TSSM
Creating VCE Success

**2017 Trial Examination**

THIS BOX IS FOR ILLUSTRATIVE PURPOSES ONLY

Letter

**STUDENT NUMBER**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |

# FURTHER MATHEMATICS

## Units 3 & 4 – Written examination 2

Reading time: 15minutes

Writing time: 1 hour and 30 minutes

### QUESTION AND ANSWER BOOK

**Structure of book**

| *Section* | *Number of questions* | *Number of questions to be answered* | *Number of modules* | *Number of modules to be answered* | *Number of marks* |
|---|---|---|---|---|---|
| A –Core | 7 | 7 | | | 36 |
| B - Modules | | | 4 | 2 | 24 |
| | | | | | Total 60 |

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

**Materials supplied**

- Question and answer book of 22 pages.

**Instructions**

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.**

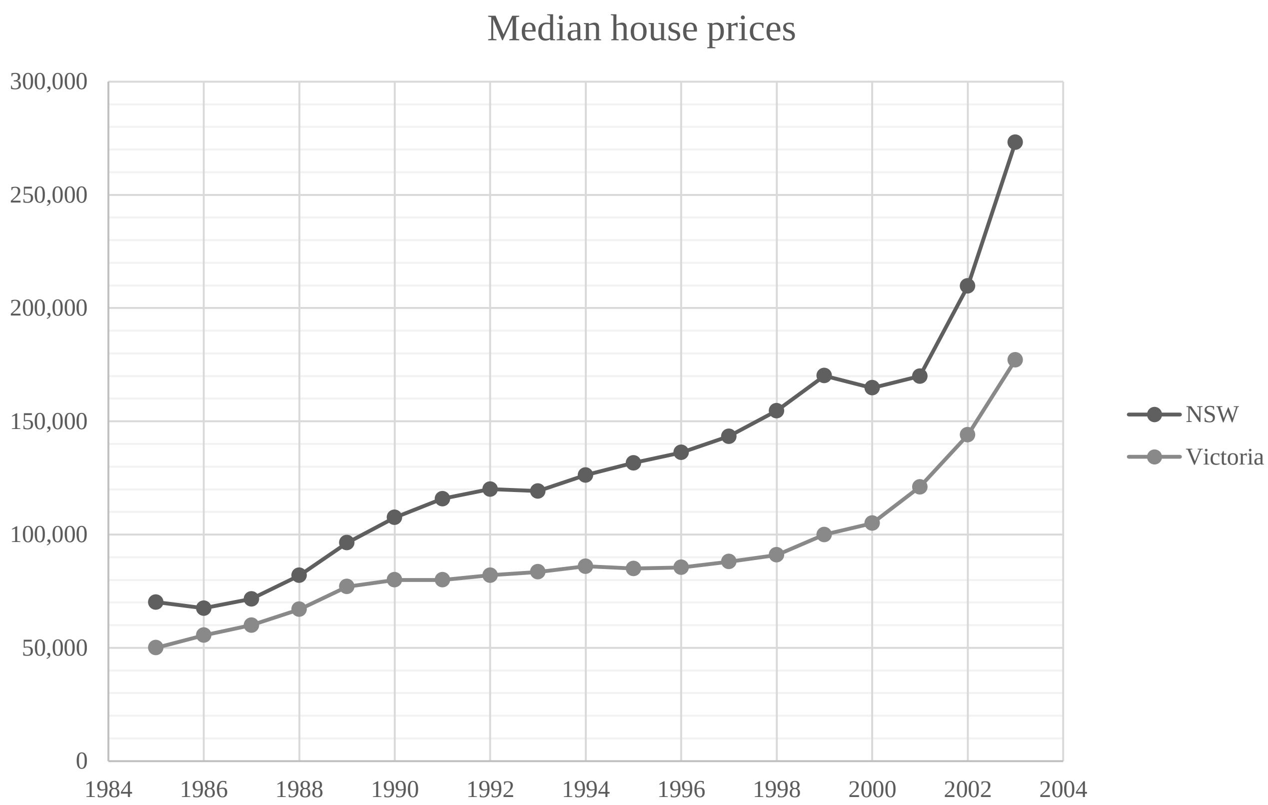
## SECTION A – Core

**Instructions for Section A**

Answer **all** questions in the spaces provided.
You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, $\pi$, surds or fractions.
In 'Recursion and financial modelling', all answers should be rounded to the nearest cent unless otherwise instructed. Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

**Data analysis**

**Question 1.**
The graph below shows median house prices across the states of NSW and Victoria from 1985 until 2003.




**a.** Which state has shown the largest change over this 18 year period?

___

1 mark

**b.** What was the percentage change in median house price for this state over the 10 year period from 1993? Give your answer to the nearest 10%

