The Mathematical Association of Victoria

Trial Examination 2018

FURTHER MATHEMATICS

Trial Written Examination 1 - SOLUTIONS

SECTION A: Core

Question	Answer	Question	Answer
1	C	**13**	B
2	C	**14**	D
3	B	**15**	A
4	D	**16**	E
5	C	**17**	E
6	D	**18**	D
7	D	**19**	C
8	A	**20**	D
9	E	**21**	C
10	B	**22**	B
11	C	**23**	E
12	B	**24**	B

SECTION B : Modules

1 – Matrices

Question	Answer
1	C
2	D
3	B
4	C
5	E
6	B
7	E
8	D

2 – Networks & decision mathematics

Question	Answer
1	C
2	B
3	C
4	B
5	A
6	D
7	D
8	A

3 – Geometry & measurement

Question	Answer
1	C
2	E
3	D
4	D
5	E
6	C
7	D
8	E

4 – Graphs & relations

Question	Answer
1	B
2	D
3	C
4	D
5	A
6	D
7	B
8	D

Data Analysis

Question 1 Answer C

There are 40 values in this histogram. The median can best be found using a cumulative frequency as shown in the table below:

Number of cups of coffee	Frequency	Cumulative Frequency
0	2	2
1	5	7
2	10	17
3	15	32
4	3	35
5	4	39
6	0	39
7	0	39
8	0	39
9	0	39
10	1	40

The Q_3 value lies between the 30th and 31st values, both of which are 3.

Question 2 Answer C

The five figure summary for this data is Min = 1, $Q_1 = 1.6$, Med = 3.25, $Q_3 = 6.3$ and Max = 24.6

IQR = 6.3 – 1.6 = 4.7

The upper fence is $6.3 + 1.5 \times 4.7 = 13.35$, 14.0 and 24.6 are both greater than 13.35 so they are outliers.

The lower fence is $1.6 - 1.5 \times 4.7 = -5.45$, there are no values below -5.45, so no lower end outliers.

Therefore there are two outliers.

Question 3 Answer B

Vinh's standardised height value $= \frac{165 - 153}{7} = 1.71$

Fawad's standardised height value $= \frac{140 - 153}{7} = -1.86$

Checking the options,

A. Standardised values as calculated above are NOT identical — NOT TRUE

B. Increasing Vinh's height by 1 cm brings him FURTHER from the mean
So his standardised height value will be 1.86, same distance from the mean as Fawad's — TRUE

C. Decreasing Fawad's height by 1 cm brings him CLOSER to the mean
So his standardised height value will be – 1.71, same distance from the mean as Vinh — NOT TRUE

D. – 1.86 is further from the mean than 1.71 — NOT TRUE

E. The standardised height values have incorrect values — NOT TRUE

Question 4 Answer D

There are four (4) columns, so the total column heights required = 6 + 9 + 2 + 2 = 19

Percentage = $\frac{19}{27} \times 100\%$ = 70.370…% ≈ 70.4%

Question 5 Answer C

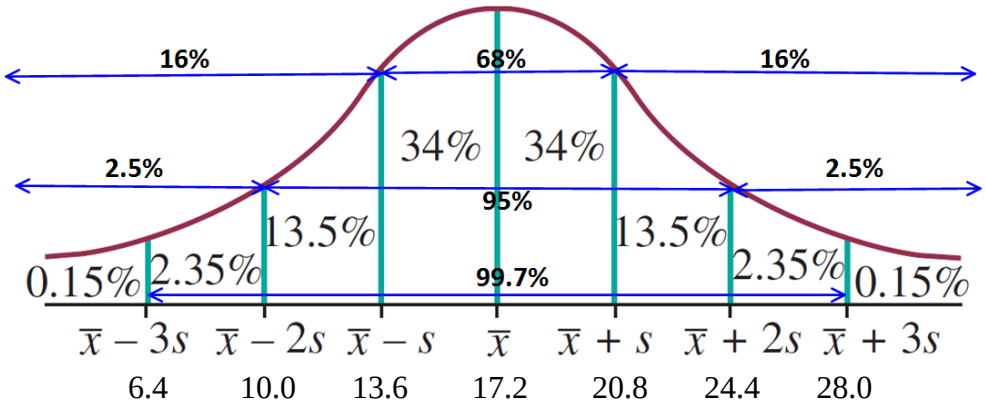
The entire data set has been moved DOWN by 4 kg, but the spread will remain the same.
The mean will decrease to 69 kg, but the standard deviation will remain at 6 kg.

Question 6 Answer D

In order the data is 75 76 79 82 84
For six values, the median lies midway between the 3rd and 4th values.
The 3rd value is 79, so for a median value of 80 the 4th value will have to be 81.

Question 7 Answer D

As the data is normally distributed, the mean and standard deviation can be fitted to a bell shaped curve using the 68-95-99.7% rule.



Using this information the answer that is not true is D, because 2.35% of maximum temperatures are between 24.4º and 28º not 13.5%.

