THE SCHOOL FOR EXCELLENCE (TSFX)
UNIT 3 & 4 FURTHER MATHEMATICS 2020
WRITTEN EXAMINATION 2 – SOLUTIONS

Errors and updates relating to this examination paper will be posted at
www.tsfx.edu.au/examupdates

SECTION A – CORE

DATA ANALYSIS

QUESTION 1

a. Positively skewed with an outlier. 1M

The data is clearly skewed as the median is closer to the minimum than to the maximum and there is an outlier shown at $120 000

b. 50 1M

From the boxplot it can be seen that Q_3 is at $50 000. Therefore 25% of the families have an *Annual Gross Income* of more than $50 000, $\frac{25}{100} \times 200 = 50$.

c. 120 000 > 83 000, so it is an outlier.

The interquartile range of *Annual Gross Income* is 50 000 – 28 000 = 22 000.

The upper fence is 50 000 + 1.5 × 22 000 = 83 000. 1M

120 000 is more than 83 000, so it is an outlier. 1M

d. **i.** $\$180 - < \$200$ 1M

The modal class interval is the most common group, represented by the peak of the histogram. This is the peak from $180 to $200, so $\$180 - \le \$200$.

ii. $\$160 - < \$180$ 1M

The median class interval is the group where the middle or 50^{th} percentile lies.

This can be found using a cumulative frequency as shown below:

Weekly grocery Bill	Frequency	Cumulative Frequency
$\$120-<\$140$	7	7
$\$140-<\$160$	23	30
$\$160-<\$180$	26	56
$\$180-<\$200$	40	96
$\$200-<\$220$	2	98
$\$220-<\$240$	1	99
$\$240-<\$260$	1	100

It can be seen that the 50th percentile is in the class interval $\$160-<\$180$.

QUESTION 2

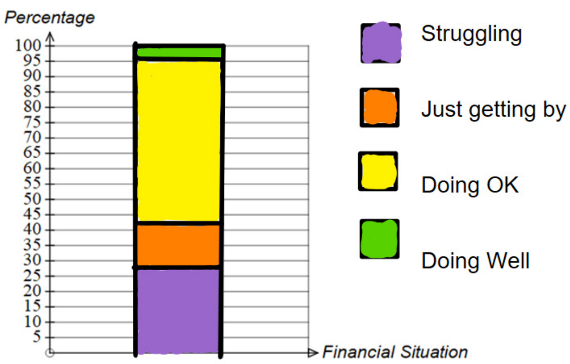
a. 2694 1M

The number of people who were "doing OK" is 5000 – (1405+706+195) = 2694.

b. 28% 1M

There were 1405 people who were struggling, so the percentage is $\frac{1405}{5000}\times 100 \approx 28\%$.

c.



The following percentages should be displayed:

$\frac{1405}{5000}\times 100 \approx 28\%$ for Struggling

$\frac{706}{5000}\times 100 \approx 14\%$ for Just getting by

$\frac{2694}{5000}\times 100 \approx 54\%$ for Doing OK

$\frac{195}{5000}\times 100 \approx 4\%$ for Doing well

Calculated percentages correct. 1M
Percentaged segmented bar chart as shown (along with appropriate key). 1M

QUESTION 3

a. Ordinal 1M

There is a built in order where below 50% is less than below 60%, so it is ordinal data.

b. Age group 1M

The level of poverty can be affected by the age group so the age group is the explanatory variable. This is confirmed by the position in the table.

c. 17.7% of children under 15 years are in the below 50% level of poverty, but a smaller percentage of 13.9% of young people 15 to 24 years are in this group.

OR

25.7% of children under 15 years are in the below 60% level of poverty, but a smaller percentage of 19.3% of young people 15 to 24 years are in this group.

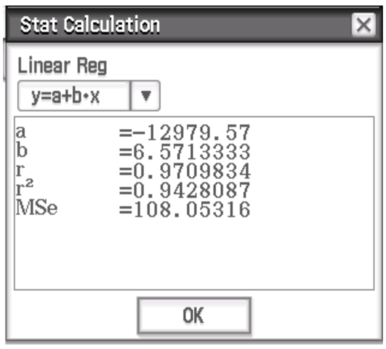
Identifying a difference in percentages across a row. 1M
Correctly stating two percentages. 1M

QUESTION 4

a. Strong, positive, linear association. 1M

b. -12 980 and 6.571 (in that order)

The question must be approached using the CAS calculator:



The values of a and b must be rounded to four significant figures, so they are -12 980 and 6.571.