___

___

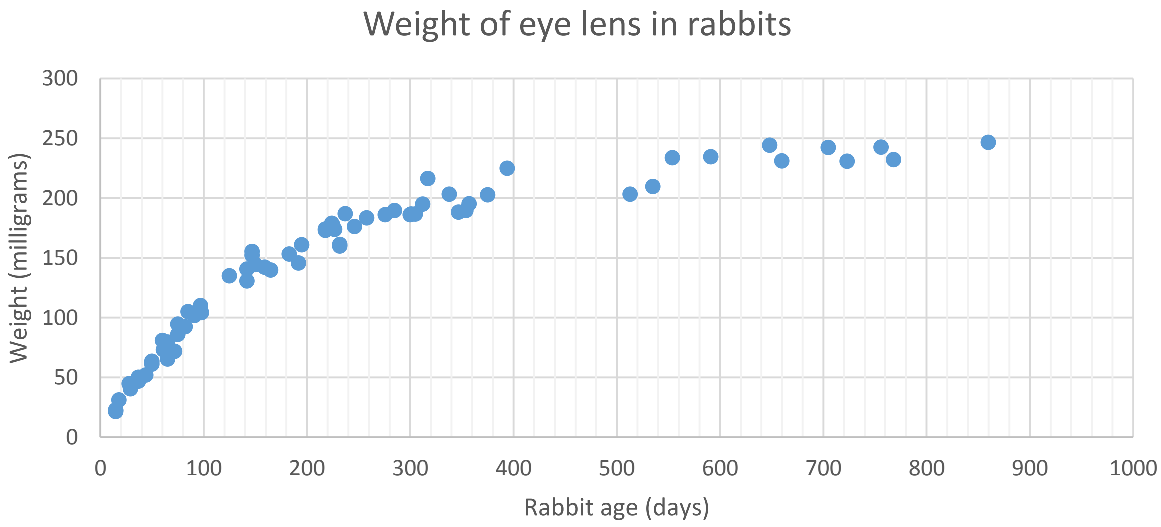
1 mark

**SECTION A – Question 1 -** continued

**c.** Compare the two sets of data identifying any differences or similarities in their trends.

2 marks

**Question 2.**




**a.** Describe the relationship between Rabbit age and weight of eye lens in terms of strength, direction and form.

1 mark

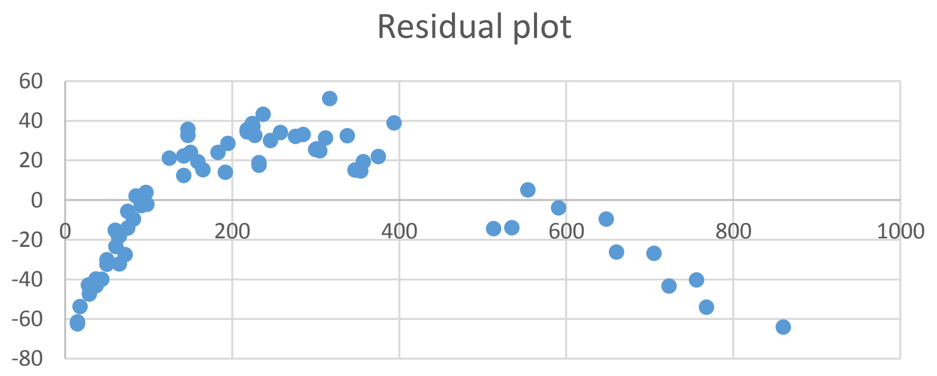
**b.** The least squares regression line for this data is $weight = 80.17 + 0.27 \times age$. Given that the correlation coefficient is 0.87, comment on the reliability of this equation for predicting the weight.

2 marks

**SECTION A – Question 2 -** continued

**TURN OVER**

**c.** The residual plot for the data is shown below. What would you recommend should be done to improve the accuracy of the least squared regression line given in the previous question?




2 marks

**d.** A log($x$) transformation was completed to linearise the data. The resulting equation is $weight = -169 + 143 \times \log(age)$. Calculate the predicted weight of a rabbit aged 215 days giving your answer correct to two decimal places and explain if the result is interpolation or extrapolation.