Question 8 Answer A

For a least squares regression line in the for $y = a + bx$ **:**

$b = \frac{r \times S_y}{S_x} = \frac{-0.6 \times 8.4}{3.6} = -1.4$

$a = \bar{y} - b \times \bar{x}$ = 34.1 – (– 1.4 × 17.2) = 58.2

So equation is *gas use in MJ* = 58.2 – 1.4 × *maximum temperature*

Question 9 Answer E

The correlation coefficient is -0.6. The correlation coefficient tells us that a moderate negative linear relationship exists between gas use and maximum temperature.
Checking the options,

A.	this implies a positive relationship.	INCORRECT
B.	this is a statement using the *r* value in an interpretation of the coefficient of determination.	INCORRECT
C.	"need" implies causality.	INCORRECT
D.	the explanatory and response variables are reversed.	INCORRECT
E.	this implies a moderate negative relationship	CORRECT

Question 10 Answer B

There were a total of 65 + 24 = 89 people over 25 years who preferred to binge watch television.
The total number of people in the survey were 410.

The percentage is therefore $\frac{89}{410} \times 100\% = 21.707\% \approx 22\%$.

Question 11 Answer C

Age group in years and preferred method of watching TV are both categorical variables. The age group is ordinal as there is a built in order of younger through to older people. There is no particular order in the preferred method of watching TV so the two variables are ordinal and nominal categorical variables respectively.

Question 12 Answer B

The shape of this graph indicates that possible transformations could be month^2, log (value) or $\frac{1}{value}$. The only available option out of these would be log (value).

Question 13 Answer B

Residual = *actual* – *predicted*
26.87 = 2044 – *predicted*
predicted = 2044 – 26.87 = 2017.13

Using equation *Value (Bitcoin)* = – 285.43 + 18.72 × *Value (Ethereum)*
2017.13 = – 285.43 + 18.72 × *Value (Ethereum)*
Value (Ethereum) = (2017.13 + 285.43) ÷ 18.72 = \$123

Question 14 Answer D

There is a clear increasing trend, so option C is incorrect. The data is repeating a pattern every four points with sales increasing from quarter 1 through to quarter 4 and then falling away again for quarter 1 the following year. It is therefore seasonal data.

Question 15 Answer A

The time period for Quarter 1 2017 can be found from the table below:

Year	Quarter 1	Quarter 2	Quarter 3	Quarter 4
2013	1	2	3	4
2014	5	6	7	8
2015	9	10	11	12
2016	13	14	15	16
2017	17	18	19	20

The deseasonalised sales can be calculated using $20.13 + 4.72 \times 17 = 100.37$.
Actual sales = deseasonalised sales × seasonal index
Actual sales = $100.37 \times 0.86 = 86.3182 \approx 86.32$

Question 16 Answer E

$D = \frac{A}{I} = \frac{100}{0.54}$ = 185% i.e. an INCREASE of 85%

Question 17 Answer E

Given that the trailer depreciates by an equal amount losing \$7000 over 5 years, it must lose

$\frac{7000}{5}$ = \$1400 per year.

Checking the options,

A. the value is gaining \$1400 each year. INCORRECT
B. D_n is being multiplied by 0.8 as well as losing \$1400 each year. INCORRECT
C. this sequence is increasing in value each year. INCORRECT
D. the sequence would represent a reducing balance depreciation with changing amounts of loss each year. INCORRECT
E. the value is decreasing by \$1400 each year CORRECT

Question 18 Answer D

A multiple of 1.003 could be an annual rate of 0.3% increase or a monthly increase of 0.3% which is equivalent to $0.3 \times 12 = 3.6\%$ per annum. This means options C and E are both incorrect.

Also $\frac{0.3}{100} \times 15\,000$ = \$45 so this must be an interest only loan as the only payment being made is equal to the interest being charged. Option D is correct.

Question 19 Answer C

Value lost in 2016 = \$27 543.74 – \$14 999.60 = \$12 544.14

Depreciation rate = $\frac{\$12544.14}{29867}$ = \$0.42/km = 42¢/km (take care reading units!)

