

YEAR 12 Trial Exam Paper

2019

FURTHER MATHEMATICS

Written examination 2

Reading time: 15 minutes Writing time: 1 hour 30 minutes

STUDENT NAME:

QUESTION AND ANSWER BOOK

Structure of book

Section A – Core	Number of questions	Number of questions to be answered	Number of marks
	6	6	36
Section B – Modules	Number of modules	Number of modules to be answered	Number of marks
Ī	4	2	24
			Total 60

- Students must write in blue or black pen.
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 35 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions

- Write your **name** in the space provided above.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All responses must be written in English.

At the end of the examination

• You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination.

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SECTION A – Core

Instructions for Section A

Answer **all** questions in the spaces provided.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π , surds or fractions.

In 'Recursion and financial modelling', all answers should be rounded to the nearest cent unless otherwise instructed.

Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Data analysis

Question 1 (9 marks)

Table 1

Destination	Flights per month	Flights delayed >5 min	Minutes delayed
Sydney	186	49	85
Perth	124	31	55
Brisbane	155	41	65
Adelaide	65	17	27
Hobart	94	24	126
Cairns	65	15	285
Darwin	31	10	314
Townsville	54	14	384
Launceston	42	12	235
Uluru	80	21	155
Ballina (Byron)	110	28	43
Sunshine Coast	125	33	62

The data in Table 1 shows a range of scheduled flights, departing from Melbourne airport, run by a local airline, Airsetter Airlines.

The four variables in this data set are:

- *destination* name of city
- *flights per month* total number of flights
- *flights delayed* > 5 min total number of flights delayed more than 5 minutes
- *minutes delayed* average minutes delayed in October 2018.

A flight is classified as 'delayed' if the departure time is greater than 5 minutes after the scheduled time.

g.

3

a. How many variables are numerical, discrete variables?

b. What percentage of flights from Melbourne to Townsville were classified as delayed flights?

Round your answer to one decimal place.

Table 2

c. Complete the following five-number summary for the variable *minutes delayed*.

1 mark

Minimum	Q_1	Median	Q_3	Maximum
27	58.5		260	

d. Using the five-number summary, sketch the boxplot for the variable *minutes delayed* on the scale provided below.

2 marks

0	100	200	300	400
		minutes delayed		

e. Describe the shape of the distribution of the total minutes delayed for the airline.

1 mark

f. Complete the following sentence by writing the correct numerical values in the appropriate boxes provided below.

For the distribution of <i>minutes delayed</i> , the middle 50% of the data lies							
between	and		minutes.				
Write down a calculation to sho) ow that t	he value 27 is not ar	n outlier for this data set.	2 marks			

1 mark

1 mark

1 mark

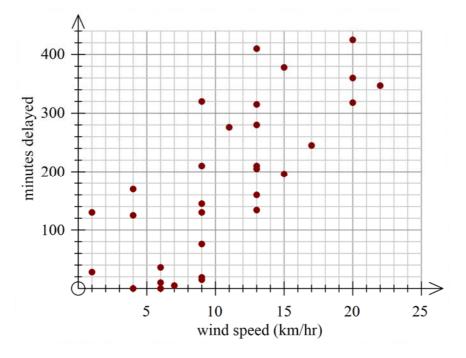
SECTION A – continued

TURN OVER

Question 2 (9 marks)

During October 2018, there was only one scheduled flight between Melbourne and Darwin per day.

The minutes delayed and the wind speed (km/h) at 9 am in Melbourne for each day in October are plotted on the scatterplot below.



A least squares line is to be fitted to the data with the aim of predicting the number of minutes delayed from the wind speed on a particular day in Melbourne.

The equation of this line is

minutes delayed =
$$-10.745 + 18.051 \times wind$$
 speed

a. Name the explanatory variable in this equation.

1 mark

b. Interpret the slope of the least squares regression line in terms of the variables *wind speed* and *minutes delayed*.

1 mark

c. Draw the line for the least squares regression equation on the scatterplot above.

