TSSM
Creating VCE Success

THIS BOX IS FOR ILLUSTRATIVE PURPOSES ONLY

2017 Trial Examination

STUDENT NUMBER									Letter

FURTHER MATHEMATICS

Units 3 & 4 – Written examination 1

Reading time: 15 minutes

Writing time: 1 hour and 30 minutes

MULTIPLE-CHOICE QUESTION BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of modules*	*Number of modules to be answered*	*Number of marks*
A – Core	24	24			24
B - Modules	32	16	4	2	16
					Total 40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 26 pages.

Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

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SECTION A – Core

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions. Choose the response that is **correct** for the question. A correct answer scores 1; an incorrect answer scores 0. Marks will **not** be deducted for incorrect answers. No marks will be given if more than one answer is completed for any question. Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data analysis

Question 1.

The Australian Census requires participants to fill in the following information regarding the address of their dwelling,

- Apartment/Flat/Unit number (if any),
- Street number,
- Street name,
- Suburb/locality,
- State/territory,
- Postcode,
- Property/building name (if any).

The number of numerical variables and the number of nominal categorical variables collected from this data respectively are:

A. 3 and 5
B. 2 and 5
C. 3 and 4
D. 0 and 5
E. 0 and 4

The following information is required for questions 2, 3, 4 and 5.

The birth weights of babies born in the local hospital are recorded over 3 months. They are represented in the stem and leaf plot below:

Birth Weights

Stem	Leaves																	
0	9																	
1																		
1	9																	
2																		
2	5	7	9															
3	0	0	1	1	2	3	3	3	3	3	3	3	4	4	4	4	4	4
3	5	5	5	7	7	7	7	7	8	8	8	9	9	9				
4	1	1	2	3	4	4	4	4										
4	6	8																
5	0	0																

Key 2|3 = 2.3 kg

Question 2.

The interquartile range for the data given is:

- **A.** 0.6
- **B.** 0.7
- **C.** 0.8
- **D.** 1.5
- **E.** 4.1

Question 3.

The distribution of the data can best be described as:

- **A.** Symmetrical
- **B.** Negatively skewed
- **C.** Negatively skewed with outliers
- **D.** Positively skewed
- **E.** Approximately symmetrical with outliers

Question 4.

The number of outliers in this data set is:

- **A.** 0
- **B.** 1
- **C.** 2
- **D.** 3
- **E.** 4

Question 5.

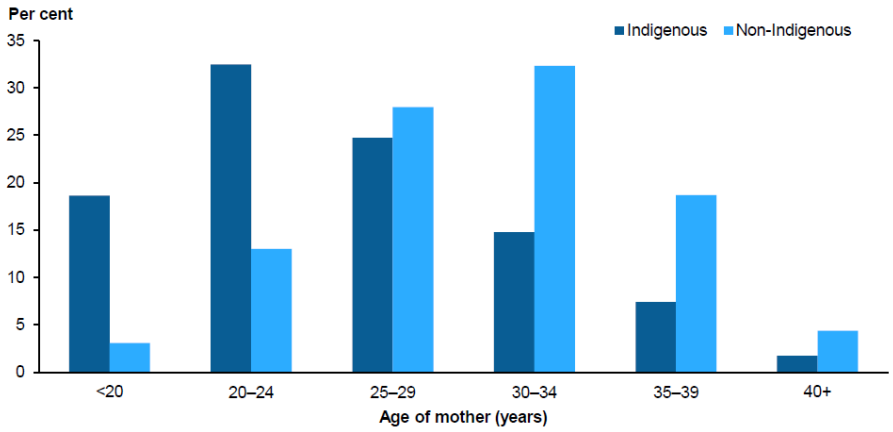
Birthweight is categorised as follows:
- very low birthweight: less than 1,500 grams
- low birthweight: less than 2,500 grams
- normal birthweight: 2,500 to 4,499 grams
- high birthweight: 4,500 grams or more.

Sources: AIHW 2012; WHO 1992.

The percentage of birthweights from this sample that are considered high birthweight is:

A. 4%
B. 8.0%
C. 8.2%
D. 8.3%
E. 16%

Question 6.



Women who gave birth, by maternal age and Indigenous status, 2011

Source: Li Z, Zedi R, Hilder L & Sullivan E 2013. Australia's mothers and babies 2011. Perinatal statistics series no. 28. Cat. No PER 59. Canberra: AIHW.

Which of the following statements is certainly true?

A. More indigenous women than non-indigenous women who gave birth in 2011 were 20-24 years of age.
B. The median age of indigenous women is higher than the median age of non-indigenous women for those who gave birth in 2011.
C. The mean age of non-indigenous and indigenous women is similar
D. Non-indigenous women who gave birth in 2011 had a mean age higher than the median age of indigenous women who gave birth in 2011.
E. The number of indigenous and non-indigenous women in the survey is the same.