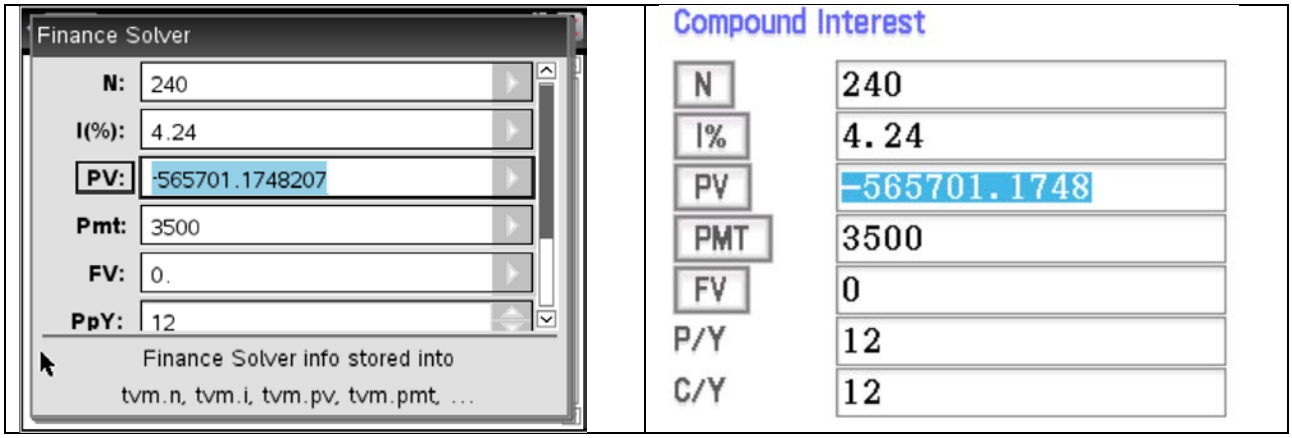
Question 20 Answer D

Interest rate per month = $\frac{10.5}{1200}$ = 0.00875
Interest charged for twelfth payment = 0.00875 × 222.73 = \$1.95
EITHER
Reduction in loan balance = \$220.00 – \$1.95 = 218.05
As \$222.73 is still owed, he must pay an extra (\$222.73 – \$218.05 =) \$4.68 to amortise the loan.
Total final payment = \$220.00 + \$4.68 = \$224.68

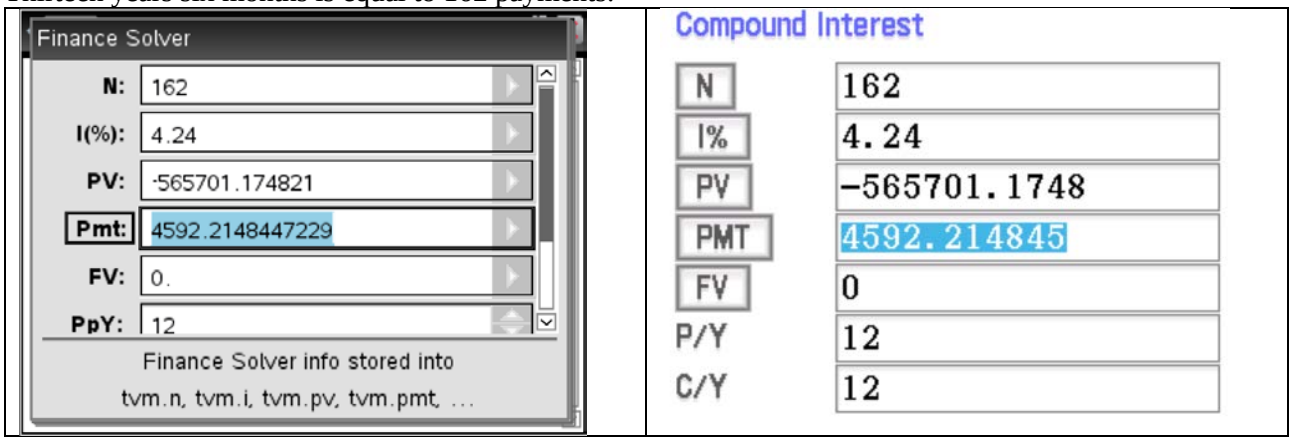
OR
Total to repay = balance owing plus interest = \$222.73 + \$1.95 = \$224.68

Question 21 Answer C

First, using Finance Solver to find the value of the superannuation lump sum



Now find the monthly payment if the money runs out in 13.5 years
Thirteen years six months is equal to 162 payments.

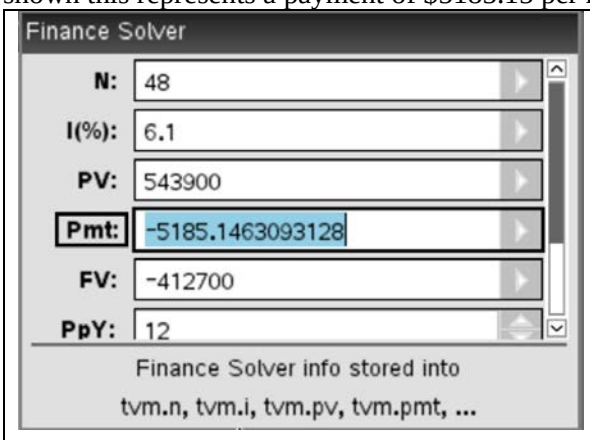


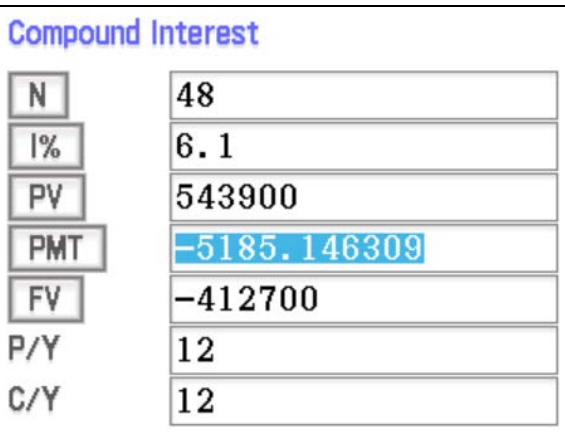
The extra amount he requires is \$4592.21 – \$3500 = \$1092.21

If the lump sum is rounded to \$565 700, the monthly amount is calculated to be \$4592.205…, which rounds to the same amount (\$4592.21).