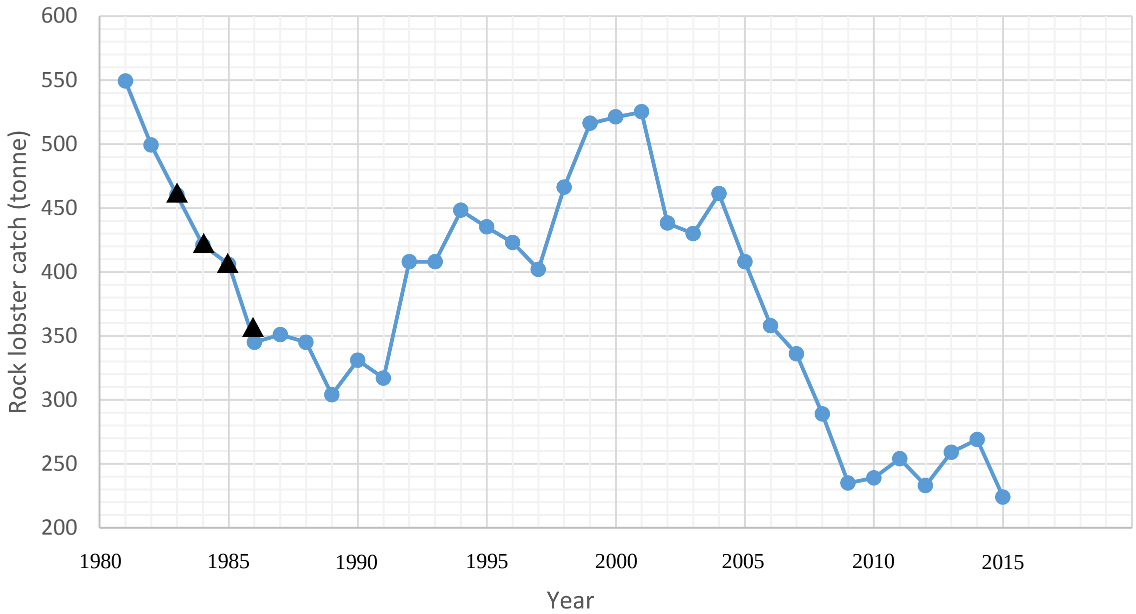
2 marks

**SECTION A** – continued

**Question 3.**

The graph below shows the commercial catch of rock lobster taken from the Western Zone (which runs from Apollo Bay to the South Australian border) annually from 1981 to 2015.

*Figures courtesy Victorian Rock Lobster Fishery Stock Assessment Report 2015.*




**a.** Five-median smoothing is to be used to smooth the time series plot. The first four smoothed values are shown on the plot. Complete the five-median smoothing by marking the remaining values on the time series plot above.

2 marks

**b.** Describe the features of the time series plot.

________________________________________________

________________________________________________

________________________________________________

________________________________________________

________________________________________________

2 marks

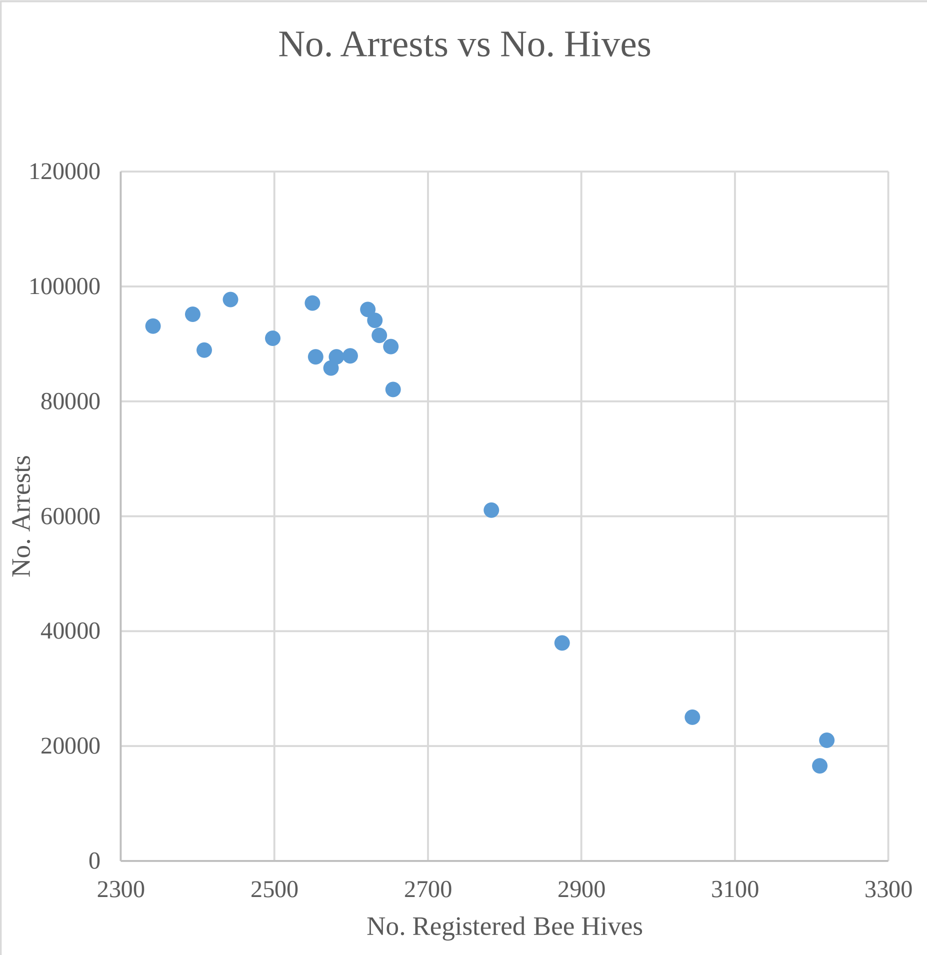
**SECTION A – Question 3 –** continued

**TURN OVER**

**Question 4.**

The table below shows the number of registered bee hives kept in the US and the number of juveniles arrested for possession of marijuana from 1990 to 2009. A scatter plot showing the relationship is also given.