Both correctly rounded in correct order. 2M

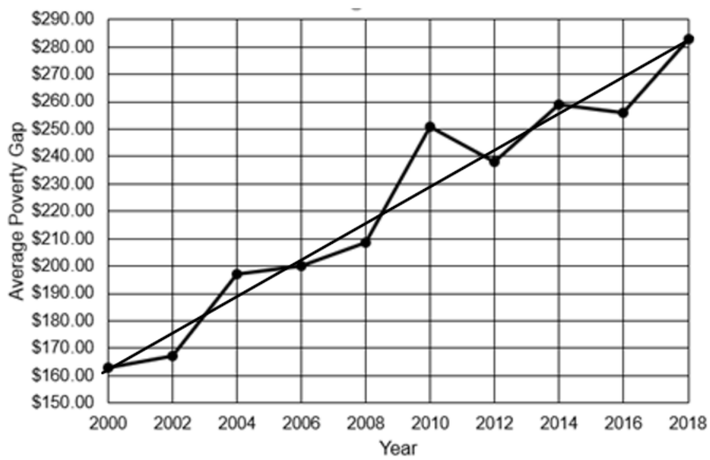
OR

Both correctly rounded in reverse order OR one value correctly rounded in correct place.

OR

Both values correct but incorrect rounding. 1M

c. Line passing though (2000, 162) and (2018, 280.3) as shown below: 1M



The points can be calculated as shown below:

For *year* = 2000: $average\ poverty\ gap = -12980 + 6.571 \times 2000 = 162$
For *year* = 2018: $average\ poverty\ gap = -12980 + 6.571 \times 2018 \approx 280.3$

d. The average poverty gap increases by $6.571 for every extra one year of time 1M

e. 94.3% of the variation in the average poverty gap can be explained by the variation in year. 1M

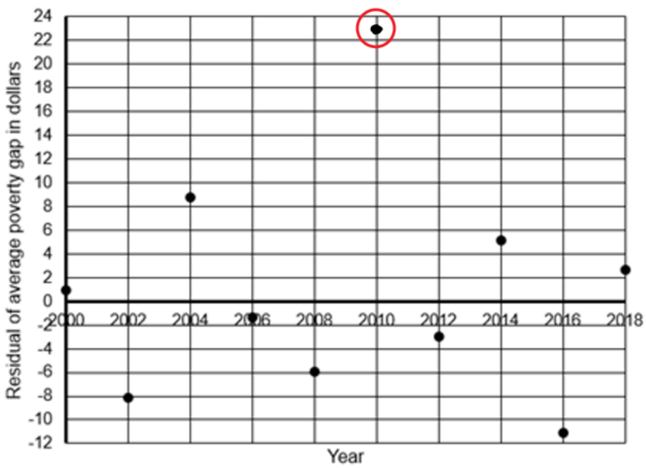
f. **i.** ($)23 1M

The actual value of the *average poverty gap* in 2000 was $250.69 from the table.

The predicted *average poverty gap* in 2010 is $-12980 + 6.571 \times 2010 = 227.71$.

The residual of *average poverty gap* is 250.69 – 227.71 = 22.98 ≈ $23.

ii. The point (2010, 23) as shown circled below: 1M



g. The linear model is suitable as the residual plot has no pattern (is randomly scattered). 1M

h. 2028 1M

The equation below must be solved:

$$350 = -12980 + 6.571 \times year$$

$$year = \frac{350 + 12980}{6.571} = 2028.61...$$

Therefore the *average poverty gap* of $350 is reached during the *year* 2028.