Question 7.
The distribution of ages of mothers who gave birth between 1998 and 2007 is approximately symmetrical with a mean age of 31.5 years and a standard deviation of 4.5years. Given that over this 10 year period there were 2,538,700 births, the approximate number of mothers over the age of 45 years that gave birth each year is closest to:

A. 800
B. 2,100
C. 7,600
D. 10,100
E. 400

The following information will be used for questions 8 to 11

Life Expectancy for Australians

Year of birth	No	Males	Females
1890	1	47.2	50.8
1900	2	51.1	54.8
1910	3	55.2	58.8
1920	4	59.2	63.3
1930	5	63.5	67.1
1940	6	66.1	70.6
1950	7	67.1	72.8
1960	8	67.6	74.2
1970	9	69.6	76.6
1980	10	72.7	79.2
1990	11	75.6	81.3
2000	12	78.7	83.5
2010	13	79.3	83.9

Question 8.
When birth year No. 1 represents being born in 1890 and No. 2 represents being born in 1900, then the equation of the least squares regression line for Male life expectancy is closest to:

A. $\textit{life expectancy} = 47.67 + 2.56 \times \textit{year}$
B. $\textit{year} = 47.67 + 2.56 \times \textit{life expectancy}$
C. $\textit{year} = -17.79 + 0.38 \times \textit{life expectancy}$
D. $\textit{life expectancy} = 51.21 + 2.76 \times \textit{year}$
E. $\textit{life expectancy} = 2.76 + 51.21 \times \textit{year}$

Question 9.
The value of Pearson's product-moment correlation coefficient for *male life expectancy* and *year of birth* is closest to:

A. 0.968
B. 0.969
C. 0.983
D. 0.984
E. 0.985

Question 10.
The least squares regression equation for *female life expectancy* is:
$female\ life\ expectancy = 51.21 + 2.76 \times year$. The predicted *life expectancy* of a woman born in 1965 would be closest to:

A. 73.29 years
B. 74.67 years
C. 76.05 years
D. 77.9 years
E. 65 years

Question 11.
The coefficient of determination for the relationship between *female life expectancy* and *year of birth* is closest to:

A. 0.968
B. 0.969
C. 0.983
D. 0.984
E. 0.985

The following information is required for Questions 12 to 14

Maximum monthly temperatures measured at the Cape Otway Light Station are summarised below:

	Jan	**Feb**	**Mar**	**Apr**	**May**	**Jun**	**Jul**	**Aug**	**Sep**	**Oct**	**Nov**	**Dec**
Mean maximum temperature (°C) for years 1864 to 2016	21.4	21.6	20.3	18.0	15.6	13.7	13.0	13.8	15.2	17.0	18.3	19.9
seasonal index		1.25	1.17	1.04	0.90	0.79	0.75	0.80		0.98	1.06	1.15

Question 12.
The seasonal index for September is closest to:

A. 1.23
B. 0.87
C. 1.24
D. 0.86
E. 0.88

Question 13.
The mean maximum temperature in March 2016 was 20.3°C. The deseasonalised temperature for March 2016 is closest to:

A. 23.8
B. 17.3
C. 23.7
D. 17.4
E. 16.5

Question 14.
The mean maximum temperature in November 2016 was 0.34°C higher than the deseasonalised historical mean for this month. The mean maximum temperature recorded in November 2016 is closest to:

A. 17.7°C
B. 18.7°C
C. 17.4°C
D. 17.6°C
E. 18.6°C

The following information is to be used in Questions 15 and 16

World population (in millions) can be modelled by the equation
$\log(\textit{population}) = 2.239 + 0.289 \times \textit{year}$ where *year* is the number of centuries since 1500.
(i.e. *year* 1 is 1600, *year* 2 is 1700, etc.)

Question 15.
Using this model, the predicted population in 2020 is closest to:

A. 3.74 million
B. 3.89 million
C. 5,520 million
D. 7,810 million
E. 8,240 million

Question 16.
If the world population in 1980 was 4,440 million, the residual for 1980 when the above model is applied (measured in millions) is closest to:

A. 210
B. 810
C. -810
D. -210
E. 520

Recursion and financial modelling

Question 17.
A business currently charges $1,180 to supply a 2000 litre tank. Prices will be raised 1.3% next month. The new price for this tank will be:

A. $1193
B. $1195
C. $1213
D. $1333
E. $1534

Question 18.
Amanda invests $15,000 for 160 days in a term deposit paying 3.5% pa, calculated daily. The value of her investment after 160 days is closest to:

A. $16705
B. $23903
C. $15220
D. $15750
E. $15232

Question 19.
The value of an asset over 3 consecutive years is $2200, $1980 and $1760. The recurrence relation for calculating the asset value is:

A. $V_{n+1} = 0.9V_n \quad V_0 = \$2200$
B. $V_{n+1} = V_n - 0.1V_n \quad V_0 = \$2200$
C. $V_{n+1} = V_n - 220 \quad V_0 = \$2200$
D. $V_{n+1} = 1.1V_n \quad V_0 = \$2200$
E. $V_{n+1} = V_n + 220 \quad V_0 = \$2200$

The following information is required for questions 20 and 21

Brendan purchases equipment to set up his own recording studio at a cost of $6900. He pays a deposit of $500, and the balance is paid in equal monthly instalments over 2 years. A compounding rate of 6% interest per annum is to be charged.