| Year | Hives | Arrests |
|---|---|---|
| '90: | 3220 | 20940 |
| '91: | 3211 | 16490 |
| '92: | 3045 | 25004 |
| '93: | 2875 | 37915 |
| '94: | 2783 | 61003 |
| '95: | 2655 | 82015 |
| '96: | 2581 | 87712 |
| '97: | 2631 | 94046 |
| '98: | 2637 | 91467 |
| '99: | 2652 | 89523 |
| '00: | 2622 | 95962 |
| '01: | 2550 | 97088 |
| '02: | 2574 | 85769 |
| '03: | 2599 | 87909 |
| '04: | 2554 | 87717 |
| '05: | 2409 | 88909 |
| '06: | 2394 | 95120 |
| '07: | 2443 | 97671 |
| '08: | 2342 | 93042 |
| '09: | 2498 | 90927 |




**a.** Determine the equation of the least squares regression line using *number of bee hives* as the explanatory variable. Round your answers to the nearest whole number.

*Number of arrests* = ______ + ______ x *no. hives*

2 marks

**SECTION A – Question 4 -** continued

**b.** State the coefficient of determination correct to two decimal places for the association in terms of the variables and comment on the validity of the relationship.

______________________________________________________________________

______________________________________________________________________

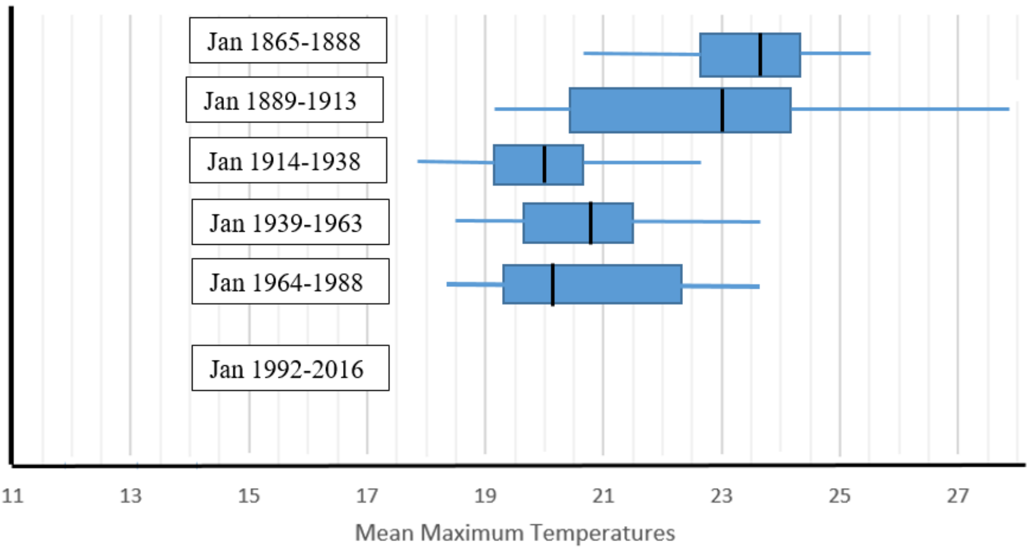
______________________________________________________________________

______________________________________________________________________

2 marks

**Question 5.**

The graph below shows parallel box plots of the average daily maximum for January in groups of 25 years.




| Year | Jan | Year | Jan | Year | Jan | Year | Feb | Year | Mar |
|---|---|---|---|---|---|---|---|---|---|
| 1992 | 18.9 | 1997 | 22.5 | 2002 | 18.5 | 2007 | 21.2 | 2012 | 22.7 |
| 1993 | 21.2 | 1998 | 21.2 | 2003 | 21.5 | 2008 | 21 | 2013 | 22 |
| 1994 | 20.1 | 1999 | 19.9 | 2004 | 18.7 | 2009 | 21.4 | 2014 | 23.7 |
| 1995 | 22.3 | 2000 | 19.3 | 2005 | 21.4 | 2010 | 21.8 | 2015 | 20.9 |
| 1996 | 19.2 | 2001 | 23.4 | 2006 | 22.1 | 2011 | 21.5 | 2016 | 21.7 |

This table contains the Mean Maximum temperatures for January over the last 25 years.

**a.** Use this data to add another box plot to the graph above.

3 marks

**SECTION A – Question 5** – continued

**TURN OVER**

**b.** Compare the box plots in terms of centre and spread.

2 marks

**Recursion and financial modelling**

**Question 6.**

Jennifer wants to save up money to use as an annuity for the time she will be studying at university. She is currently in Year 10 and hopes to continue working her part time job which provides her with a take home income of $110 per week. Her parents give her an allowance of $20 per week to encourage her to save.

**a.** How much will Jennifer have been paid by her employer over the 3 years of work assuming her wage remains the same?

1 mark

**b.** Jennifer sets up an annuity account. Each week she pays $130 into this account which pays 1.83% per annum plus an additional 1.83% bonus interest if more than $200 is added to the account each calendar month. Interest is **calculated daily** and **paid monthly**. Calculate the total amount in the account after 3 years (a total of 156 deposits of $130 including the initial deposit) and determine how much interest she has earned over this period.

2 marks

**c.** How much could Jennifer withdraw from her savings each month if the annuity is to last the 3 years of her degree when she will no longer be making deposits?