(Answer on the scatterplot above.)

d. Using the equation of the least squares regression line, show that the number of minutes delayed is 242 minutes when wind speed is 14 km/h.

5

2 marks

e.	On a particular day in October, the wind speed was 15 km/h and the flight from Melbourne to Darwin was delayed by 196 minutes.	
	Determine the residual value when the equation of the least squares line is used to predict the minutes delayed when the wind speed is 15 km/h.	
	Round your answer to one decimal place.	
	5 1	2 marks
The	value of the correlation coefficient r is 0.747.	

f. Describe the strength of the relationship between *wind speed* and *minutes delayed*.

1 mark

g. What percentage of the variation in the number of minutes delayed for flights from Melbourne to Darwin can be explained by the variation in wind speed in Melbourne? Round your answer to the nearest whole number.

Question 3 (6 marks)

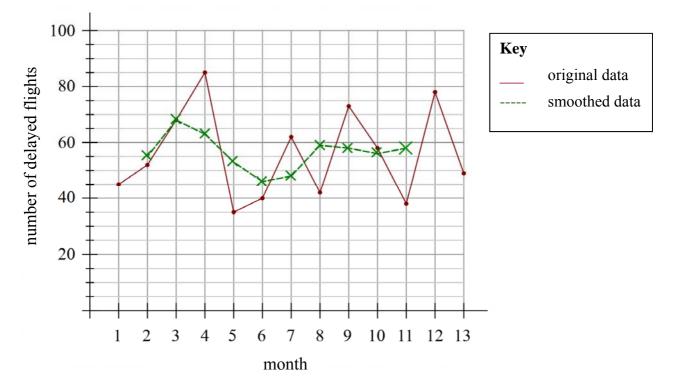
Airsetter Airlines wants to investigate the relationship between month and number of delays.

Table 3 shows the number of delays recorded for each month, from October 2017 to October 2018, for the flight from Melbourne to Sydney.

Table 3

	1	2	3	4	5	6	7	8	9	10	11	12	13
Month	Oct '17	Nov '17	Dec '17	Jan '18	Feb '18	Mar '18	Apr '18	May '18	June '18	July '18	Aug '18	Sept '18	Oct '18
Number of delays	45	52	68	85	35	40	62	42	73	58	38	78	49

This data is shown on the time series plot below, along with an incomplete graph for the data that has been smoothed using three-mean smoothing.



b.	A least squares line is used to model the trend in the time series plot for the original
	data.

7

The equation is

```
number of delays = 54.58 + 0.17 \times month
```

i. Use the equation of the least squares line to find the month and year in which it is predicted there will be 58 delays.

1 mark

1 mark

ii. The correlation coefficient, *r*, for this least squares regression line is 0.04062.Using the *r*-value, explain why the prediction in **part b.i.** would not be reliable.

Table 4 shows the three-mean smoothed values for this time series plot.

This table is incomplete.

	1	2	3	4	5	6	7	8	9	10	11	12	13
Month	Oct '17	Nov '17	Dec '17	Jan '18	Feb '18	Mar '18	Apr '18	May '18	June '18	July '18	Aug '18	Sept '18	Oct '18
Number of delays	45	52	68	85	35	40	62	42	73	58	38	78	49
Three- mean smoothing		55	68	63	53	46	48	59	58	56	58		

c. Calculate the three-mean smoothed value for September 2018.

1 mark

1 mark

1 mark



d. Plot your answer from **part c.** on the time series plot on page 6.

(Answer on the time series plot on page 6.)

e. When a least squares line is used to model the three-mean smoothed data, the *y*-intercept for this line is approximately 58.69.

Use the data in Table 4 to determine the equation of the least squares line that can be used to model the trend for the smoothed data for the number of delays for the flight between Melbourne and Sydney airports.

Write you answer in the appropriate boxes provided below.

Round the values for slope and intercept to two significant figures.



