FURTHER MATHEMATICS

Units 3 & 4 – Written examination 2

TSSM™
Creating VCE Success

2017 Trial Examination

SOLUTIONS

SECTION A: Core

Question 1.

a. NSW

1 mark

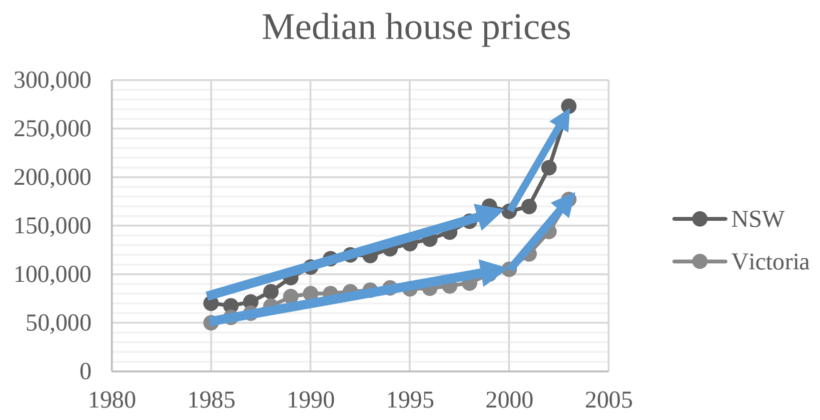
b. 130%

10 year period 1993 ($120,000) to 2003 ($275,000) an increase of $155000. Calculate as a percentage of the 1993 house price.

$$\frac{155000}{12000} \times 100 = 129.167$$

This rounds to 130%

c.



NSW and Victorian median house prices show a steady increasing trend from 1983 to 2000. NSW prices increased from $70,000 to $165,000 (a rate of approx. $6,300 per year), compared with Victorian median house prices increased from $50,000 to $105,000 over the same period (a rate of approx. $3,600 per year). Both states then showed much stronger growth in median house prices over the years 2000 to 2003, with NSW prices increasing to $275,000 (a rate of $39,600 per year), compared with Victorian prices increasing to $175,000 (a rate of $23,000 per year). The difference between the median house prices has increased from $20,000 in 1985, to $60,000 in 2000, to a whopping $100,000 by 2003.

1 mark for each of two comparisons.

Question 2.

a. Strong, positive and non-linear correlation shown in graph.

1 mark

b. The correlation coefficient suggests a strong positive relationship between the variables, but because the relationship is non-linear, a linear regression equation will not be very reliable for predicting the weight of eye lens.

1 mark for strength of relationship, 1 mark for reliability

c. The residual plot shows an obvious pattern which confirms the relationship is non-linear. The data should be linearised by using either $\log(x)$, y^2 or $\frac{1}{x}$ transformation.

1 mark for stating non-linear,   1 mark for stating possible transformations.