i. It is an example of extrapolation. 1M

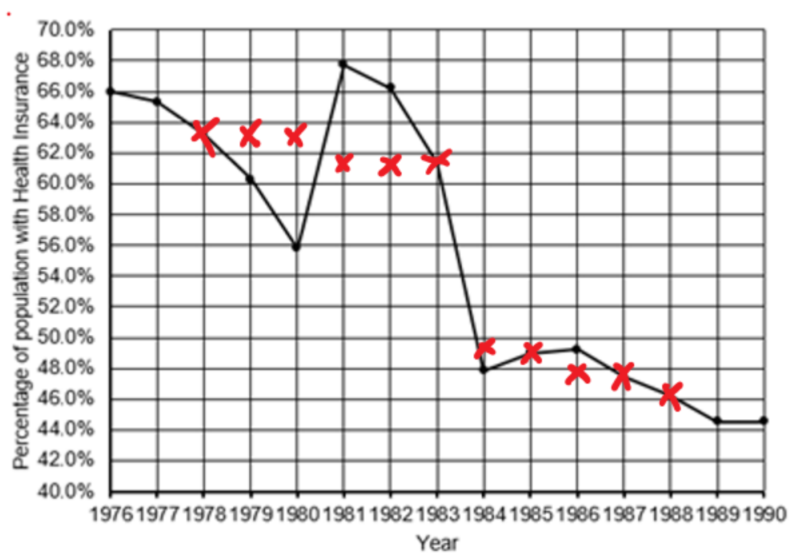
Predicting the value in 2028 is an example of extrapolation beyond the avaiable data and as there is no evidence that the relationship continues past 2018, the prediction is unreliable.

QUESTION 5

a. The trend is decreasing from 1976 through to 1998/1999 after which the trend is stable. 1M

From 1976 through to 1998/1999 the percentage of population with health insurance clearly decreases with only one increase from 1980 to 1981. As trend considers the big picture, there is an overall decrease. In the year 2000 the percentage increases dramatically, but from then until 2019 the trend neither increases or decreases.

b. Points shown as below with no additional points:



All points correctly placed 2M
At least six points correctly placed 1M

c. Calculations as shown below:

$$\frac{63.3+60.3+55.8+67.8+66.2+61.4}{6}=62.4\dot{6}$$

$$\frac{60.3+55.8+67.8+66.2+61.4+47.8}{6}=59.88\dot{3}$$ Both calculations 1M

$$\frac{62.4\dot{6}+59.88\dot{3}}{2}=61.175\approx 61.2\%$$ 1M

RECURSION AND FINANCIAL MODELLING

QUESTION 6

a. \$470 000 1M

In the recurrence relation $S_0 = 470\,000,\ S_{n+1} = 0.99 \times S_n$ the intital value, S_0, is \$470 000.

b. 1% 1M

The recurrence relation has a multiple of 0.99. This means that the balance is multiplying by $1-\frac{1}{100}=0.99$ so it represents 1% reduction per day.

c. The two calculations as shown: $S_1 = 0.99 \times 470000 = \$465\,300$, $S_2 = 0.99 \times 465300 = \$460\,647$ 1M

Both calculations must be fully shown as above, including values of both S_1 and S_2.

d. 26% 1M

After 30 days the mutiple that would be applied to Tracey's account is $0.99^{30} = 0.739700...$

This means the account will reduce to 73.97% of the original amount, a reduction of 100 – 73.97 = 26.03 ≈ 26% .

QUESTION 7

a. 0.35% 1M

The interest rate is 4.2% per annum, which is equivalent to $\frac{4.2}{12} = 0.35\%$ per month.

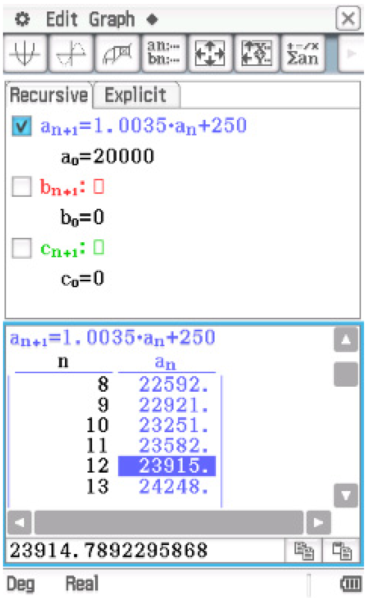
b. $T_0 = 20000,\ T_{n+1} = 1.0035 \times T_n + 250$ 1M

The recurrence relation must reflect a multiple of $1 + \frac{r}{100} = 1 + \frac{0.35}{100} = 1.0035$ as well as a monthly deposit of $250.