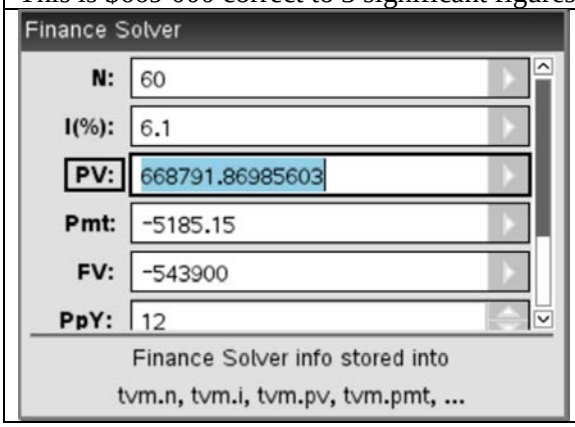
Question 22 Answer B

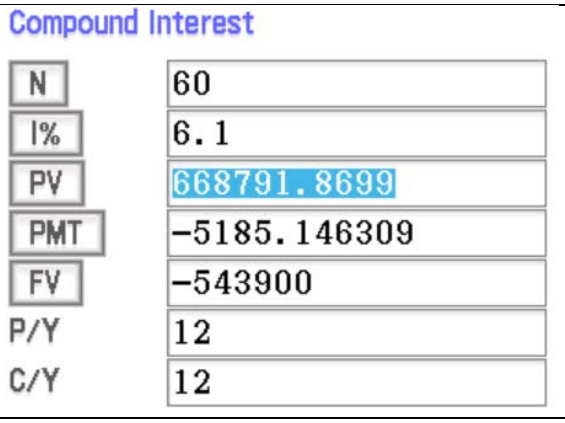
Over a 4 year period (from 5 to 9 years) Ian's loan balance has decreased from \$543 900 to \$412 700. As shown this represents a payment of \$5185.15 per month to the loan.





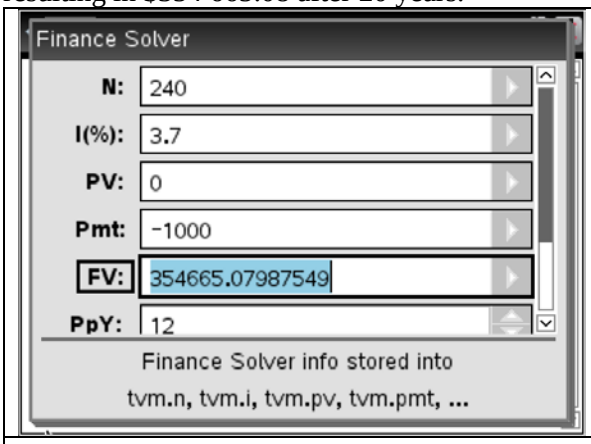
Using the payment of \$5185.15 and a future balance owed after 5 years of \$543 900, it can be determined that the initial loan was \$668 791.87 correct to the nearest cent.
This is \$669 000 correct to 3 significant figures.

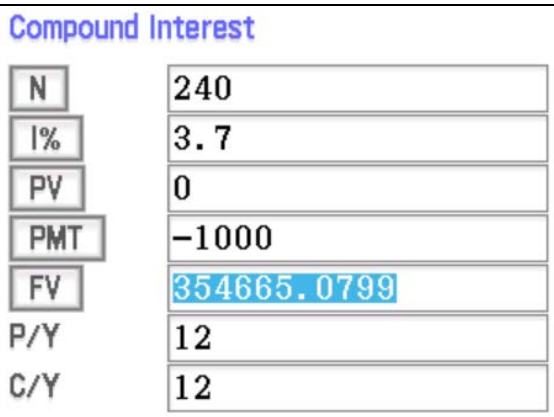




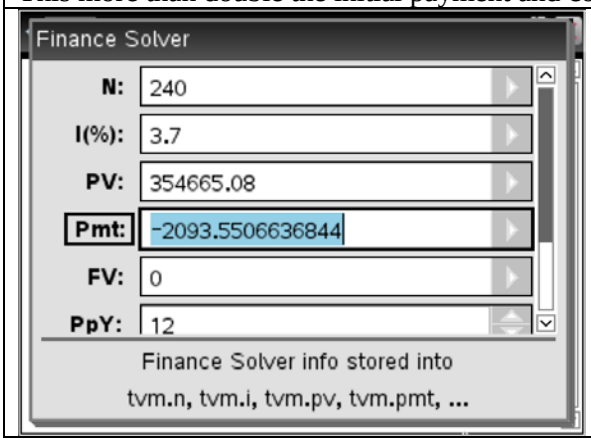
Question 23 Answer E

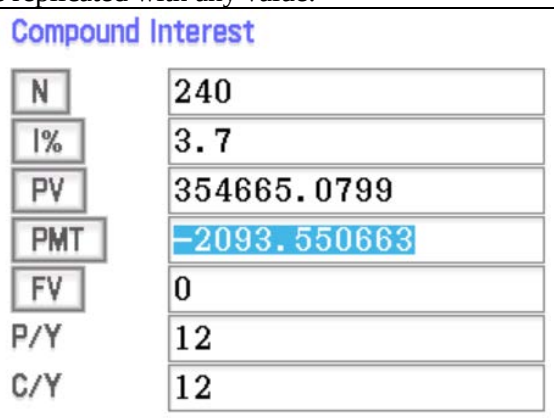
This question is best explored using a trial amount. Here an investment of $1000 per month has been used, resulting in $354 665.08 after 20 years.