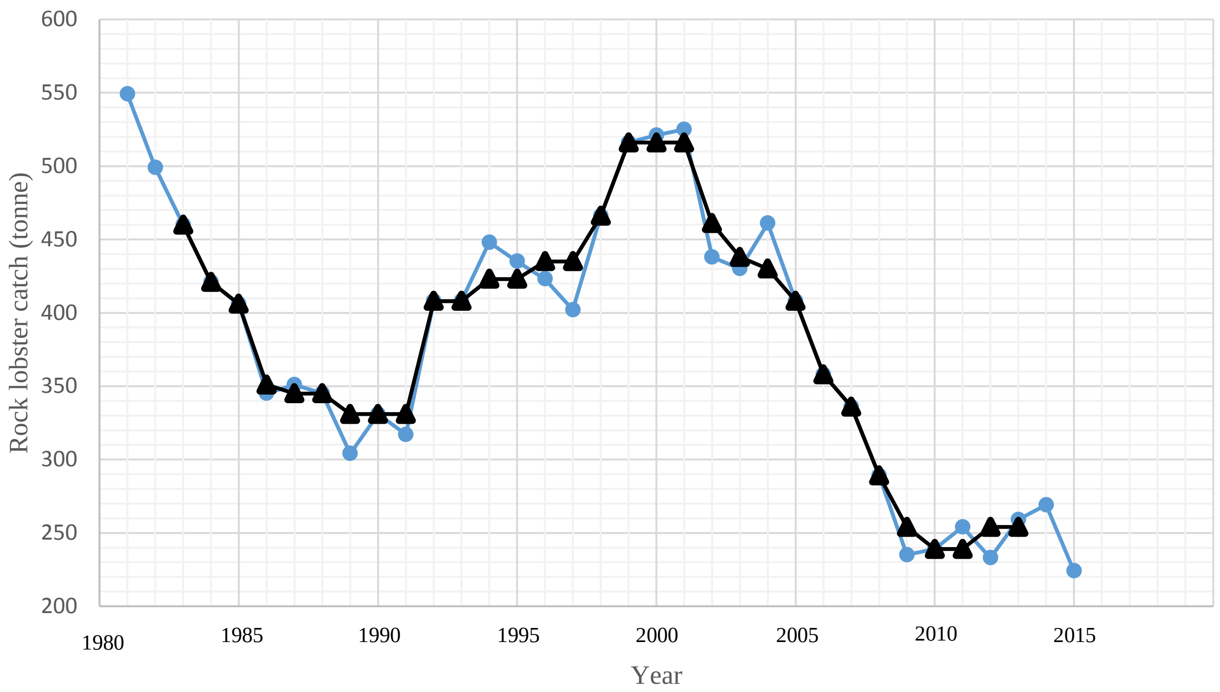
d. $-169+143\log(215)=164.54$

Weight would be164.54 grams and this is an example of interpolation

1 mark for each.

Question 3.

a.



2 marks

b. Rock lobster catch has a decreasing trend from 1981 to 1990, an increasing trend from 1990 to 2001, decreasing trend from 2001 to 2009 after which catch tonnage has been fairly constant and the lowest level over the period 1981 to 2015.

2 marks

Question 4.

a. Linear regression equation is
$\textit{Number of arrests} = 352131 + -104 \times \textit{no.hives}$

LinRegBx *hives*,*arrests*,1: CopyVar *stat.RegE*

"Title"	"Linear Regression (a+bx)"
"RegEqn"	"a+b·x"
"a"	352131.
"b"	-103.546
"r²"	0.871216
"r"	-0.933389
"Resid"	"{...}"

1 mark for each coefficient

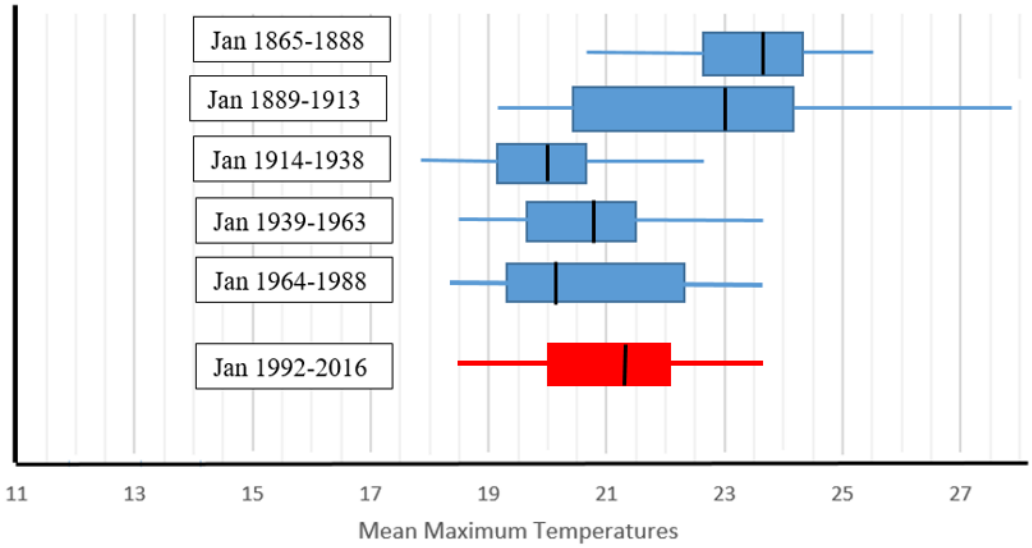
b. $r^2 = 0.87$ which would indicate that 87% of variation in number of arrests can be explained by variation in the number of registered hives. BUT, since these two variables have absolutely nothing to do with each other this relationship is purely coincidental. A high correlation coefficient or coefficient of determination does not automatically mean that there is a relationship between two variables. It is highly unlikely that variation in the number of registered bee hives will influence the number of arrests of juveniles for possession of marijuana.