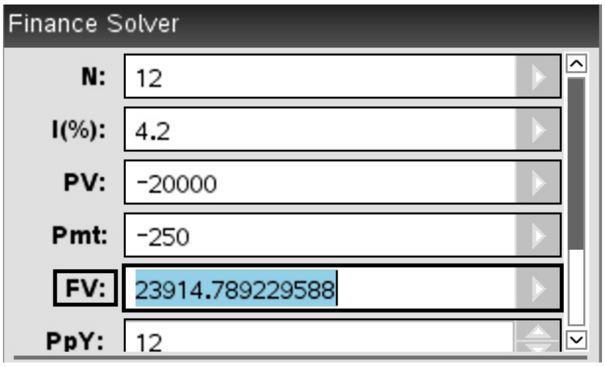
c. $23 914.79 1M

After 12 months the balance of the account is $23 914.79. This can be determined using sequencing on the ClassPad or using Finance on the TI-Nspire:

ClassPad



TI-Nspire



d. $914.79 1M

After 12 months the balance of the account is $23 914.79 as shown in part c.

The interest is the amount in the account more than the amount deposited so it is:

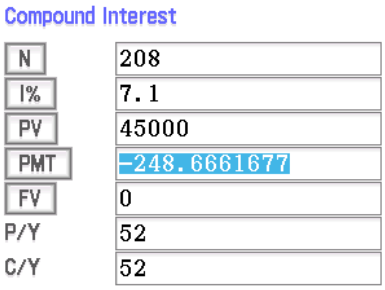
$23914.79 - (20000 + 12 \times 250) = \$914.79$

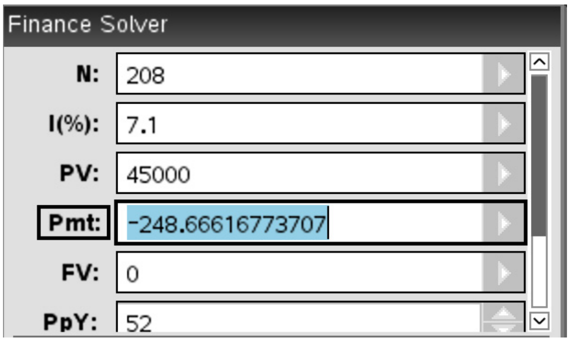
This answer could be consequential using $ans - (20000 + 12 \times 250)$.

QUESTION 8

a. $248.67 1M

This should be approached using the CAS function:



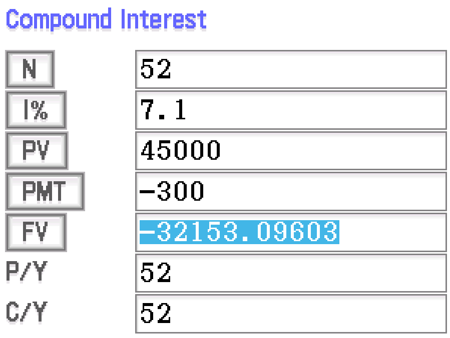


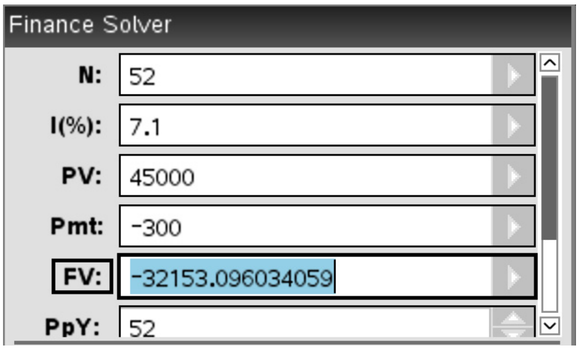
b. $6723 1M

The total paid is $248.67 \times 208 = \$51\,723.36$
The interest is. $51\,723.36 - 45\,000 = \$6723.36 \approx \$6723$.