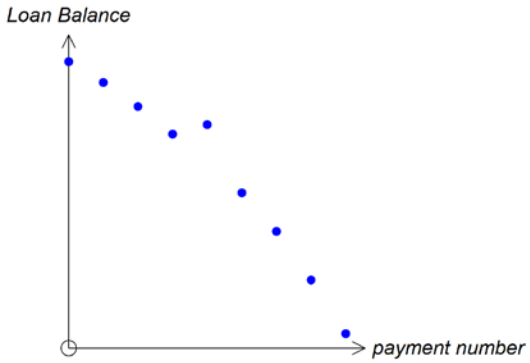
Changing the balance into an annuity investment and setting the future value as $0 from an investment of $354 665.08, it can be seen that the monthly payment is $2093.56.
This more than double the initial payment and could be replicated with any value.





Question 24 Answer B

The missed repayment means that interest is still added without the associated reduction due to payment so there will be an increased balance from the previous balance after 3 months. Options C, D and E are incorrect as they do not reflect the added interest.
Both option A and B show an increased balance.
Option A is incorrect as the increase is too great (interest is not equal to payment in a reducing balance loan) and the missed payment will result in additional interest, meaning that there will be a small adjustment at the end of the loan. This is reflected in option B where the balance is not quite zero after 8 payments.



Module 1 – Matrices

Question 1 Answer C

$$3\begin{bmatrix} 3 & 7 \\ 6 & 2 \end{bmatrix} + 2\begin{bmatrix} 3 & 7 \\ 6 & 2 \end{bmatrix} = 5\begin{bmatrix} 3 & 7 \\ 6 & 2 \end{bmatrix} = \begin{bmatrix} 5\times 3 & 5\times 7 \\ 5\times 6 & 5\times 2 \end{bmatrix} = \begin{bmatrix} 15 & 35 \\ 30 & 10 \end{bmatrix}$$

Question 2 Answer D

The equations are $3p + 4b = 20.65$ and $4p + 3b = 20.30$.

In matrix form $\begin{bmatrix} 3 & 4 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} p \\ b \end{bmatrix} = \begin{bmatrix} 20.65 \\ 20.30 \end{bmatrix}$

Determinant $= 3 \times 3 - 4 \times 4 = -7$

And hence, $\begin{bmatrix} p \\ b \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 4 & 3 \end{bmatrix}^{-1}\begin{bmatrix} 20.65 \\ 20.30 \end{bmatrix}$ (not offered as an option)

Or $\begin{bmatrix} p \\ b \end{bmatrix} = -\frac{1}{7}\begin{bmatrix} 3 & -4 \\ -4 & 3 \end{bmatrix}\begin{bmatrix} 20.65 \\ 20.30 \end{bmatrix}$ by determining the value of the inverse

Question 3 Answer B

The final matrix is a (1×1) matrix, containing the sum of the five given numbers.
Check the orders of the product matrices of each option,

A. $(5 \times 1) \times (1 \times 5) = (5 \times 5)$
B. $(1 \times 5) \times (5 \times 1) = (1 \times 1)$
C. $(5 \times 5) \times (5 \times 1) = (5 \times 1)$
D. $(5 \times 5) \times (5 \times 5) = (5 \times 5)$
E. $(5 \times 5) + (5 \times 5) = (5 \times 5)$

Only Option B provides a product matrix of the correct order.

Question 4 Answer C

Checking each of the options in turn using the rule :

A. $m_{11} = 3 \times 1 - 2 \times 1 + 1 = 2$ INCORRECT
B. $m_{11} = 3 \times 1 - 2 \times 1 + 1 = 2$, $m_{12} = 3 \times 1 - 2 \times 2 + 1 = 0$ INCORRECT
C. $m_{11} = 3 \times 1 - 2 \times 1 + 1 = 2$, $m_{12} = 3 \times 1 - 2 \times 1 + 1 = 0$
$m_{21} = 3 \times 2 - 2 \times 1 + 1 = 5$, $m_{22} = 3 \times 2 - 2 \times 2 + 1 = 3$ CORRECT
D. $m_{11} = 3 \times 1 - 2 \times 1 + 1 = 2$, $m_{21} = 3 \times 2 - 2 \times 1 + 1 = 5$,
$m_{31} = 3 \times 3 - 2 \times 1 + 1 = 8$ INCORRECT
E. $m_{11} = 3 \times 1 - 2 \times 1 + 1 = 2$, $m_{12} = 3 \times 1 - 2 \times 2 + 1 = 0$,
$m_{13} = 3 \times 1 - 2 \times 3 + 1 = -2$ INCORRECT

Only Option C gives ALL elements correct

Question 5 Answer E

The calculation could be carried out the calculator, $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} S \\ T \\ A \\ R \end{bmatrix} = \begin{bmatrix} A \\ R \\ T \\ S \end{bmatrix}$.

OR, looking at the permutation matrix, we see that the third letter originally will become the first letter with one multiplication. The third letter is A, and as there is only one Option with a column starting with A, this must be the correct answer (Option E). Note also that the fourth letter (R) becomes the second , the second letter (T) becomes the third and the first letter (S) moves to the end (spelling out ARTS).