Question 20.
The total amount that Brendan will pay (including deposit) by the end of the 2 year agreement is:

A. $7668
B. $7284
C. $7168
D. $7784
E. $7308

Question 21.
The effective annual rate of interest paid by Brendan is closest to:

A. 6%
B. 6.2%
C. 11.1%
D. 6.5%
E. 11.5%

Question 22.
The recurrence relation $V_0 = 20000 \quad V_{n+1} = 1.021V_n - 500$ could be used to model:

A. A perpetuity with periodic payments of $500
B. A reducing balance loan with annual interest rate of 8.4%, compounding quarterly
C. An interest-only loan of $20000
D. A reducing balance depreciation of an asset initially valued at $20000
E. A reducing balance loan with an annual interest rate of 2.1%, compounding monthly

The following information is required for questions 23 and 24

Dexter has invested in an annuity which earns interest at the rate 4.9% per annum compounding monthly. He receives monthly payments from the annuity. After 1 year the balance is $11183.75, and after 2 years it is $1925.67.

Question 23.
The monthly payments received by Dexter are closest to:

A. $800
B. $1114
C. $9303
D. $13155
E. $415

Question 24.
The initial investment in Dexter's annuity is closest to:

A. $20450
B. $2210
C. $20000
D. $11184
E. $22300

END OF SECTION A

SECTION B - Modules

Instructions for Section B

Select **two** modules and answer **all** questions within the selected modules.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

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SECTION B – continued
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Module 1 – Matrices

Question 1.

Three matrices, *A*, *B* and *C* can be multiplied in the following ways:
$A \times B \times C$ and $C \times A \times B$, but $B \times A \times C$ is not defined. If matrix *B* has order 2x3, and $A \times B \times C$ results in a square matrix then:

A. *A* must be a 3x2 matrix
B. $C \times A \times B$ must be a square matrix
C. The number of rows in ***A*** must equal the number of columns in *B*
D. $A \times C$ must be defined
E. $C \times A$ must be a square matrix

Question 2.

The matrix product $\begin{bmatrix} s & t & e & m \end{bmatrix} \times \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ is equal to

A. $\begin{bmatrix} t & m & s & e \end{bmatrix}$
B. $\begin{bmatrix} e & s & m & t \end{bmatrix}$
C. $\begin{bmatrix} m & e & t & s \end{bmatrix}$
D. $\begin{bmatrix} e & s & t & m \end{bmatrix}$
E. $\begin{bmatrix} s & m & e & t \end{bmatrix}$

Question 3.
A school with three campuses (A, B and C) has a uniform shop at each site. The number of jumpers, blazers and shirts in stock at each is shown in the table below:

	Campus		
	A	B	C
Jumper	12	3	7
Blazer	8	2	4
Shirt	14	9	15

This information is placed in a stock matrix S. The prices of jumpers, blazers and shirts is represented in matrix *P*. The row matrix *R* and column matrix *C* are represented by:

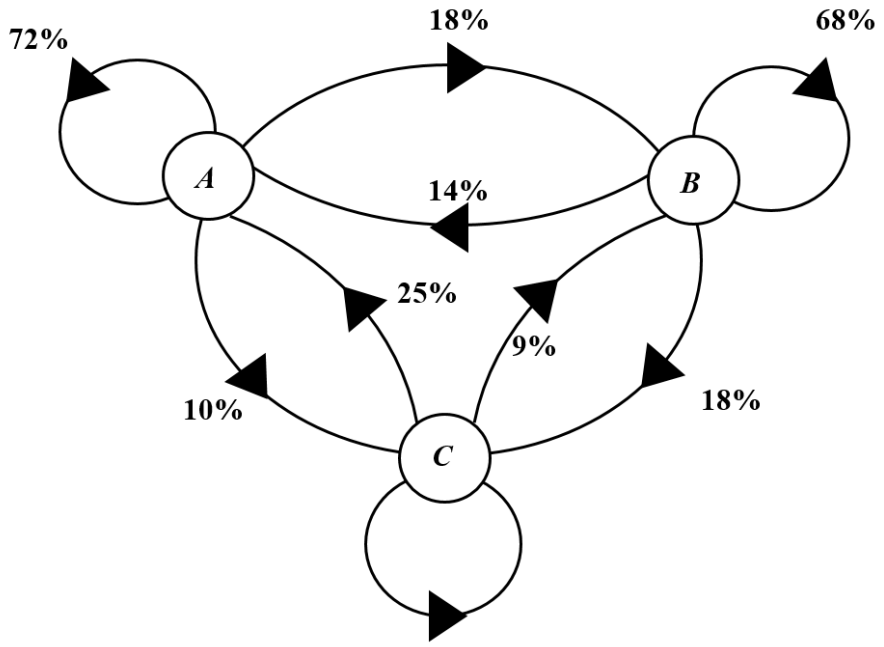
$$P = \begin{bmatrix} 68 & 140 & 30 \end{bmatrix} \quad R = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

In order to calculate the total number of jumpers, blazers and shirts in stock campuses and the total value of all stock respectively you would need to complete the following matrix multiplications:

A. $R \times S$ and $P \times S \times C$
B. $S \times R$ and $P \times C \times S$
C. $S \times C$ and $S \times C \times P$
D. $P \times S$ and $C \times P \times S$
E. $S \times C$ and $P \times S \times C$

The following information is required for Questions 4 to 6

A bicycle business has three locations ***A***, ***B*** and ***C*** from which patrons can hire a bicycle. Cyclists may deliver their bike to any of the three locations. Over the period of a week, the number of bicycles at each location changes according to the transition diagram below:



Question 4.