Recursion and financial modelling Question 4 (5 marks)

Alison invests her money into a savings account that will pay compound interest every month. The balance of Alison's account, after n months, V_n , can be modelled by the recurrence relation shown below.

 $V_0 = 8000, \quad V_{n+1} = 1.00225 V_n$

a. What is the annual interest rate for Alison's savings account?

1 mark

- **b.** Recursion can be used to calculate the balance of the account after each month.
 - i. Write down a calculation to show that the balance in the account after two months, V_2 , is \$8036.04.

1 mark

ii. After how many months will the balance of Alison's account first exceed \$8500?

c. A rule of the form $V_n = a \times b^n$ can be used to determine the balance of Alison's account after *n* months.

Write the rule for Alison's investment below.

1 mark

d. The value of Alison's investment after 12 months is \$8219 when rounded to the nearest dollar.

What is the equivalent flat rate interest rate, per annum, for Alison's investment? Round your answer to two decimal places.

Question 5 (3 marks)

After four years, Alison withdraws \$5000 from her account to purchase a new photocopier for her business.

11

The value of the photocopier will be depreciated using the reducing balance method.

The value of Alison's photocopier, in dollars, after n years, C_n , can be modelled by the recurrence relation shown below.

 $C_0 = 5000, \quad C_{n+1} = R \times C_n$

a. The annual rate of depreciation for this photocopier is 18%.What is the value of *R* in the recurrence relation?

1 mark

b. How much has the photocopier depreciated in value after five years of use?

1 mark

c. The value of the photocopier after one year of use is \$4100.

Alison's company prints approximately 35 000 pages per month on this photocopier.

Alison chooses to calculate the depreciation of her photocopier using unit cost depreciation.

By how much does the photocopier depreciate per page, using this method? Give your answer in cents to one decimal place.

Question 6 (4 marks)

Alison sells her business for the value of \$780 000. She will invest her money in an annuity. The annuity earns interest at a rate of 3.82% per annum, compounding monthly.

a. Initially, Allison does not add any of her own money to this investment.

What is the value of her investment at the end of the first two years?

1 mark

After three years, the balance of Alison's investment is \$874 552. Alison would like her investment to reach a value of \$950 000. She is able to contribute \$1500 per month.

b. How many months, in total, will it take for Alison's investment to first exceed \$950 000?

1 mark

c. Show that the interest earned in the fourth year of Alison's investment is \$34 318.

2 marks

SECTION B – Modules

Instructions for Section B

Select two modules and answer all questions within the selected modules.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π , surds or fractions.

Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Content

Contents	Page
Module 1 – Matrices	14
Module 2 – Networks and decision mathematics	19
Module 3 – Geometry and measurement	24
Module 4 – Graphs and relations	30

13

Module 1 – Matrices

Question 1 (3 marks)

A local produce store sells three types of tomatoes: Roma (R), cherry (C) and heirloom (H).

The cost, in dollars, to purchase one kilogram of each type of tomato is shown in matrix *A* below.

 $A = \begin{bmatrix} 7.98\\9.78\\4.39 \end{bmatrix} R$

a. What is the cost of one kilogram of cherry tomatoes?

b. Write down the order of matrix *A*.

c. Jack would like to purchase one kilogram of Roma tomatoes and two kilograms of heirloom tomatoes.

The total cost of Jack's purchases can be found by the matrix product $B \times A$. Write down the row matrix *B*.

1 mark

B =

1 mark

Question 2 (2 marks)

A local farmer grows all three types of tomatoes, and at the beginning of each season assigns a certain number of square metres to grow each.

The number of square metres assigned for each crop in 2018 is shown in matrix C_{2018} below.

$$C_{2018} = \begin{bmatrix} 48 \\ 25 \\ 45 \end{bmatrix} \begin{bmatrix} R \\ C \\ H \end{bmatrix}$$

Each year, the farmer can expect to lose approximately 10% of each crop due to drought, disease or other factors.

a. Find the expected number of remaining square metres at the start of 2019 for the cherry tomato crop.