Question 6 Answer B

Looking at the second and third rows of the communication matrix, we see that there is NO communication in either direction between Leanne (L) and Maree (M). This eliminates Options A, D and E as these Options show communication in either direction between these two managers.
Looking at the fourth row, we see that Noni (N) communicates with ALL the other managers. Option C is eliminated since it does not show communication from Noni to Leanne.
Option B remains as the correct communication matrix.

Question 7 Answer E

$S_1 = TS_0$, i.e. $\begin{bmatrix} 24 \\ 32 \\ 39 \end{bmatrix} = \begin{bmatrix} 0.5 & 0.2 & Q \\ 0.3 & 0.6 & P \\ 0.2 & 0.2 & R \end{bmatrix} \begin{bmatrix} 30 \\ 25 \\ 40 \end{bmatrix}$.

Looking at each of the first two calculations involved :
$24 = 0.5 \times 30 + 0.2 \times 25 + Q \times 40$, i.e. $24 = 15 + 5 + 40Q$, giving $Q = 0.1$
$32 = 0.3 \times 30 + 0.6 \times 25 + P \times 40$, i.e. $32 = 9 + 15 + 40P$, giving $P = 0.2$
Because T is a regular transition matrix, $R = 1 - 0.1 - 0.2 = 0.7$.
Option E is therefore the only true expression.

OR SOLVE $\left(\begin{bmatrix} 0.5 & 0.2 & Q \\ 0.3 & 0.6 & P \\ 0.2 & 0.2 & R \end{bmatrix} \times \begin{bmatrix} 30 \\ 25 \\ 40 \end{bmatrix} = \begin{bmatrix} 24 \\ 32 \\ 39 \end{bmatrix}, P, Q, R \right)$ gives $P = 0.2$, $R = 0.7$ and $Q = 0.1$.

Option E is therefore the only true expression.

Question 8 Answer D

Clearly multiplication of these two matrices is required, but should it be *TN* or *NT* ?
When we multiply two matrices, the answer matrix has its number of rows, and their labels, from the first matrix and the number of columns, and their labels, from the second matrix.
The labels required in this answer matrix are *A*, *B* and *C* for the service people and *U*, *V* and *W* for the schools.
The multiplication *TN* will give the labels *D*, *H* and *G* for BOTH the rows and the columns.
The multiplication *NT* will give the labels *U*, *V* and *W* for the rows, and *A*, *B* and C for the columns.

Hence $NT = \begin{bmatrix} 2 & 2 & 12 \\ 1 & 1 & 8 \\ 2 & 2 & 10 \end{bmatrix} \times \begin{bmatrix} 13 & 14 & 15 \\ 15 & 17 & 12 \\ 21 & 18 & 20 \end{bmatrix} = \begin{bmatrix} 308 & 278 & 292 \\ 196 & 175 & 187 \\ 266 & 242 & 254 \end{bmatrix}$ (Option D)

Module 2 – Networks and decision mathematics

Question 1 Answer C

The statements are explained below:

There is a loop at vertex *H* – there is an edge from H back to itself	CORRECT
There are three faces in this network – there are 3 enclosed faces plus the face around the network, a total of 4 faces	INCORRECT
The network is a simple network – a simple network has no loops or multiple edges	INCORRECT
DE is a bridge in the network – removing DE would separate the network into two distinct disconnected sections	CORRECT
The degree of vertex *H* is three – there are three ends of edges attached at H	CORRECT

Three (3) statements are correct.

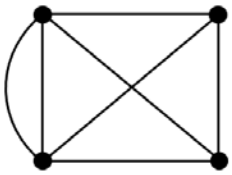
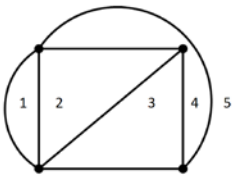
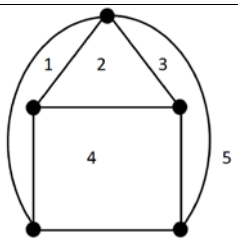
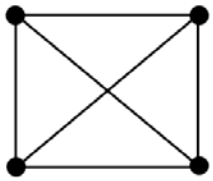
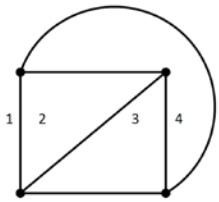
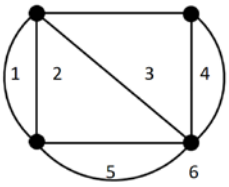
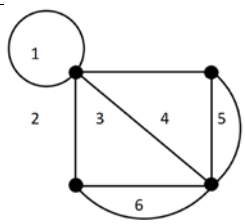
Question 2 Answer B

This question relies on the understanding of walks (sequence of edges through successive vertices), trails (walk not repeating edges), circuits (trail that returns to starting point), paths (walk not repeating vertices) and cycles (path returning to starting point).
Checking the options,

A.	*ABCFEDA* is both a circuit and a cycle.	TRUE
B.	*FEDAEDCF* is not a walk of any kind as there is no edge from *D* to *C* directly.	NOT TRUE
C.	*ABCFE* is both a trail and a path.	TRUE
D.	*DAEBA* is a trail, but not a path as *A* is repeated.	TRUE
E.	*EADEBCF* is a trail, but not a path as *E* is repeated.	TRUE

Question 3 Answer C

Regions or faces are enclosed spaces in a network as well as the region around the outside of the network. The number of faces can be determined if the networks are drawn in a planar manner.