c. 138 weeks 2M

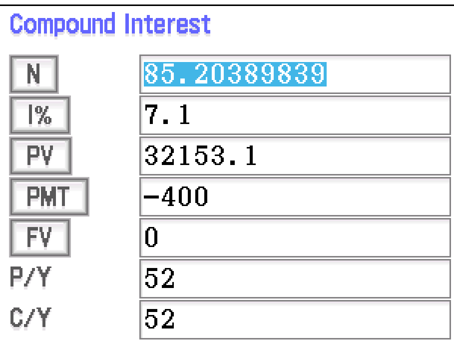
There are two stages to this loan. The balance after the first 52 weeks is calculated below:

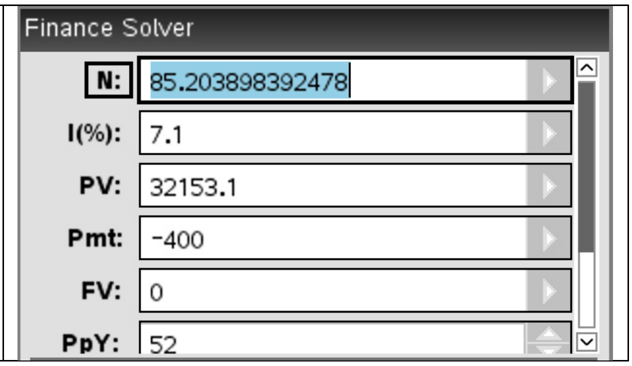




Determining the balance after 12 months of $32 153.10 1M

The time taken when the payment is increased to $400 must be calculated using a new starting balance of $32 153.10:



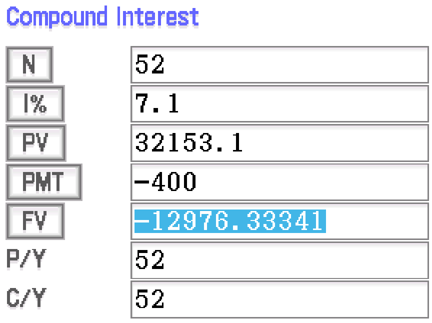


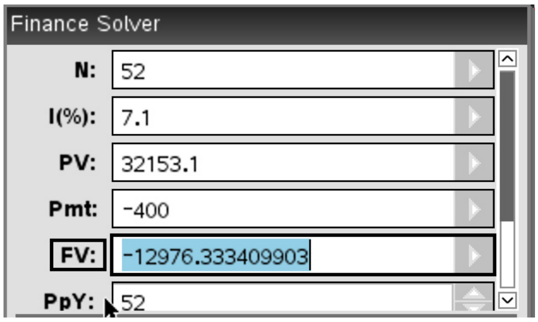
The second part of the loan will take 86 weeks as there will be 85 weeks at $400, followed by another samller payment.

The total time will be 52 + 86 = 138 weeks. 1M

d. $14 401.67 2M

The first step is to calculate how much she owes after two years. Following on from the balance of $32 153.10 at the end of the first year, it can be seen that she would owe $12 976.33 after two years: 1M





The value of the car after two years with 22% per annum reducing balance depreciation can be determined as $45000 \times 0.78^2 = \$27378$.

The amount that Tracey will receive is 27 378 – 12 976.33 = $14401.67.

e. (52 × 300) + (52 × 400) – 14 401.67 = $21 998.33 1M

Over the two year period, Tracey paid (52 × 300) + (52 × 400) = $36 400.

She received an insurance payout of $14 401.67, therefore the cost to her over the two year period was (52 × 300) + (52 × 400) – 14 401.67 = $21 998.33.

MODULE 1: MATRICES

QUESTION 1

a. 40 1M

From the matrix *J*, it can be seen that 28 + 12 = 40 tour groups went to Victoria.

b. $x = 1.25$ 1M

To increase by 25% the multiple is $1+\frac{25}{100}=1.25$.

c. $A=\begin{bmatrix}1 & 1\end{bmatrix}$, $B=\begin{bmatrix}1\\1\end{bmatrix}$ 1M

Matrix A will sum the columns and matrix B will sum the rows so the entire matrix J is summed to one value.

QUESTION 2

a. $\begin{bmatrix}120 & 85\\94 & 72\end{bmatrix}$ 1M

The two equations to be solved are $120T+85M=40950$ and $94T+72M=33540$.