Round your answer to the nearest whole number.

1 mark

b. The expected number of square metres remaining for each type of tomato at the start of 2019 can be determined from the matrix product

$$C_{2019} = D \times C_{2018}$$

where D is a diagonal matrix. Write down matrix D.

1 mark

D =

Question 3 (5 marks)

Once a year, Johanna uses an old family recipe to make homemade sauce using heirloom tomatoes.

In order to purchase enough tomatoes for her sauce, she must shop at four different produce suppliers: *A*, *B*, *C* and *D*.

In 2018 she purchased 3.5 kg from each of the four suppliers.

Let S_0 be the state matrix that shows the amount of tomatoes, in grams, purchased by Johanna, from each supplier in 2018.

$$S_{0} = \begin{bmatrix} 3500 \\ 3500 \\ 3500 \\ 3500 \\ 2500 \end{bmatrix} D$$

The quality of the tomatoes at each supplier varies from year to year, and as such Johanna does not always purchase an equal amount of tomatoes from each.

The transition matrix T below can be used to predict the weight, in grams, of heirloom tomatoes that Johanna will purchase from each supplier.

			С			
	0.80	0.05	0.07	0.11	A	next year
т_	0.05	0.75	0.03	0.06	B	nort your
1 =	0.10	0.12	0.86	0.05	C	nexi year
	0.05	0.08	0.04	0.78	D	

a. What percentage of Johanna's purchases from supplier *A* in 2018 will be lost to other suppliers in 2019?

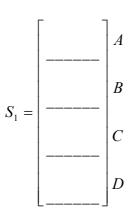
1 mark

b. How many grams of tomatoes will be purchased from supplier *C* in 2019?

The state matrix describing the number of tomatoes, in grams, purchased from each supplier in the *n*th year is given by

$$S_{n+1} = T \times S_n$$

c. Complete the state matrix, S_1 , below for the number of tomatoes, in grams, purchased from each supplier in 2019.



d. How many tomatoes, in grams, is Johanna expected to purchase from supplier *B* in 2021?

Round your answer to the nearest whole number.

1 mark

e. In the long term, what percentage of Johanna's total tomato purchases will come from supplier *D*?

Round your answer to one decimal place.

Question 4 (2 marks)

In 2022, supplier *D* informs Johanna that they can only supply her with 2 kilograms of tomatoes due to issues with their own supply.

Johanna still requires 14 kilograms of tomatoes in total.

Her purchases in 2022 are given by R_0 .

Johanna's purchases in the following years can be calculated using the matrix equation below.

$$R_{n+1} = T \times R_n + G$$

where

$$R_{0} = \begin{bmatrix} 4000 \\ 4000 \\ 4000 \\ 2000 \end{bmatrix} A \qquad A \qquad B \qquad C \qquad D \\ T = \begin{bmatrix} 0.80 & 0.05 & 0.07 & 0.11 \\ 0.05 & 0.75 & 0.03 & 0.06 \\ 0.10 & 0.12 & 0.86 & 0.05 \\ 0.05 & 0.08 & 0.04 & 0.78 \end{bmatrix} A \qquad G = \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix}$$

1.

a. How many grams of tomatoes is Johanna expected to purchase from supplier *B* in 2022?

1 mark

b. Johanna has decided that her purchases will remain consistent moving into the future. Complete the matrix *G*, below.

Module 2 – Networks and decision mathematics

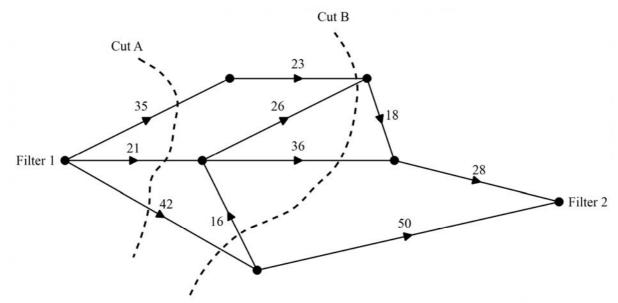
Question 1 (2 marks)

The graph below shows the possible paths that orange juice can be sent through a system of tubes between the first and second major filters at a local factory.