1 mark

**SECTION A – Question 6 -** continued

**d.** Jennifer realises that this will not be enough money to cover the cost of rent in Melbourne. Rent will cost her \$175 per week and she will need \$25 per week for other costs. She will also receive some funds through Youth Allowance which she estimates will cover all other costs. How much money would she need to have invested at the start of her university studies so that she can make fortnightly withdrawals from her annuity for this amount.

1 mark

**Question 7.**

Laurence has decided to extend his hobby of bee keeping into a business. He borrows \$20000 to invest in equipment which will be depreciated using the reducing balance method.

**a.** If after 5 years the value of the equipment is \$13181.63, determine the rate of depreciation.

1 mark

**b.** Write a recurrence relation for the value of the equipment where $V_{n+1}$ and $V_n$ represents the value of the equipment after $n$+1 years and $n$ years respectively.

1 mark

**SECTION A – Question 7 –** continued

**TURN OVER**

Laurence sells his honey at local markets and finds that demand for his honey increases as more people hear about his new business. The number of customers from one market to the next increases by 15%, and he finds that 10 regulars do not purchase honey the following market as they still have enough to keep them until next time.

**c.** If he had 96 customers at his first market, set up a recurrence equation and use it to calculate the expected number of customers at market number 10.

2 marks

Laurence estimates that he makes an average profit of $8 from each customer, and there is just 1 market each month. His loan for $20000 is at a fixed 3.9% per annum, compounding monthly. Laurence puts all of his profit from the first market into paying off his loan and continues to make regular payments each month of this same amount.

**d.** How long will it take Laurence to pay out his loan in full?

1 mark

**e.** Because this is not a whole number of months, how much will his final payment be to amortise his loan?

2 marks

**END OF SECTION A**

## SECTION B – Modules

**Instructions for Section B**

Select **two** modules and answer **all** questions within the selected modules.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example $\pi$, surds or fractions.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

## Contents



**SECTION B –** continued

**TURN OVER**

**Module 1 – Matrices**

**Question 1.**

$$A = \begin{bmatrix} -2 & 5 \\ 3 & -7 \end{bmatrix} \quad B = \begin{bmatrix} 4 & -1 & 2 & 5 & 2 \\ -3 & 6 & 4 & 0 & 0 \\ 0 & -2 & 3 & -1 & 0 \end{bmatrix}$$

**a.** State the order of matrices $A$ and $B$.

2 marks

**b.** Find $A^{-1}$, the inverse of matrix $A$.

1 mark

**c.** $C$ and $D$ are two matrices such that $C \times A + B \times D$ is defined. Determine the orders of matrix $C$ and matrix $D$.

2 mark



**Question 2.**

Prior to an election, voters in one particular electorate, when surveyed, will align themselves with Party A, or Party B or are undecided. The way people respond to the survey can change from week to week. The following transition matrix shows how people's votes vary from week to week.

$$\begin{array}{cccc} & \text{A} & \text{N} & \text{B} & \\ T = & \left[\begin{array}{ccc} 0.8 & 0.25 & 0.05 \\ 0.15 & 0.4 & 0.2 \\ 0.05 & 0.35 & 0.75 \end{array}\right] & & & \begin{array}{c} \text{A} \\ \text{N} \\ \text{B} \end{array} \end{array}$$

In a preliminary survey, 1328 people said they would vote for Party A, 704 said they were undecided and 1210 said they would vote for Party B.

**a.** Write an initial state matrix representing the results of the preliminary poll.

$$S_0 = \left[\begin{array}{c} \\ \\ \\ \end{array}\right]$$

1 mark

**b.** Write a recurrence model where $S_n$ is a matrix that represents the way people said they would vote $n$ surveys after the preliminary survey.

___

___

___

2 marks

**SECTION B – Module 1 –** continued

**TURN OVER**

**Question 3.**

A squash competition was played so that each person played each other twice. The results are shown in the dominance matrix below:

$$D_1 = \begin{array}{c} \\ \\ \end{array}\begin{array}{c} \begin{matrix} A & B & C & D & E \end{matrix} \\ \begin{bmatrix} 0 & 2 & 1 & 0 & 1 \\ 0 & 0 & 2 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 2 & 2 & 1 & 0 & 0 \\ 1 & 1 & 1 & 2 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}$$

A – Aiden, B – Barry, C – Chloe, D – Donna, E - Ernest

**a.** This dominance matrix shows that two players have both won 5 games. Calculate the two step dominance matrix and fill it in below:

$$D_2 = \begin{bmatrix} \quad & \quad & \quad & \quad & \quad \\ \\ \\ \\ \\ \end{bmatrix}$$

1 mark

**b.** By adding the one and two step dominance matrices together, determine the final ranking of the five players, stating the total number of one and two step wins.

| Position | Player | No wins (1 and 2 step) |
|---|---|---|
| 1st | | |
| 2nd | | |
| 3rd | | |
| 4th | | |
| 5th | | |

