. The difference in longitude is $88^\circ - 72^\circ = 16^\circ$.

The actual time difference = $\frac{16^{\circ}}{15^{\circ}} \times 60 = 64$ minutes.

64 minutes before 6.20 pm will be 5.16 pm.

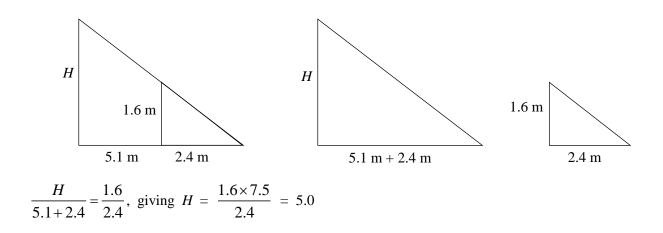
Question 3 Answer B

Total volume of ice-cream = volume of hemisphere (r = 3) + volume of cone (r = 3, h = 12)

$$V = \frac{1}{2} \times \frac{4 \times \pi \times 3^3}{3} + \frac{1}{3} \times \pi \times 3^2 \times 12 = 169.646... \approx 170$$

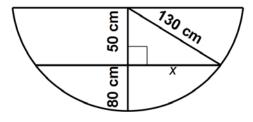
Question 4 Answer D

Using similar triangles, and letting H = height of streetlight



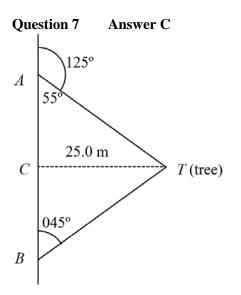
Question 5 Answer E

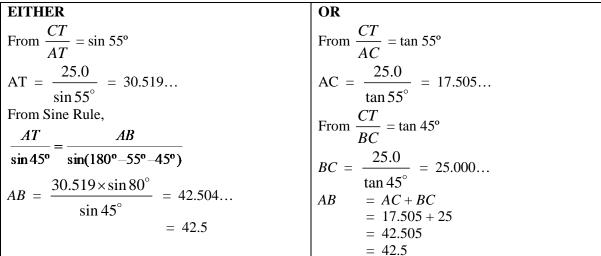
The following two dimensional diagram can be drawn:



Using the right angled triangle shown and Pythagoras' theorem: $x = \sqrt{130^2 - 50^2} = 120$ The diameter has been asked for, so as the radius is 120 cm, the diameter is 240 cm.

Question 6 Answer C $BO = \sqrt{37^2 - 35^2} = 12$ (Pythagorean triple) CO = BO (diagonals of square) $BC = \sqrt{12^2 + 12^2} = 16.9705... \approx 17.0$ cm





Question 8 Answer E

EITHER

Ratio flag heights Jeremy : Amy is 280 : 198 which is a multiple of 1.414...

The radii of the two circles will be in the same proportion.

The areas of the two circles will be the ratio of the squares of the lengths (since the figures are similar).

Ratio areas = $280^2 : 198^2 \approx 1.99979 : 1 \approx 2.00 : 1$ (Same as $1.414...^2$)

OR

Radius Amy circle = $0.5 \times 0.6 \times 198 = 59.4$ mm

Radius Jeremy circle = $0.5 \times 0.6 \times 280 = 84.0$ mm

Ratio areas =
$$\frac{\pi \times 84.0^2}{\pi \times 59.4^2}$$
 = 1.99979... \approx 2.00

Module 4: Graphs and Relations

Question 1 Answer B

The line shown has a y-intercept of -4 and a gradient of $\frac{rise}{run} = \frac{4}{6} = \frac{2}{3}$.

 $y = \frac{2}{3}x - 4$ 3y = 2x - 12 -2x + 3y = -123y = 2x - 12

Question 2 Answer C

At zero time the temperature is 25°, so there is a y-intercept of 25. When it is turned on it increases in temperature by 30° for every minute, representing the gradient. As the temperature becomes stable the relationship is no longer the same so the upper limit is 15.

Question 3 Answer B

The rate is below 4% for about one year from 09-10 and then a further three years from 12 -15. This is a total of 4 years.

Question 4 Answer B

The interest rate has changed from about 13% on 1st January, 1985 to about 2.5% on 1st January, 2015. This is an average change of $\frac{13-2.5}{30} = 0.35\%$ per year. Some variation is expected here as the values on the graph are not perfectly clear, but other responses are not close to this value.

Question 5 Answer D

Each point must be tested to determine whether it meets the requirement 3x - 4y < -7.

Point	Calculation	Less than – 7?
(-1, 4)	$3 \times -1 - 4 \times -4 = 13$	No
(7, -2)	$3 \times 7 - 4 \times -2 = 29$	No
(3, 1)	$3 \times 3 - 4 \times 1 = 5$	No
(2, 6)	$3 \times 2 - 4 \times 6 = -18$	Yes
(-1, 1)	$3 \times -1 - 4 \times 1 = -7$	No

Question 6 Answer C

Each statement can be written as an equation:

Between them Xavier and Younnis work no more than 50 hours per week	$x + y \le 50$
Xavier works a minimum of 15 hours	$x \ge 15$
Younnis works a minimum of 10 hours	$y \ge 10$
Xavier must work at least twice as many hours as Younnis	$y \leq \frac{x}{2}$

Question 7 Answer A

The line shown represents the relationship $y = 5x^3$. This is because the axes are labelled x^3 and y and the relationship is of the form $y = kx^3$. The value of k is given by $\frac{40}{8} = 5$.

Options D and E can be eliminated as they do not represent the appropriate shape for a cubic graph of this type.

Option A is the correct response as the points (1, 5) and (2, 40) can be seen. Each of these is appropriate for the equation:

$$y = 5x^{3}$$
$$y = 5 \times 1^{3} = 5$$
$$y = 5 \times 2^{3} = 40$$

Question 8 Answer E

The shaded feasible region is to the right of the line x = 3 so $x \ge 3$ and above the line y = 5 so $y \ge 5$. The region is below the line with a *y*-intercept of 12 and a gradient of $-\frac{12}{20} = -\frac{3}{5}$. This line is written

as:

 $y = -\frac{3}{5}x + 12$ 5y = -3x + 60 3x + 5y = 60As the region is below this line $3x + 5y \le 60$.

The region is also below the line with a y-intercept of 20 and a gradient of $-\frac{20}{10} = -2$. This line is

written as: y = -2x + 202x + y = 20

As the region is below this line $2x + y \le 20$.

END OF SOLUTIONS