19

The unmarked vertices represent other, smaller filters that the juice also passes through along the way.

The weighting of each edge represents the maximum number of litres that can pass through each tube per hour.



Cut A, shown on the graph above, has a capacity of 98.

a. Write down the capacity of Cut B.

1 mark

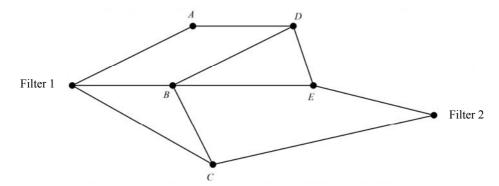
b. Determine the maximum number of litres of orange juice that can be sent from Filter 1 to Filter 2 per hour.

Question 2 (3 marks)

At the end of each day, the juice production line is cleaned thoroughly.

The factory has purchased a robot that will clean the tubes automatically.

The network of tubes is displayed below with each filter labelled.



a. How many of the vertices on the graph have degree 3?

1 mark

The robot would need to pass through each pipe exactly once as it moves through the system. It starts at Filter 1 and can finish at any of the other filters.

It is currently not possible for the robot to check each pipe without retracing its steps.

b. An additional tube can be added to the system to allow the robot to pass through each pipe exactly once.

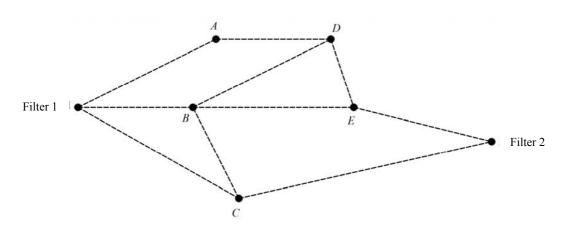
Between which two vertices must this tube be added?

1 mark

1 mark

c. A separate robot will work its way through the system, checking the function of each filter.

On the diagram below, draw a possible Hamiltonian path for the second robot to follow.

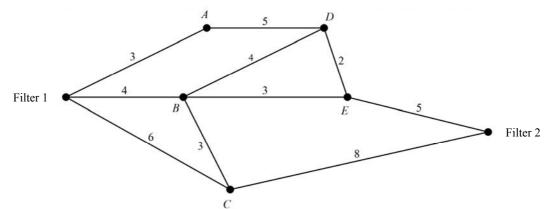


Question 3 (2 marks)

It is sometimes necessary to send the filter repair robot directly to a particular filter.

The network below shows the time, in seconds, that it will take the robot to travel between filters.

21



a. Write down the shortest path that the robot can take between Filter 1 and Filter 2.

1 mark

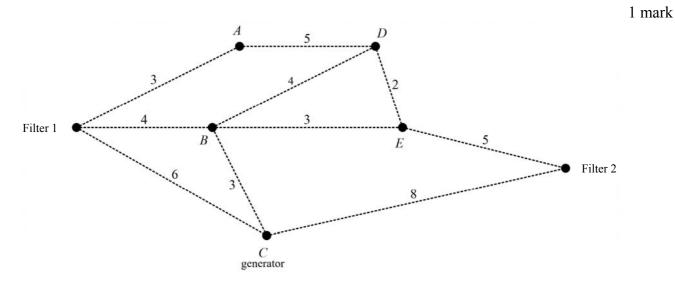
A backup generator has been installed at the factory to allow filters to continue to run in the case of a power outage.

The generator is installed at the location of filter C.

Wiring must be placed through the tubing, so that the backup power reaches each filter.

The weights on the edges indicate the distance, in metres, between each filter.