3 marks

**END OF MODULE 1 – SECTION B -** continued

**Module 2 – Networks and decision mathematics**

**Question 1.**
Swimming coach Deb, has four swimmers in her squad who will be competing at Country Championships this year. She wishes to enter them in a medley relay but needs to work out which swimmer should swim which stroke to achieve the fastest time.

| **Table 1** | PB in seconds | | | |
|---|---|---|---|---|
| **Swimmer** | **Free** | **Back** | **Breast** | **Fly** |
| Kara | 36.38 | 41.24 | 46.31 | 42.25 |
| Jess | 31.20 | 37.04 | 47.18 | 33.73 |
| Andrea | 32.24 | 40.76 | 45.05 | 33.99 |
| Joanne | 35.53 | 42.99 | 48.32 | 39.34 |

**a.** Fill in the missing values from Table 2, the result of a row reduction being applied to the data in Table 1:

**Table 2**

| **Swimmer** | **Free** | **Back** | **Breast** | **Fly** |
|---|---|---|---|---|
| Kara | 0 | 4.86 | 9.93 | |
| Jess | 0 | 5.84 | 15.98 | 2.53 |
| Andrea | 0 | | 12.81 | 1.75 |
| Joanne | 0 | 7.46 | 12.79 | 3.81 |

1 mark

**b.** Explain why the first column of this table contains zeros.

_______________________________________________

_______________________________________________

1 mark

**SECTION B – Module 2 – Question 1 –** continued

**TURN OVER**

**c.** Table 3 is the result of a column reduction, the second step of the Hungarian algorithm:

**Table 3**

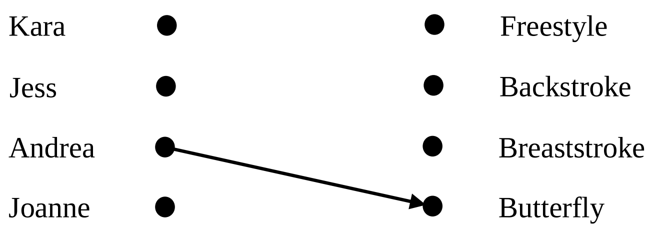
| **Swimmer** | **Free** | **Back** | **Breast** | **Fly** |
|---|---|---|---|---|
| Kara | 0.00 | 0.00 | 0.00 | 4.12 |
| Jess | 0.00 | 0.98 | 6.05 | 0.78 |
| Andrea | 0.00 | 3.66 | 2.88 | 0.00 |
| Joanne | 0.00 | 2.60 | 2.86 | 2.06 |

The dotted lines above have been drawn to cover the zeros in Table 3. These indicate that an optimal allocation cannot be made yet. Give a reason why.

1 mark

**d.** Complete the next iteration of the Hungarian algorithm then fill in the bipartite graph below.

| **Swimmer** | **Free** | **Back** | **Breast** | **Fly** |
|---|---|---|---|---|
| Kara | | | | |
| Jess | | | | |
| Andrea | | | | |
| Joanne | | | | |




2 marks

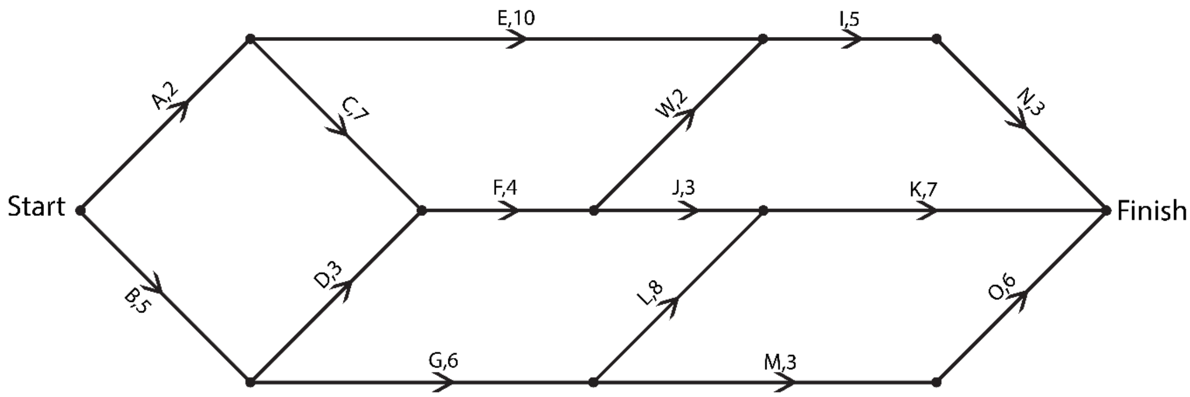
**e.** The order of strokes in the medley is given below. Fill in the optimum medley relay team and calculate their entry time. Give your answer in minutes and seconds.

| | **Swimmer** | **Time** |
|---|---|---|
| **Back** | | |
| **Breast** | | |
| **Fly** | | |
| **Free** | | |