2 marks

Question 5.

a. Five figure summary is:

1992-2016
18.5
20
21.4
22.05
23.7



0.5 marks each for correctly locating min, max, Q1, Q3 and median. 0.5 marks for using a **ruler.**

b. The median average daily maximum temperatures for January fell from values above 23^{o}C in earlier years (1865 to 1913) to much lower medians from 1914 to 1988. The median daily maximum for 1992 to 2016 is roughly 1 degree Celsius higher than for 1964 to 1988. The most variable years were 1889 to 1913 with a range of 8.5 degrees compared to ranges of roughly 5 degrees for all other 25 year periods.

1 mark for comparison of medians, 1 mark for comparison of either ranges or interquartile ranges**.**

Recursion and financial modelling

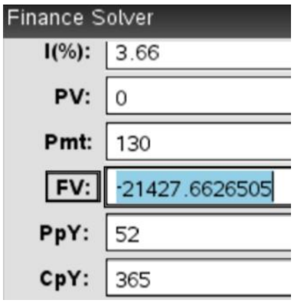
Question 6.

a. $17,160

1 mark

Final value of investment $21,427.66.

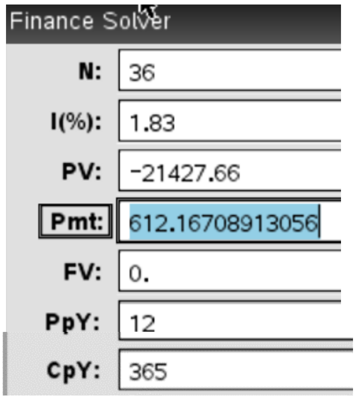
1 mark



Total interest earned
$21427.66 - 156 \times 130 = \$1147.66$

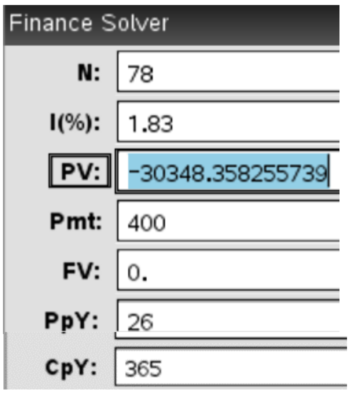
1 mark

b. Total invested in annuity is $21,427.66. Interest rate will drop to 1.83% because deposits are no longer being made. Using finance solver Jennifer would receive $612.15 each month.



1 mark

c. Annuity needs to have $30,348.63 for Jennifer to cover her costs of $200 per week, ($400 per fortnight).



1 mark

d. $30488.63 - 21427.66 = \$8920.70$

1 mark

Question 7.

a. Rate of depreciation is 8%

1 mark

solve$\left(r^5 \cdot 20000 = 13181.63, r\right)$

$r = 0.919999993523$

b. $V_0 = 20000 \quad V_{n+1} = 0.92 \times V_n$

1 mark

c. $N_0 = 96 \quad N_{n+1} = 1.15N_n - 10$

1 mark

Using the recursion equation, the tenth market there will be 170 customers.

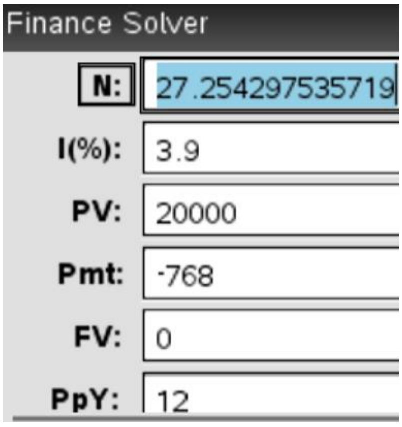
1 mark

96	96
1.15· 96−10	100.4
1.15· 100.4−10	105.46
1.15· 105.46−10	111.279
1.15· 111.279−10	117.97085
1.15· 117.97085−10	125.6664775
1.15· 125.6664775−10	134.516449125
1.15· 134.516449125−10	144.693916494
1.15· 144.69391649375−10	156.398003968
1.15· 156.39800396781−10	169.857704563

d. 28 months or 2 years and 4 months.

$\$8 \times 96 = \$768$ repayments. Use finance solver to determine period of the loan.