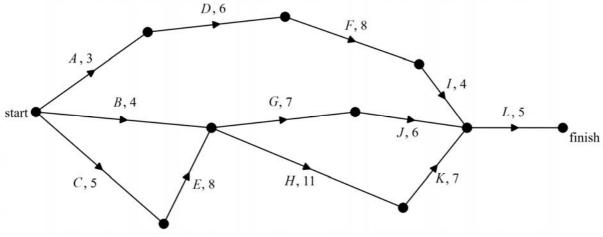
b. Sketch the minimum spanning tree, representing the wiring from the backup generator on the diagram below.



Question 4 (5 marks)

The factory will install a second system in order to increase the production of orange juice. This project involves 12 activities, A to L.

The directed network below shows these activities and their completion time, in days.



a. Determine the earliest starting time, in days, for activity *J*.

b. The minimum completion time for this project is 36 days.Write down the critical path.

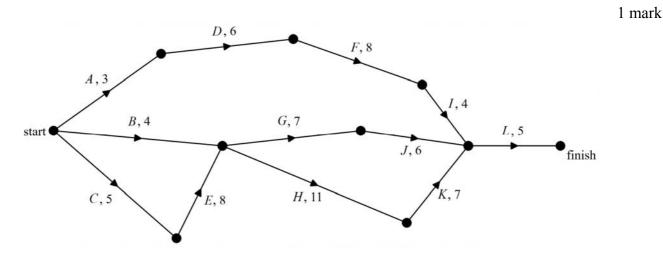
1 mark

1 mark

c. How many activities in this network have a float time of 10 days?

d. An additional activity, *X*, will be added with duration of three days, an Earliest Start Time (EST) of 13 and a Latest Start Time (LST) of 24.

Draw this additional activity on the directed graph below.



e. The completion time of the project can be decreased by reducing the completion time of activities *E* and *K* by 1 hour each.

The additional activity, X, will not be included in this new project.

What is the new completion time?

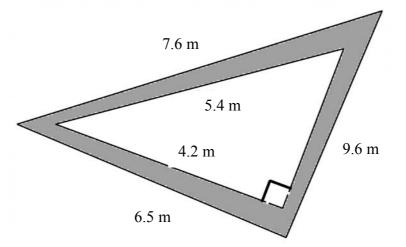
Module 3 – Geometry and measurement

Question 1 (4 marks)

A company invests money into creating an outdoor leisure area for staff.

This area will be triangular in shape, and will have a grass area, surrounded by a garden around the perimeter.

The garden is shaded below. Dimensions are not shown to scale.



a. Show that the area of the entire leisure area (grass and garden included) is 24.62 m^2 .

2 marks

b.	Find the area of the garden. Round your answer to the nearest whole number.			

c. A paved area will be placed in the middle of the grass, which will be in the shape of a circle with a diameter of 2.3 metres.

If paving stones cost \$35 per square metre, how much will it cost to pave this area?

1 mark

Question 2 (3 marks)

The landscaper who designed the staff leisure area, Nathan, has travelled to Japan for work; however, he assured the company that he will be available for communication if any concerns arise.

Nathan travelled from Melbourne (38° S, 145° E) at 9.10 am on Tuesday, 8 January 2019. He arrived in Tokyo, Japan (36° S, 140° E) at 6.35 pm, on the same day.

The time difference between Melbourne and Tokyo is two hours.

a. Nathan calls the manager of the company to check on progress two hours after he lands in Tokyo.

What time will it be in Melbourne when he calls?

1 mark

Tokyo is located at latitude 36° N and longitude 140° E. Assume that the radius of the Earth is 6400 km.

b. What is the radius of the small circle of the Earth at latitude 36°? Round your answer to the nearest kilometre.

1 mark

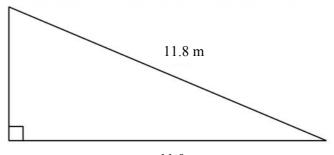
c. Athens in Greece is located at latitude 36° N and longitude 24° E.
Find the shortest small circle distance between Tokyo and Athens.
Round your answer to the nearest kilometre.

Question 3 (5 marks)

Alongside the staff leisure area, a children's play area will be built.