2 marks

**SECTION B – Module 2 –** continued

**Question 2.**




**a.** Determine the earliest finishing time in days for this project.

1 mark

**b.** State the critical path for this activity network.

1 mark

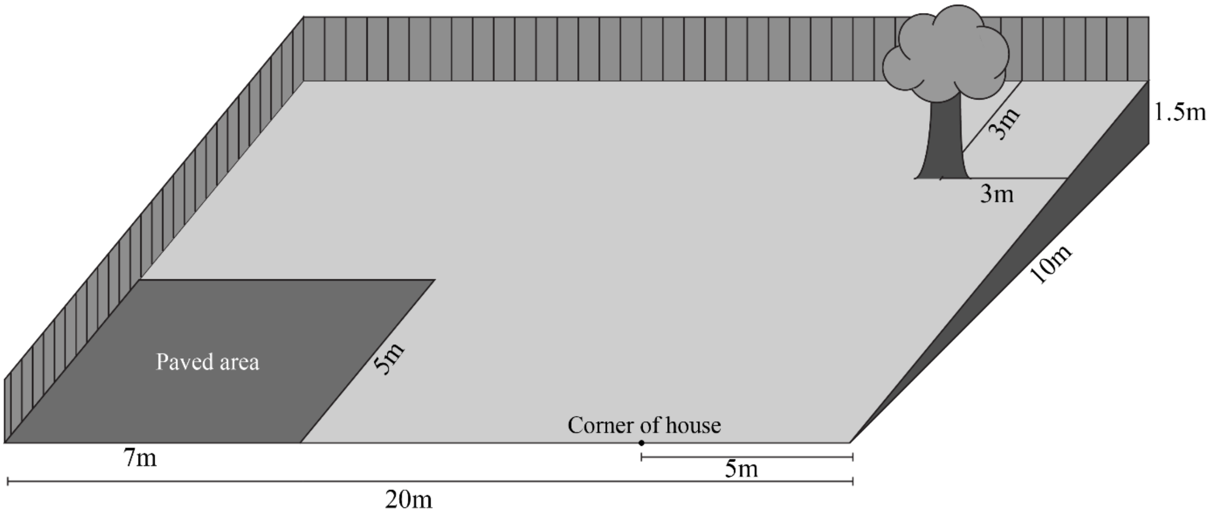
**c.** If the activities L and E can be reduced by up to 3 days, and activity G can be reduced by up to 2 days at a cost of $540 per day, determine the earliest finishing time possible and the associated cost to reduce it.

3 marks

**END OF MODULE 2 – SECTION B –** continued

**TURN OVER**

**Module 3 – Geometry and measurement**
**Question 1.**




A family has built their new home on a housing block of dimensions 20 m by 29 m. Their house is on a flat section of the block and their back yard is sloped towards the house, with the far end of the block 1.5 m higher than the house slab. There is a paved area running down one side of the house to a paved outdoor entertaining area measuring 7 m by 5 m as shown on the diagram above.

**a.** The owner is concerned about the amount of water running off the paved area towards the house during very heavy rain events. Given that 1 mm of rain over 1 m$^2$ of area produces 1 litre of water and extreme rain events result in 60 mm of rain per hour, determine how many litres of water will be shed from the paved area each hour, and thus the number of litres per second that will need to be dispatched by the drainage pipe.

___

___

___

2 marks

**b.** The flow rate through the drainage pipe can be calculated using this formula

$\textit{flow rate} = \frac{A}{18}$, where $A$ is the cross-sectional area of the pipe measured in square centimetres, and the flow rate is measured in litres per second. Pipe comes in sizes 1” (approx. 2.54cm in diameter), 1.5” (3.8 cm), 2” (5 cm), 2.5” (6.4 cm) and 3” (7.6 cm). What is the smallest size pipe that would be required to ensure the drain will cope with very heavy down pours? Show mathematics to support your answer.

___

___

___

2 marks

**SECTION B – Module 3 – Question 1 -** continued

**c.** The owners have planted a tree 3m from the back and side fences as shown in the diagram. They wish to determine how high the tree can grow before it is in danger of hitting the house. Calculate the horizontal distance of the tree from the corner of the house.

3 marks

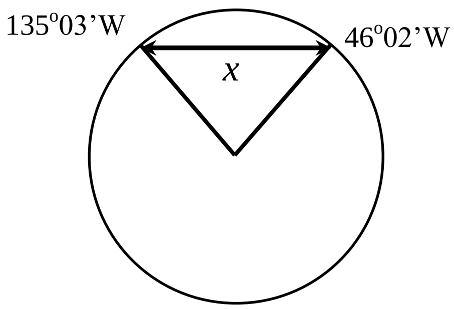
**Question 2.**

The cities of Qaqortoq in Denmark (Latitude $60^{o}43'$N, Longitude $46^{o}02'$W) and Whitehorse in Canada (Latitude $60^{o}43'$N, Longitude $135^{o}03'$W**)** have the same latitude.

**a.** Calculate the distance between these two towns if you travel along the parallel of latitude $60^{o}43'$N. Assume the earth's radius is 6400 km. Give your answer to the nearest 10 km.