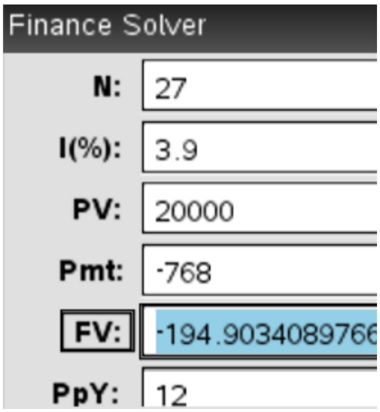
1 mark



e. $195.53

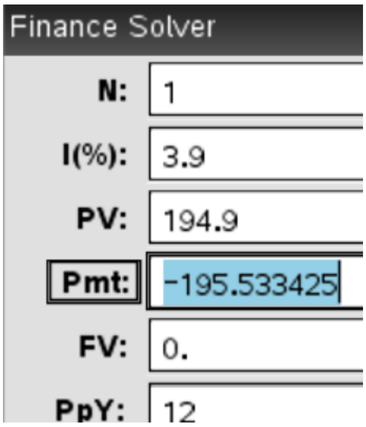
Calculate amount due after 27 months.

1 mark



Then calculate amount of last payment.

1 mark



SECTION B: Modules

Module 1 – Matrices

Question 1.

a. A has order 2×2 and B is 3×5

2 marks (1 each)

b. $A^{-1} = \begin{bmatrix} -2 & 5 \\ 3 & -7 \end{bmatrix}^{-1} = \begin{bmatrix} 7 & 5 \\ 3 & 2 \end{bmatrix}$

1 mark

c. C is 3×2 and D is 5×2

2 marks (1 each)

Question 2.

a. $S_0 = \begin{bmatrix} 1328 \\ 704 \\ 1210 \end{bmatrix}$

1 mark

b. $S_0 = \begin{bmatrix} 1328 \\ 704 \\ 1210 \end{bmatrix}, \quad S_n = T \times S_{n-1}$

2 marks (1 each)

Question 3.

a. $D_2 = (D_1)^2 = \begin{bmatrix} 0 & 2 & 1 & 0 & 1 \\ 0 & 0 & 2 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 2 & 2 & 1 & 0 & 0 \\ 1 & 1 & 1 & 2 & 0 \end{bmatrix}^2 = \begin{bmatrix} 2 & 1 & 5 & 3 & 3 \\ 3 & 1 & 1 & 4 & 2 \\ 3 & 5 & 3 & 2 & 1 \\ 1 & 4 & 6 & 1 & 5 \\ 5 & 6 & 5 & 1 & 3 \end{bmatrix}$

1 mark

b.

$$D_1 + D_2 = \begin{bmatrix} 0 & 2 & 1 & 0 & 1 \\ 0 & 0 & 2 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 2 & 2 & 1 & 0 & 0 \\ 1 & 1 & 1 & 2 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 5 & 3 & 3 \\ 3 & 1 & 1 & 4 & 2 \\ 3 & 5 & 3 & 2 & 1 \\ 1 & 4 & 6 & 1 & 5 \\ 5 & 6 & 5 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 6 & 3 & 4 \\ 3 & 1 & 3 & 4 & 3 \\ 4 & 5 & 3 & 3 & 2 \\ 3 & 6 & 7 & 1 & 5 \\ 6 & 7 & 6 & 3 & 3 \end{bmatrix}$$

Remove the digits in the leading diagonal (since you do not count double dominance over self) and add up the values in each row.

$$\begin{bmatrix} 0 & 3 & 6 & 3 & 4 \\ 3 & 0 & 3 & 4 & 3 \\ 4 & 5 & 0 & 3 & 2 \\ 3 & 6 & 7 & 0 & 5 \\ 6 & 7 & 6 & 3 & 0 \end{bmatrix} \quad \begin{bmatrix} 16 \\ 13 \\ 14 \\ 21 \\ 22 \end{bmatrix}$$

Complete the ranking table

Position	Player	Number of wins (1 and 2 step)
1st	Ernest	22
2nd	Donna	21
3rd	Aiden	16
4th	Chloe	14
5th	Barry	13

3 marks

Module 2 – Networks and decision mathematics

Question 1.

a.

Swimmer	Free	Back	Breast	Fly
Kara	0	4.86	9.93	**5.87**
Jess	0	5.84	15.98	2.53
Andrea	0	**8.52**	12.81	1.75
Joanne	0	7.46	12.79	3.81

1 mark

b. Because all of the swimmers fastest times are for freestyle, the row reduction reduces all elements in the first column to zero.