The main feature in the play area will be a giant slide.

The dimensions of one possible design for the giant slide are shown below.





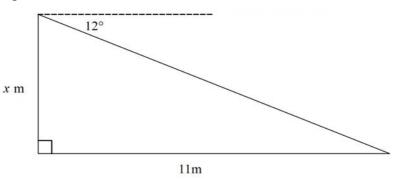
a. If a child is standing at the top of the slide, what is the angle of depression if they were to look at their parent standing at the bottom of the slide?Round your answer to the nearest degree.

1 mark

The leisure centre is concerned that the slide may be too steep for children, and they are looking into various designs that could change the length of the slide.

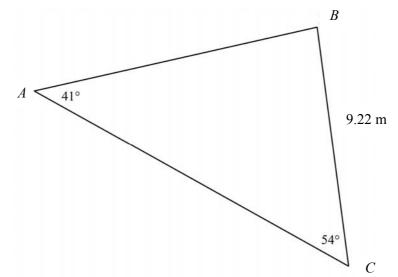
It is suggested that the angle of depression should be reduced to 12 degrees, as shown below.

The horizontal length of the slide will remain 11 metres.



b. Find the height of the slide if the angle of depression is reduced to 12 degrees. Round your answer to two decimal places.

The play area will be constructed in the shape of a triangle with bench seats at *A*, *B* and *C* for parents to supervise at each vertex. The play area is shown below.



- **c.** Find the remaining angle, $\angle ABC$.
- **d.** Find the length of the side between bench seats *A* and *B* in metres. Round your answer to two decimal places.

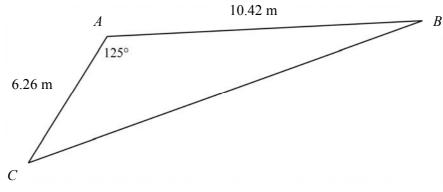
1 mark

1 mark

28



An alternative set up for the children's play area is shown below.



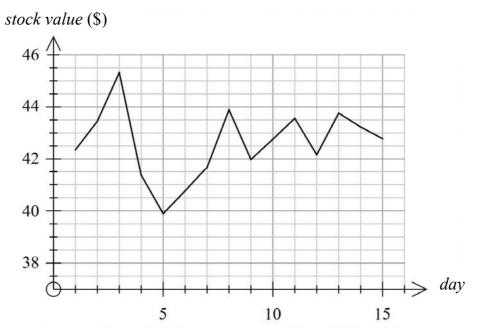
e. Find the distance between *B* and *C* in metres. Round your answer to the nearest whole number.

Module 4 – Graphs and relations

Question 1 (2 marks)

The following chart displays the change in daily stock value for a sauce company over a 15–day period.

Stock prices are recorded at the end of the business day.



a. On which day during the 15–day period had stocks in the sauce company reached their highest value?

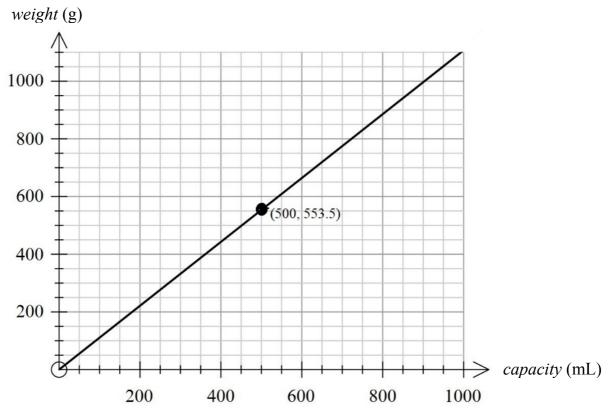
1 mark

b. Between which two consecutive days does the smallest increase in stock prices occur?

Question 2 (4 marks)

For shipping purposes, the company needs to calculate the weight of the sauce.

The following graph shows the relationship between the weight in grams and capacity in millilitres.