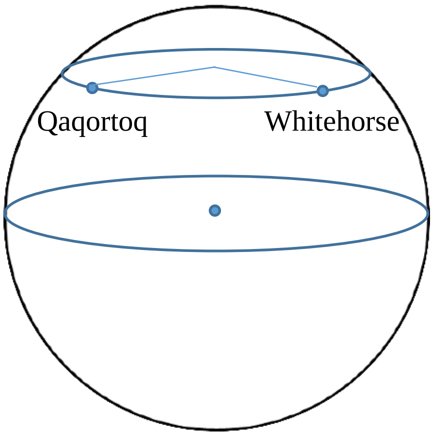
2 marks

**b.** Using the triangle shown in the diagram, calculate the length *x* shown on the diagram. Give answer to one decimal place.




1 mark

**c.** Assuming the earth's radius to be 6400 km, now calculate the shortest distance between Qaqortoq and Whitehorse if the Great Circle connecting them is travelled, and how much distance is saved by travelling this way?




2 marks

**END OF MODULE 3 – SECTION B -** continued

**Module 4 – Graphs and relations**

**Question 1.**

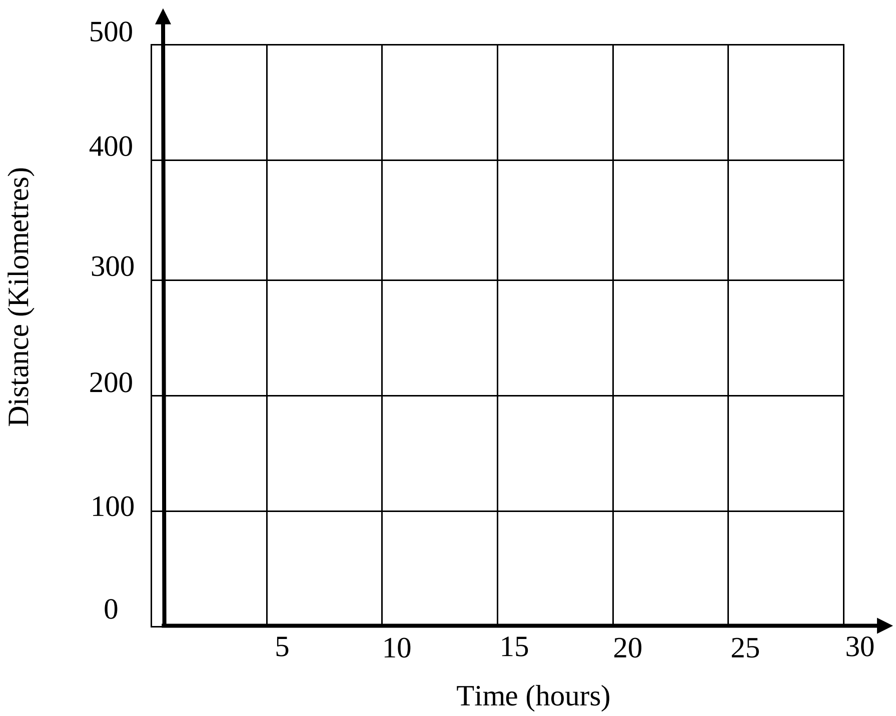
A family company sells their home remedy “Sneeze Eaze” at \$5.80 per unit. The cost of production (\$*C*), is given by the rule: $C = 5x + 48$, where *x* is the number of units produced. Find the value of *x* for which the cost of production of *x* units is equal to the revenue received by the company for selling *x* units.

1 mark

**Question 2.**

Three power boats compete in a 500 km handicap race. The boats leave at 5 hourly intervals. Boat A leaves first and has a speed for the race of 20 km/hr. Boat B leaves next and travels at an average speed of 25 km/hr. Boat C leaves last and has an average speed of 40 km/hour.

**a.** On the axes below draw the graph of each boats’ journey.




3 marks

**b.** Use your graph to determine which boat won the race.

1 mark

**SECTION B – Module 4 –** continued

**TURN OVER**

**Question 3.**

Origin Energy have the following fee structure for use of electricity at peak times:

| Residential Peak Anytime (GD/GR) | Unit | Excl GST | Inc GST |
|---|---|---|---|
| Consumption of first 10.9589 kWh/Day | cents per kWh | 26.27 | 28.897 |
| Consumption - Balance kWh/Day | cents per kWh | 28.14 | 30.954 |
| Daily Supply Charge | cents per day | 113.29 | 124.619 |

**a.** Write down how you would be charged if you used exactly 10.9589 kWh of power in a day. Give your answer in dollars, including GST.

1 mark

**b.** How much would it cost, in dollars, if you used 50 kWh of electricity in 1 day?

1 mark

**c.** Write an equation for calculating the cost of electricity, **in cents**, assuming you use more than 11 kWh per day, where $x$ represents the total number of kWh used.

2 marks

**d.** The average Australian home uses 15.6 kWh per day. Calculate the daily cost in dollars based on Origin's fee structure.

1 mark

**e.** Assuming that there are 91 days in a quarter, estimate the price of a quarterly electricity bill for the average Australian family, and the annual cost.

2 marks

**END OF QUESTION AND ANSWER BOOK**