1 mark

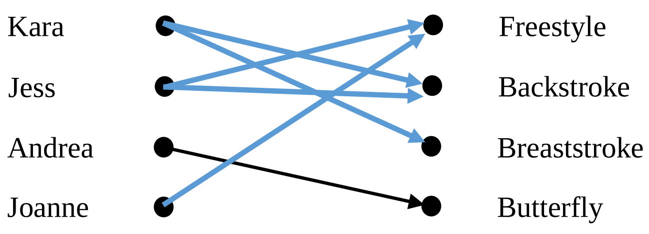
c. Optimum allocation is reached when the minimum number of lines required to cover the zeros is equal to the number of jobs to be allocated.

1 mark

d.

Swimmer	Free	Back	Breast	Fly
Kara	0.98	0	0	5.10
Jess	0	0	5.07	0.78
Andrea	0	2.68	1.90	0
Joanne	0	1.62	1.88	2.06

1 mark



1 mark

e.

	Swimmer	Time
Back	Jess	37.04
Breast	Kara	46.31
Fly	Andrea	33.99
Free	Joanne	35.53

1 mark

Entry time: 152.87 seconds = 2:32.87 (2 min 32.87 sec)

1 mark

Question 2.

a. Earliest finish time is 26 days

1 mark

b. Critical path is $B \to G \to L \to K$

1 mark

c. New earliest finish time is 23 days

1 mark

Total cost is $1620

1 mark

The jobs to be reduced would be G and L by a total of 3 days between them. After this the critical path becomes $A \to C \to F \to W \to I \to N$, and none of these jobs can be reduced.

1 mark

Module 3 – Geometry and measurement

Question 1.

a. $7 \times 5 \times 60 = 2100$ litres of water per hour from paved area

1 mark

$\frac{2100}{60 \times 60} = 0.583$ litres per second

1 mark

b. $\frac{\pi r^2}{18} = 0.583$

$r = \sqrt{\frac{18 \times 0.583}{\pi}} = 1.83 \text{ cm}$ OR

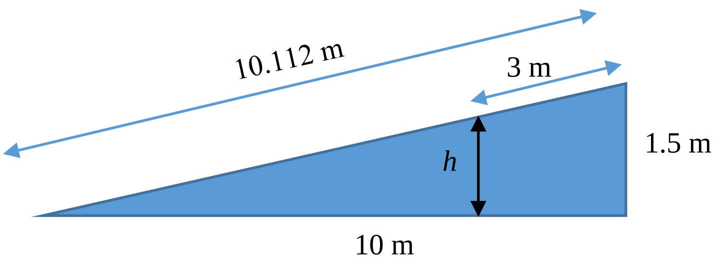
$\text{solve}\left(0.58333333333333 = \frac{\pi \cdot r^2}{18}, r\right)$

$r = -1.82818319786$ or $r = 1.82818319786$

Pipe diameter would need to be a minimum of $2 \times 1.83 = 3.66 \text{ cm}$ so 1.5” (3.8 cm) pipe is required.

2 marks

c. Calculate sloping length of yard

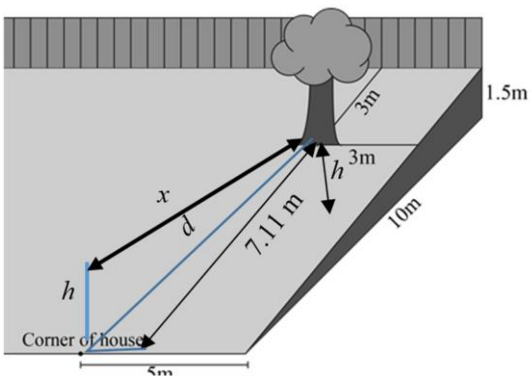


$\sqrt{1.5^2 + 10^2} = 10.1119 \text{ m}$

Calculate sloping length to tree

$10.1119 - 3 = 7.1119 \text{ m}$

1 mark

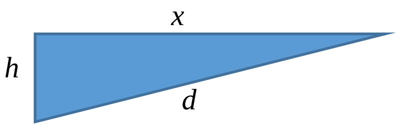


$\frac{h}{7.1119} = \frac{1.5}{10.112}$

$h = \frac{1.5 \times 7.1119}{10.1119} = 1.0550 \text{ m}$

1 mark

$d = \sqrt{2^2 + 7.1119^2} = 7.3877 \text{ m}$



$x = \sqrt{7.3877^2 - 1.0550^2} = 7.31 \text{ m}$

Tree is 7.31 m horizontally from the house.