In matrix format this will be $\begin{bmatrix}120 & 85\\94 & 72\end{bmatrix}\begin{bmatrix}T\\M\end{bmatrix}=\begin{bmatrix}40950\\33540\end{bmatrix}$.

b. $\frac{1}{650}$ 1M

The value of a will be $\frac{1}{\text{determinant}}$. The determinant is given by $120\times 72-85\times 94$ so

$$a=\frac{1}{120\times 72-85\times 94}=\frac{1}{650}.$$

c. \$24 000 1M

The equation can be solved using $\begin{bmatrix}T\\M\end{bmatrix}=\frac{1}{650}\begin{bmatrix}72 & -85\\-94 & 120\end{bmatrix}\begin{bmatrix}40950\\33540\end{bmatrix}=\begin{bmatrix}150\\270\end{bmatrix}$.

Therefore T = \$150 and M = \$270.

A profit of \$150 was made per person in a tent and a profit of \$270 was made per person in a motel so, therefore the profit in May will be $70\times 150+50\times 270=\$24\,000$.

QUESTION 3

a. Charis. 1M

Dino can communicate with Etienne, Charis and Farina. The only one of these who is a team leader is Charis.

b. Communication is two way. If A can speak to B then B can speak to A. 1M

c.

$$M + M^2 = \begin{array}{c} \textit{from} \\ \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{bmatrix} 2 & 2 & 2 & 1 & 1 & 1 \\ 2 & 3 & 2 & 2 & 2 & 1 \\ 2 & 2 & 4 & 2 & 2 & 3 \\ 1 & 2 & 2 & 3 & 3 & 2 \\ 1 & 2 & 2 & 3 & 3 & 2 \\ 1 & 1 & 3 & 2 & 2 & 3 \end{bmatrix} \end{array} \begin{array}{c} A \\ B \\ C \\ D \\ E \\ F \end{array} \quad to$$

Aiden is correct. $M + M^2$ has no zero entries.(1M) therefore everyone in the company can communicate with everyone else in the company using either one or two step communications (1M).

QUESTION 4

a. $G_{2019} = \begin{bmatrix} 31000 \\ 17\,000 \\ 14\,000 \\ 18\,000 \end{bmatrix} \begin{matrix} V \\ N \\ Q \\ R \end{matrix}$ 1M

$$G_{2019} = \begin{bmatrix} 0.5 & 0.2 & 0 & 0 \\ 0.1 & 0.4 & 0 & 0 \\ 0.1 & 0.3 & 1 & 0 \\ 0.3 & 0.1 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 50000 \\ 30000 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 31000 \\ 17\,000 \\ 14\,000 \\ 18\,000 \end{bmatrix}$$

b. $0.1 \times 31000 + 0.4 \times 17000 + 0.2 \times 14000 + 0.1 \times 18000 = 14\,500$ 1M

The calculation required will produce the element in row 2 and column 1 of G_{2020}.
Therefore, it will be row 2 of T multiplied by column 1 of G_{2019}.

c. 2020 1M

The following matrices are under consideration. The values for Queensland are in red:

$$G_{2019} = \begin{bmatrix} 31000 \\ 17000 \\ 14000 \\ 18000 \end{bmatrix}, G_{2020} = \begin{bmatrix} 23500 \\ 14500 \\ 15600 \\ 26400 \end{bmatrix}, G_{2021} = \begin{bmatrix} 20410 \\ 13910 \\ 15580 \\ 30100 \end{bmatrix}, G_{2022} = \begin{bmatrix} 19113 \\ 13731 \\ 15456 \\ 31700 \end{bmatrix}$$

It can be seen that the maximum value for Queensland is in 2020.

d. a = 1297, b = 179, c = 124, d = −1600 2M

1M for all values with reverse signs or 1M for any two values correct.

As shown in part c. $G_{2021} = \begin{bmatrix} 20410 \\ 13910 \\ 15580 \\ 30100 \end{bmatrix}$ and $\begin{bmatrix} 0.5 & 0.2 & 0.2 & 0.1 \\ 0.1 & 0.4 & 0.2 & 0.1 \\ 0.1 & 0.3 & 0.4 & 0.1 \\ 0.3 & 0.1 & 0.2 & 0.7 \end{bmatrix} \times \begin{bmatrix} 20410 \\ 13910 \\ 15580 \\ 30100 \end{bmatrix} = \begin{bmatrix} 19113 \\ 13731 \\ 15456 \\ 31700 \end{bmatrix}$.