The percentage of bicycles that are hired from and returned to Depot ***C*** each week is:

A. 81%
B. 72%
C. 66%
D. 57%
E. 38%

Question 5.

The transition matrix is:

A. $\begin{bmatrix} 72 & 14 & 25 \\ 18 & 68 & 18 \\ 10 & 9 & 66 \end{bmatrix}$

B. $\begin{bmatrix} 72 & 18 & 10 \\ 14 & 68 & 9 \\ 25 & 18 & 66 \end{bmatrix}$

C. $\begin{bmatrix} 0.72 & 0.18 & 0.1 \\ 0.14 & 0.68 & 0.09 \\ 0.25 & 0.18 & 0.66 \end{bmatrix}$

D. $\begin{bmatrix} 0.72 & 0.14 & 0.25 \\ 0.18 & 0.68 & 0.18 \\ 0.10 & 0.09 & 0.66 \end{bmatrix}$

E. $\begin{bmatrix} 0.72 & 0.14 & 0.25 \\ 0.18 & 0.68 & 0.09 \\ 0.10 & 0.18 & 0.66 \end{bmatrix}$

Question 6.

Initially there are 100 bicycles located at each of the three depots. Every 5 weeks some bicycles are relocated to other depots so that there are once again 100 bicycles at each depot. The number of bicycles that need to be moved from depot ***A*** is closest to:

A. 122
B. 21
C. 22
D. 15
E. 17

Question 7.

Which one of the following is a singular matrix given that $m = -1$?

A. $\begin{bmatrix} -2 & 3m \\ -4 & 6 \end{bmatrix}$

B. $\begin{bmatrix} 2m & 3 \\ 4 & 6 \end{bmatrix}$

C. $\begin{bmatrix} -1 & 6m \\ 2 & -12 \end{bmatrix}$

D. $\begin{bmatrix} 4 & 2m \\ 6 & -3m \end{bmatrix}$

E. $\begin{bmatrix} -2m & 4m \\ 3m & 6 \end{bmatrix}$

Question 8.

A new gym has opened in town and it is found that similar numbers of patrons attend daily. Members are found to either attend aerobics sessions or use the gym equipment each day and the transition matrix below can be used to model the number of members attending the gym or aerobics sessions from one day to the next.

		Today	
		Gym	Aerobics
Next Day	Gym	0.68	0.4
	Aerobics	0.32	0.6

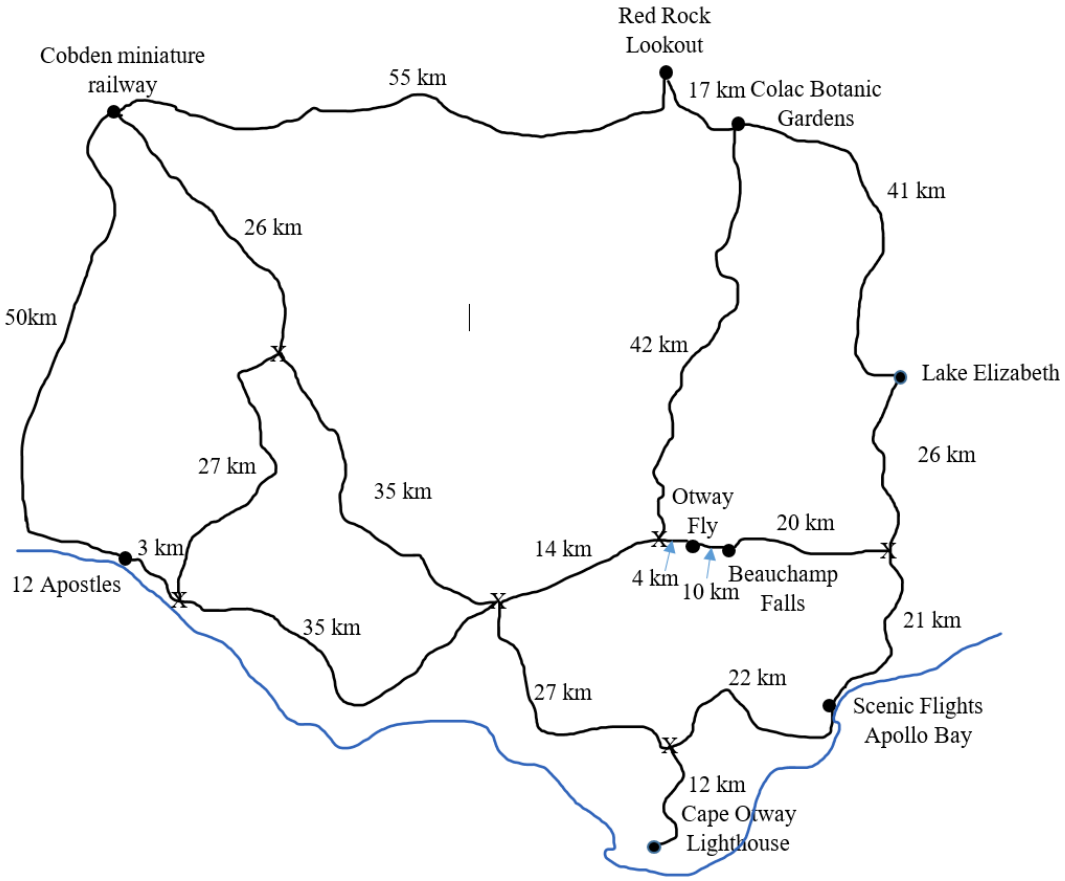
If on Monday of one week there are 72 members that attend aerobics classes and 163 members that use the gym; the number of members expected to attend aerobics in the long run would be closest to:

A. 105
B. 130
C. 95
D. 140
E. 100

END OF MODULE 1 – SECTION B - continued

Module 2 – Networks and decision mathematics

The following information is required for Questions 1 and 2



Melanie is planning a trip to the Otway region and has identified 9 places of interest that she wants to visit. The map above shows the points of interest and the distances of connecting roads. X symbols identify intersections from which distances have been measured.

Question 1.

The number of different routes that Melanie can take to travel from the 12 Apostles to the Cobden miniature railway without passing another point of interest is:

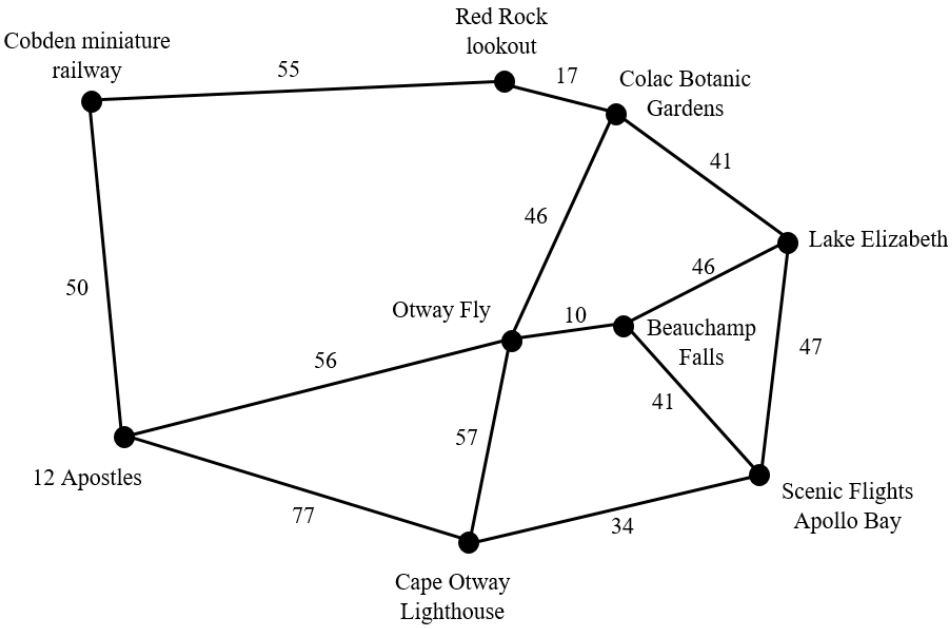
A. 1
B. 2
C. 3
D. 4
E. 5

Question 2.
This map can be converted into a network where points of interest are the vertices and each edge represents a different possible journey between points of interest, without passing another point of interest. No road can be repeated or intersection used more than once in each journey. The number of edges required to create this network is:

A. 19
B. 29
C. 32
D. 24
E. 27

The following is required for Questions 3 to 6

A simplified version of this network is shown below. It contains some of the shortest distances connecting points of interest.



Question 3.
The minimum spanning tree for this network would have a total length of:

A. 294 km
B. 288 km
C. 293 km
D. 299 km
E. 305 km

Question 4.
Which of the following is true for this simplified network?

- **A.** An Eulerian circuit circuit.
- **B.** There are 3 possible Hamiltonian cycles which start at Lake Elizabeth.
- **C.** The network contains four odd vertices.
- **D.** There are only 5 possible Hamiltonian paths which start at Apollo Bay.
- **E.** The minimum distance for a Hamiltonian cycle is 395 km.

Question 5.
How many edges need to be removed from the simplified network so ensure an Eulerian Circuit is possible?

- **A.** 0
- **B.** 1
- **C.** 2
- **D.** 3
- **E.** 4

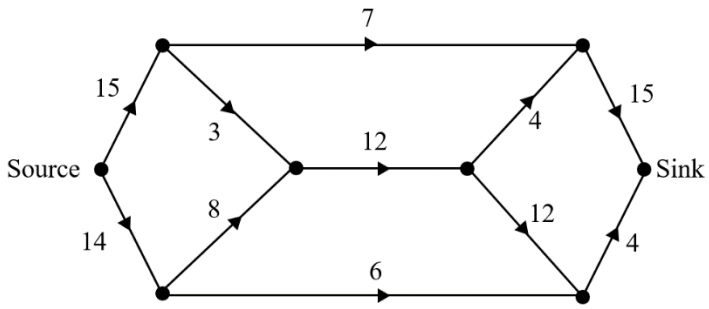
Question 6.
When using Dijkstra's algorithm to determine the shortest distance from the Cobden miniature railway to Beauchamp Falls the first row of the table would look like this:

	Red Rock	Colac Botanic Gardens	Lake Elizabeth	Apollo Bay	Cape Otway	Beauchamp Falls	Otway Fly	12 Apostles
Cobden Miniature Railway	55	*	*	*	*	*	*	50

The third row in this table should contain:

- **A.** Red Rock
- **B.** Apollo Bay
- **C.** Otway Fly
- **D.** Cape Otway
- **E.** 12 Apostles

Question 7.