1 mark

Question 2.

a. Radius of the parallel of latitude is $6400\cos 60°43' = 3130.42$ km

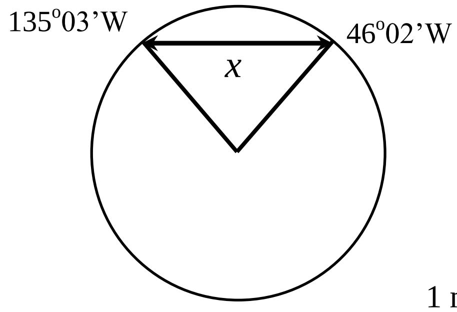
1 mark

$$d = \frac{135°03' - 46°02'}{360°} \times 2\pi \times 3130.42 = 4863.5$$

Distance along parallel of latitude is 4860 km

1 mark

b. Use the cosine rule. Both of the other 2 sides are the radius of the parallel of latitude.



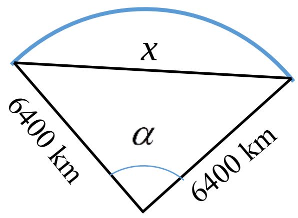
$x = \sqrt{2\times 3130.42^2 - 2\times 3130.42^2 \cos 89°1'}$

$x = 4388.9$ km

1 mark

c. Calculate angle α first using the cosine rule

$\alpha = \cos^{-1}\left(\frac{2\times 6400^2 - 4388.9^2}{2\times 6400^2}\right) = 40.105°$



Then calculate great circle distance

$$d = \frac{40.105°}{360°} \times 2\pi \times 6400 = 4480km$$

Distance = 4480 km

1 mark

Distance saved is $4860 - 4480 = 380$ km

1 mark

Module 4 – Graphs and relations

Question 1.

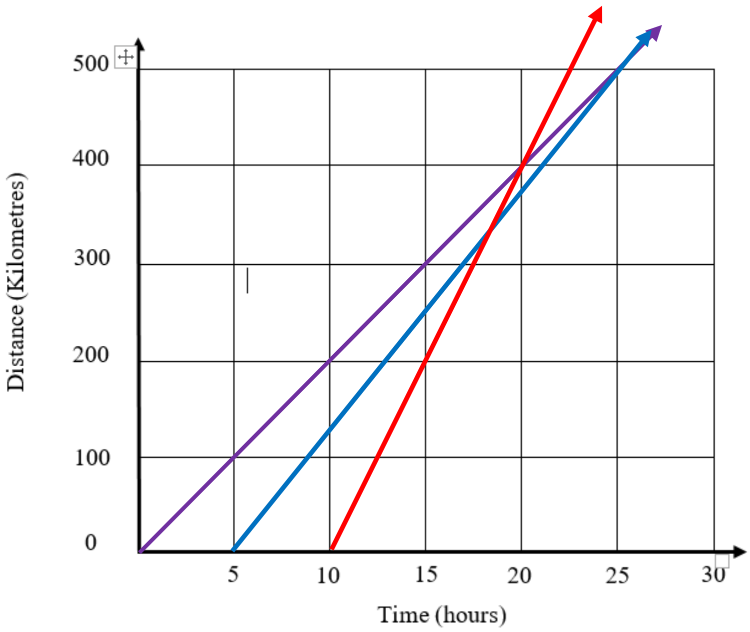
Break even at $x = 60$ $\quad$ solve$(5\cdot x+48=5.8\cdot x,x)$ $\quad$ $x=60$

$C = 5x + 48$ $\quad R = 5.8x$ $\quad$ R is revenue and C is the cost

1 mark

Question 2.

a. 1 mark for each line



1 mark for each line

b. Boat C won the race by 2.5 hours

1 mark

Question 3.

a. $\text{Cost} = 10.9589 \times 0.28897 = \$3.17$

1 mark

b. Cost $= (50 - 10.9589) \times 0.30954 + 3.17 =$ \$15.25

1 mark

c. Equation $C = \left(x - 10.9589\right) \times 30.954 + 316.679$

1 mark

$C = 30.954x - 22.542$

1 mark

d. $C = 30.954 \times 15.6 - 22.542 = 460.34$

Cost = \$4.6034 or \$4.60

1 mark

e. $\$4.6034 \times 91 = \$418.909$

Cost per quarter = \$418.91

1 mark

$\$4.6034 \times 365 = \$1680.241$

Annual Cost = \$1,680.24

1 mark