Network	Redrawn if necessary	Number of faces
		5 faces
		5 faces
		4 faces
		6 faces
		6 faces

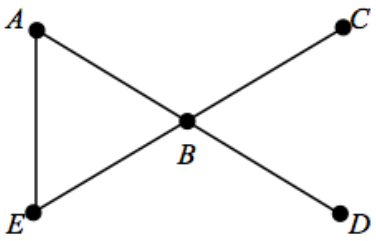
There are therefore two networks that have 5 faces.

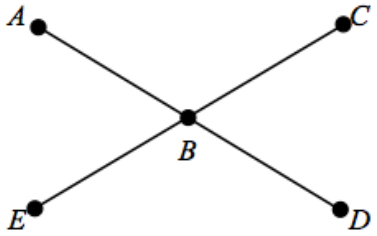
Question 4 Answer B

The minimum spanning tree is required so Prim's algorithm should be used. Dijkstra's algorithm finds the shortest path between two points, the Hungarian algorithm finds the minimum allocation for two groups, an activity network finds the minimum time to complete a project and a Hamiltonian cycle is a walk that passes through every vertex exactly once returning to the starting point.

Question 5 Answer A

This network has the same number of vertices as edges. If one edge is removed then the remaining network has one less edge than the number of vertices, so it could now be a tree.
As the network is connected and planar, using $V + F = E + 2$, because $V = E$, F must be equal to 2. This means that there is only one enclosed space and the region on the outside.
If there is a Hamiltonian cycle, it would use every vertex and therefore every edge too, so it would also be an Eulerian circuit, so option B is not true.
If there is a Hamiltonian path, it would use every vertex, but as it does not return to the start the last edge would not be used, so there would be not be an Eulerian trail, option C is not true.
A spanning tree may not be a Hamiltonian path as there may not be successive edges. This is illustrated in the diagram below where a network with the same number of vertices and edges has had an edge removed to form a spanning tree, but no Hamiltonian path exists, so option D is not true.





The original network above does not have an Eulerian circuit as the degrees of vertices C and D are both odd, so option E is incorrect.

Question 6 Answer D

The first steps to allocation are a row reduction followed by a column reduction:

$\begin{bmatrix} 2 & 7 & 8 & 6 \\ 5 & 7 & 9 & 6 \\ 3 & 4 & 8 & 3 \\ 3 & 5 & 7 & 7 \end{bmatrix}$	$\rightarrow$	$\begin{bmatrix} 0 & 5 & 6 & 4 \\ 0 & 2 & 4 & 1 \\ 0 & 1 & 5 & 0 \\ 0 & 2 & 4 & 4 \end{bmatrix}$	$\rightarrow$	$\begin{bmatrix} 0 & 4 & 2 & 4 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 4 \end{bmatrix}$	This matrix is not ready for allocation as the zeros can be covered with 3 lines, so another step is required
$\begin{bmatrix} 0 & 4 & 2 & 4 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 4 \end{bmatrix}$	$\rightarrow$	$\begin{bmatrix} 0 & 3 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$		This matrix is ready for allocation as all zeros require 4 lines to be covered.	$\begin{bmatrix} 0 & 3 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$

Question 7 Answer D

There are 3 paths through this network, *ABEF* with a duration of $a + b + e + f$, *ABDF* with a duration of $a + b + d + f$, and *ACDF* with a duration of $a + c + d + f$.
Checking the options,

A.	there is no reference to the length of b compared to c.	INCORRECT
B.	there are three paths.	INCORRECT
C.	no path *ABCDEF* exists.	INCORRECT
D.	all three paths have A and F and if the durations of $b = c$ and $d = e$, all paths will have the same length and must therefore all be critical.	CORRECT
E.	does not consider the lengths of d and e.	INCORRECT

Question 8 Answer A

The following earliest and latest starting times and float times are calculated for this network:

Activity	EST	LST	Float	Activity	EST	LST	Float
A	0	0	0	I	15	23	8
B	3	20	17	J	15	22	7
C	3	25	22	K	15	15	0
D	3	6	3	L	23	31	8
E	3	3	0	M	27	27	0
F	10	27	17	N	27	36	9
G	8	8	0	P	33	33	0
H	8	23	15				

The float time for activity *D* is 3.

Module 3 – Geometry and measurement

Question 1 Answer C

Total angle at centre of the coin is 360º.
$\theta = 360 \div 12 = 30^\circ$

Question 2 Answer E

Shanghai (122º E) is further east than Chengdu (103º E), so sunset will occur in Shanghai BEFORE Chengdu. The difference in longitude is 122º – 103º = 19º.

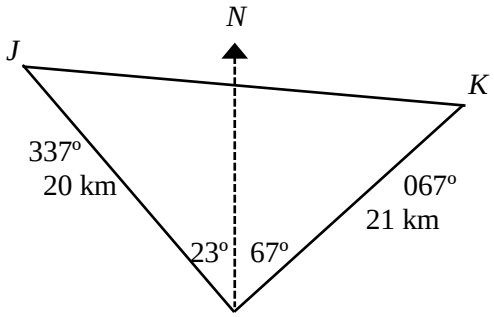
The actual time difference = $\frac{19^\circ}{15^\circ} \times 60$ = 76 minutes.
76 minutes after 5.18 pm will be 6.34 pm.