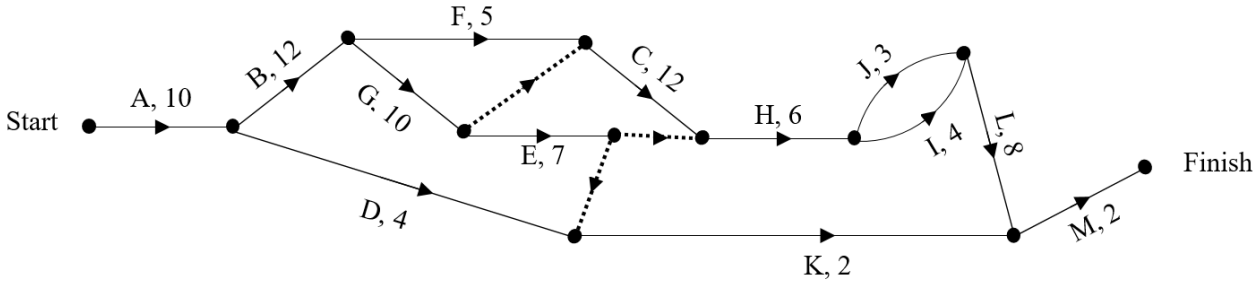


The maximum flow of the network diagram shown above is:

- **A.** 15
- **B.** 19
- **C.** 25
- **D.** 29
- **E.** 24

Question 8.

The following activity network shows the time in days to complete each task.



The earliest finish time for this project is:

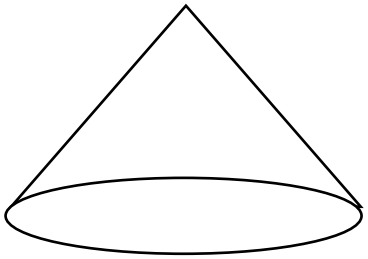
- **A.** 18 days
- **B.** 56 days
- **C.** 59 days
- **D.** 64 days
- **E.** 70 days

END OF MODULE 2 – SECTION B - continued

Module 3 – Geometry and measurement

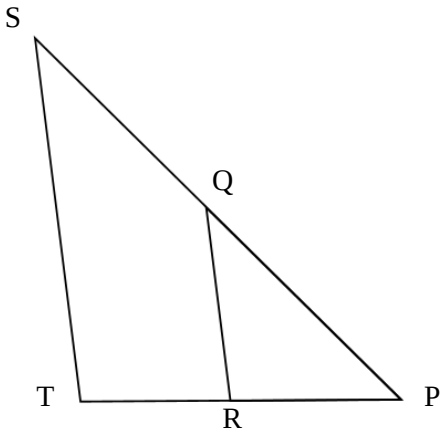
Question 1.

A right circular cone has a height of 12 cm and a diameter of 15 cm. The length of the slant edge is closest to:



A. 14.1 cm
B. 19.2 cm
C. 19.5 cm
D. 14.2 cm
E. 17.3 cm

Question 2.



Triangles PQR and PST are similar. Given that the length PR is equal to the length RT, and the area of triangle PST is 256 square centimetres, then the area of triangle PQR is:

A. 128 square centimetres
B. 512 square centimetres
C. 64 square centimetres
D. 45 square centimetres
E. 69 square centimetres

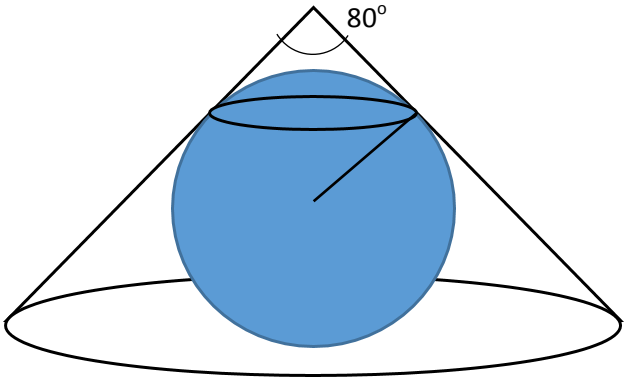
Question 3.

A man leaves his base camp and travels 2.4 kilometres on a bearing of 127°T, he then travels a further 3.7 kilometres on a bearing of 064°T. His distance from base camp is now closest to:

A. 27.51 km
B. 5.24 km
C. 6.1 km
D. 5.25 km
E. 4.41 km

The following information is required for Questions 4 to 6

A tennis ball has a diameter of 6.5 cm. A cone fits over the tennis ball just touching the surface of the ball and the table on which it stands, as shown in this diagram.