The relationship between the weight measured in grams and the capacity in millilitres is shown in the equation

weight of sauce $(g) = M \times capacity (mL)$

a. Find the value of *M* in the equation above.

b. What is the weight, in grams, of sauce with a capacity of 150 millilitres?

1 mark

The cost of shipping the bottle	s of sauce can be calculated	l using the following equation.	

```
cost = 0.0015 \times weight(g) - 10
```

c. Show that the cost of shipping 18 bottles of sauce, each weighing 450 g, is \$2 when rounded to the nearest dollar.

2 marks

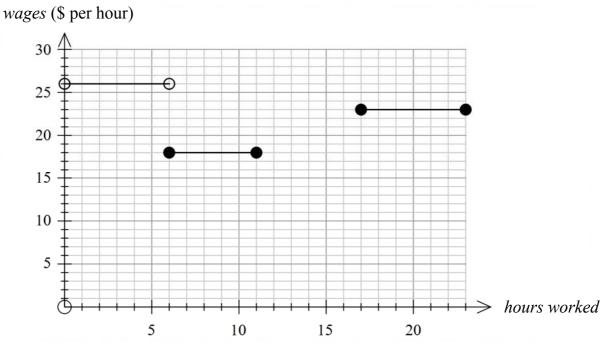
Question 3 (3 marks)

The wages to be paid to workers at the sauce factory are based on the shift that they work, as the factory runs 24 hours a day (to include cleaning and maintenance on machines).

Below is the relation that describes the wages, in dollars, for the hours worked (in 24-hour time).

 $wages (\$) = \begin{cases} \$26 \ 24 < hours \ worked < 6 \\ \$18 \ 6 \le hours \ worked \le 11 \\ \$16 \ 11 < hours \ worked < 17 \\ \$23 \ 17 \le hours \ worked \le 24 \end{cases}$

The step graph below representing this relation is incomplete.



a. Complete the **step graph above** by sketching the missing information using the relation provided.

(Answer on the step graph above.)	2 marks
Jason works a 5-hour shift, from 15.00 until 20.00.	
How much will he be paid for his shift?	1 mark

b.

Although the actual taste of the sauce produced may vary slightly from batch to batch, the sauce factory sets standards around the quantity for each of the main ingredients, within a 450 mL bottle.

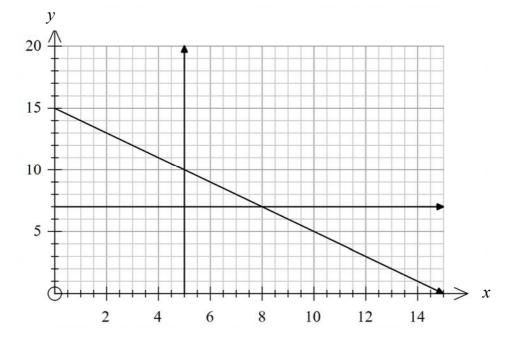
34

- Let *x* be the amount (volume) of salt added to the mixture, in grams.
- Let *y* be the volume of sugar added to the mixture, in grams.
- The maximum volume of salt and sugar in each jar of sauce is 15 grams.
- The maximum amount of salt that can go into the mixture is 5 grams, while the minimum is 2 grams.
- A minimum amount of 7 grams of sugar is required.

These constraints can be represented by the following.

Constraint 1	$x \leq 5$
Constraint 2	$x \ge 2$
Constraint 3	$y \ge 7$
Constraint 4	$x + y \le 15$

The graph below shows the lines representing Constraints 1, 3 and 4.



a. On the **graph above**, sketch the line for Constraint 2.

(Answer on the graph above.)

b. On the **graph above**, shade the region that satisfies Constraints 1 to 4.

(Answer on the graph above.)

1 mark

The cost for one gram of salt is five cents. The cost for one gram of sugar is one cent.

c. What is the minimum cost of the salt and sugar contents in a bottle of sauce?

1 mark

END OF QUESTION AND ANSWER BOOK