Question 3 Answer D

Apart from the Prime Meridian/International Date Line, all great circles of longitude will have one semicircle with a longitude west and the other semicircle with a longitude east. The sum of the longitude angles will always be 180º.
If Palmerston North is on 176º E, then the other semicircle of this great circle will be on 180º – 176º = 4º W.
(Madrid is actually directly on the other side of the Earth to Palmerston North!)

Question 4 Answer D

Angle between travellers = 90º
(360º – 337º = 23º
23º + 67º = 90º)
From Pythagoras,
distance between = $\sqrt{20^2 + 21^2}$ = 29.0 km



Question 5 Answer E

Use Cosine Rule to find the angle between the boat paths, with 13.2 km opposite the angle required.

Angle = $\cos^{-1}\left(\frac{14.8^2 + 16.3^2 - 13.2^2}{2 \times 14.8 \times 16.3}\right)$ = 49.944…º ≈ 50º.

Bearing boat B = 125º + 50º = 175º T.

Question 6 Answer C

Horizontal distance to nearest flagstick = $\frac{25}{\tan 25^o}$ = 53.6 m

Horizontal distance to furthest flagstick = 53.6 + 40.0 = 93.6 m

Angle of depression = $\tan^{-1}\left(\frac{25}{93.6}\right)$ = 14.95… ≈ 15.0°

Question 7 Answer D

Volume of two hemispheres = $\frac{4 \times \pi \times 1.5^3}{3}$ = 14.14 m^3.

50 000 L is the same as 50 m^3.

Volume cylinder part = 50 – 14.14 = 35.86 m^3

Length cylinder = $\frac{35.86}{\pi \times 1.5^2}$ = 5.073… ≈ 5.1 m

Question 8 Answer E

Diameter \$2 coin = 1.5 × diameter × \$1 coin = 1.5 × 20.0 = 30.0 mm

Thickness \$2 coin = 1.5 × thickness × \$1 coin = 1.5 × 2.0 = 3.0 mm

Volume \$1 coin = $\frac{1}{2} \times \frac{4 \times \pi \times R^3}{3}$ = 0.6283… cm^3

Volume \$2 coin = $\frac{1}{2} \times \frac{4 \times \pi \times R^3}{3}$ = 2.1205… cm^3

Ratio \$2 : \$1 :: 2.1205 : 0.6283 :: 3.375 : 1

With five \$2 coins the total weight is 5 × 3.375 = 16.875

Total weight = 16.875 + 4 = 20.875 × the weight of \$1 coin.

≈ 21 × the weight of \$1 coin.

Module 4 – Graphs and relations

Question 1 Answer B

Option B is correct because the axial intercepts for $2x - 3y = 12$ are $x = \frac{12}{2} = 6$ and $y = \frac{12}{-3} = -4$.

Question 2 Answer D

The average rate of change is given by the gradient between (5, 0.15) and (12, 0.55).

$m = \frac{0.55 - 0.15}{12 - 5} = 0.05714 \approx 0.06$ m/hr

Question 3 Answer C

Christie can keep 35 animals so $x + y = 35$ or $y = 35 - x$.
\$8 for cats and \$10 for dogs with a total of \$330 is written as $8x + 10y = 330$.

Question 4 Answer D

The profit is the Revenue – Cost, so revenue = profit + cost.
The cost for 80 coffees is \$350 from the graph.
The profit for 80 coffees is \$30, so the revenue is 350 + 30 = 380.
Each cup of coffee is $\frac{380}{80} =$ \$4.75.

Question 5 Answer A

The relationship shown is $y = kx^3$ as the axes are labelled y and x^3.

The value of k is the gradient of the line m = $\frac{12 - 0}{5 - 0} = 2.4$, so the relationship is $y = 2.4x^3$.

Question 6 Answer D

Using the relationship $P = km^2$ the point (5, 150) can be substituted into the equation to find k.

$150 = k \times 5^2$, giving $k = \frac{150}{25} = 6$

Therefore $P = 6m^2$.

So $P = 6 \times 9^2 =$ \$486

Question 7 Answer B

The statements given relate to equations as shown below:

Every shift there must be at least six staff on duty	$x + y \geq 6$
At least three trainers working per shift	$y \geq 3$
The number of trainers must be more than or equal to half the number of assistants.	$y \geq \frac{x}{2}$
Every trainer can work with four people at a time and every assistant can work with two people at a time. The gym guarantees that they will have staff to work with at least 20 people at a time.	$2x + 4y \geq 20$

There is an implied constraint that the number of assistants cannot be negative, so $x \geq 0$.

Question 8 Answer D

Checking the options,

A. if $m = n$, the gradient of the objective function would be -1 and equal to the line passing through A and B. TRUE

B. if $m = 2n$, the gradient of the objective function is -2, equal to the line through passing through C and D. TRUE

C. if $m > 2n$, the gradient of the objective function would be less than -2 and hence would pass through C last if slid upwards. TRUE

D. if $m < n$, the gradient of the objective function would between 0 and -1 and would pass through B last as the line is slid down. NOT TRUE

E. if $n < m < 2n$, the gradient of the objective function would be between -2 and -1 and would pass through D as the line is slid upwards. TRUE

END OF SOLUTIONS