The apex at the top of the cone is 80^o. The point of contact of the ball with the slant surface of the cone makes an angle of 90^o to the radius of the ball.

Question 4.
The vertical height of the cone is:

- **A.** 8.31 cm
- **B.** 7.12 cm
- **C.** 7.75 cm
- **D.** 8.30 cm
- **E.** 11.00 cm

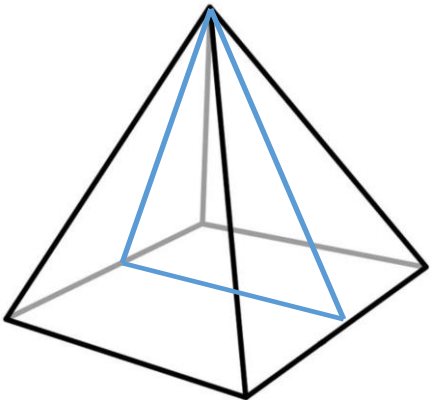
Question 5.
The radius of the base of the cone is:

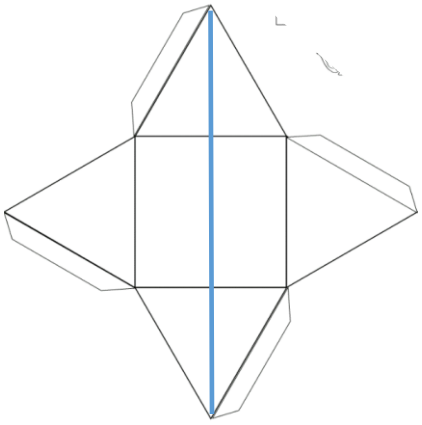
- **A.** 9.23 cm
- **B.** 8.31 cm
- **C.** 7.12 cm
- **D.** 6.97 cm
- **E.** 5.97 cm

Question 6.
The inside of the cone has been freshly painted, and when placed over the tennis ball leaves a line of paint on the surface of the ball. The length of the line of paint on the ball is:

- **A.** 15.64 cm
- **B.** 20.42 cm
- **C.** 18.8 cm
- **D.** 24.34 cm
- **E.** 21.90 cm

The following information is required for questions 7 and 8





Allan wants to create a scale model of the Great Pyramid of Giza. He wants to make a net from cardboard which he will fold up to create his model. He wishes to make his model as large as possible from a cardboard sheet with dimensions 61cm by 91cm. The central line drawn down the net has been shown on the pyramid to assist.

Question 7.
Given that the Great Pyramid of Giza has a square base of 230 m and a vertical height of 146 m, the length of the line inscribed on the Great Pyramid would be:

A. 601.70 m
B. 185.85 m
C. 486.70 m
D. 415.85 m
E. 606.00 m

Question 8.
Given that this same length on the model will be 61 cm, the height of the constructed model should be:

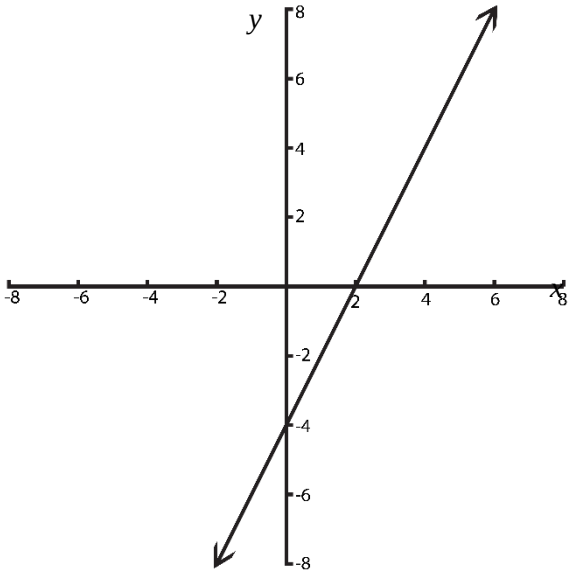
A. 47.9 cm
B. 18.3 cm
C. 14.8 cm
D. 21.4 cm
E. 14.6 cm

END OF MODULE 3 – SECTION B - continued

TURN OVER

Module 4 – Graphs and relations

Question 1.



The equation of the graph above is:

A. $2x+4y=8$

B. $4x+2y=8$

C. $4x+2y=-8$

D. $2x-y=4$

E. $-2x+y=4$

Question 2.

A straight line has an x intercept of -2, and a y intercept of 3. The coordinates of another point that the line goes through are:

A. $(1,5)$

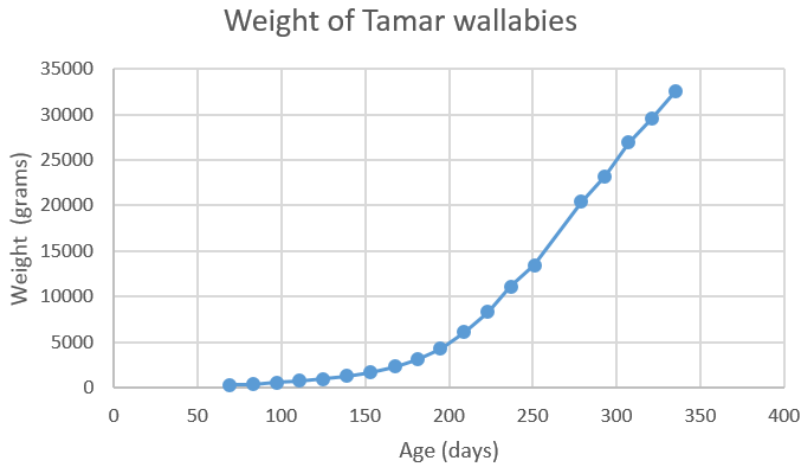
B. $(-3,-1)$

C. $(14,25)$

D. $(12,20)$

E. $(-12,-15)$

The weight of a Tamar wallaby measured in grams is shown in the graph below:



The wallaby's weight at 280 days is closest to:

A. 2000 grams
B. 2 kg
C. 19 kg
D. 21 kg
E. 24 kg

Question 3.

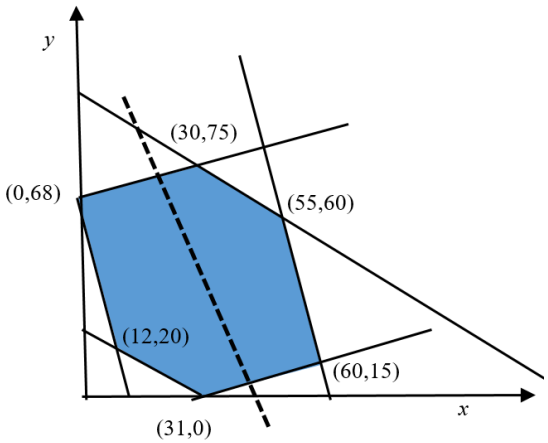
It takes a seamstress 1 hour to make an art smock, and 2.5 hours to make a school dress. She can work up to a maximum of 42 hours per week. Let x represent the number of art smocks and y represent the number of school dresses made. An inequality that represents this situation is:

A. $2.5x + y \leq 42$
B. $42 - x \leq 2.5y$
C. $x \leq 42 - 2.5y$
D. $x + 2.5y \geq 42$
E. $-y - 2.5x \leq -42$

Question 4.

The following graph shows the feasible region for a linear programming problem. The dotted line represents the objective function. The values of x and y that will maximise the function are:

A. $x = 60$ and $y = 75$
B. $x = 30$ and $y = 75$
C. $x = 60$ and $y = 55$
D. $x = 60$ and $y = 15$
E. $x = 55$ and $y = 60$



SECTION B – Module 4 – continued
TURN OVER

Question 5.

The graph below has the form $y = kx^n$. A possible equation for this graph is:

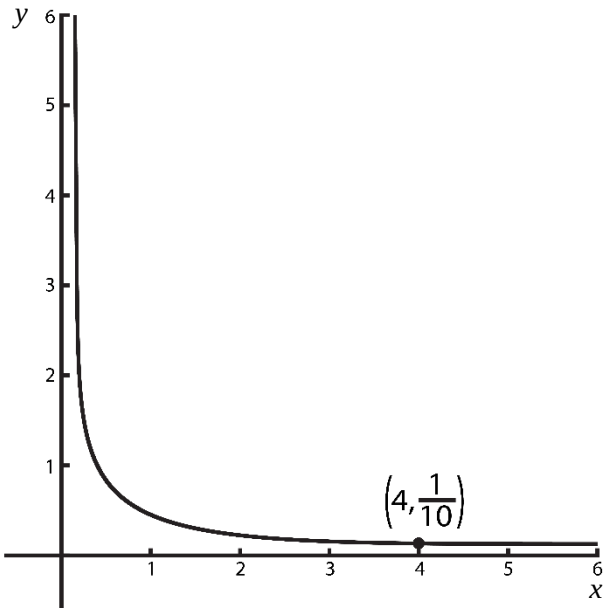
A. $y = \frac{1}{5x}$

B. $y = \frac{8}{5x^2}$

C. $y = \frac{2}{5x}$

D. $y = \frac{3}{5x}$

E. $y = \frac{2}{5x^2}$



Question 6.

A store keeper sold his entire stock of shirts and ties in a sale for $10,000. The shirts were priced at 2 for $50 and the ties $15 each. If he had sold only half of the shirts and three-fifths of the ties he would have received $5,150. The number of shirts sold in the sale was:

A. 400
B. 340
C. 276
D. 200
E. 300

Question 7.

The cost of renting a car is given by a rule of the form $C = a \times n + b$ where C is the cost, n is the number of kilometres travelled, a is the cost per kilometre and b is a fixed cost. If you travel 300 km in a hire car the cost of the car rental is $450. For a person travelling 280 km, the cost is $426. The values of a and b are:

A. $a = 90$ and $b = 1.2$
B. $a = 1.2$ and $b = 90$
C. $a = 1$ and $b = 146$
D. $a = 1.5$ and $b = 0$
E. $a = 1.15$ and $b = 100$

END OF MULTIPLE CHOICE QUESTION BOOK