Therefore the change that occurs (and must be reversed by the matrix $\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$)

is $\begin{bmatrix} 20410 \\ 13910 \\ 15580 \\ 30100 \end{bmatrix} - \begin{bmatrix} 19113 \\ 13731 \\ 15456 \\ 31700 \end{bmatrix} = \begin{bmatrix} 1297 \\ 179 \\ 124 \\ -1600 \end{bmatrix}$.

MODULE 2: NETWORKS AND DECISION MATHEMATICS

QUESTION 1

a. 1M

Staff Member	Activity
Peta	Ballooning
Quentin	Kayaking
Renato	Orienteering
Suki	Stand-up paddleboarding

Only Quentin can do kayaking and Suki can only do Stand-up paddleboarding, so Peta must do ballooning and Renato orienteering.

b. Suki should train in Kayaking 1M

Suki could only do one activity and only Quentin can do kayaking. So training Suki to do kayaking means that there would be the alternative allocation below:

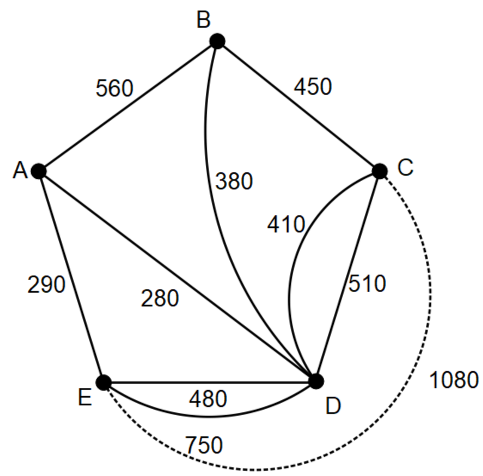
Staff Member	Activity
Peta	Stand-up paddleboarding
Quentin	Orienteering
Renato	Ballooning
Suki	Kayaking

c. **i.** Hamiltonian cycle 1M

The route is a Hamiltonian cycle as the staff member must visit each vertex exactly once, starting and finishing at the same vertex.

c. **ii.** ABCDEA or ADBCEA or ABDCEA or reverse of any already listed. 1M

d. Edge shown dotted in network below and length of 1080 1M



There is a direct path from E to C consisting of two sections 660 + 420 = 1080 m.

e. The staff member must complete an Eulerian circuit as they must use every edge exactly once, starting and finishing at A. 1M

This cannot be done as the degrees of the vertices at A (3) and B (3) are both odd. 1M
To have an Eulerian circuit the degrees of all vertices in the network must be even.

f. AB 1M

The degree of A is three and the degree of B is also three. If the staff member starts at A and takes an Eulerian trail, they will end up at B, as A and B are the only two vertices with odd degrees. Therefore they will need to repeat AB to get back to A.

QUESTION 2

A table of earliest and latest start times and float times for each activity is listed below:

Activity	EST	LST	Float
A	0	0	0
B	30	30	0
C	30	40	10
D	50	50	0
E	50	140	90
F	90	90	0
G	210	210	0
H	210	225	15
I	260	260	0

a. 330 minutes 1M

I must be the last activity and it has an earliest start time of 260 and a duration of 70, so the earliest completion time is 260 + 70 = 330 minutes.

b. ABDFGI 1M

The activities on the critical path have zero float time. From the table the critical path would be ABDFGI.

c. 40 minutes 1M

The latest start time for activity F after C is 90 minutes so the latest start time for C is 90 – 50 = 40 minutes.

d. Activity E 1M

Activity E has the largest float time of 90 minutes.

e. 10 minutes 1M

Reducing B or D by 10 minutes would match the length of time for C making a second critical path. This can also be seen by C's float time of 10 minutes.

f. $229 2M

The addition of activities J and K does not change the critical path of the network.
If all activities were reduced by the maximum amount, using a forward scan it can be seen that the new minimum time would be reduced to 288 minutes.

New time 288 minutes 1M

Taking each section of the network:

From start to beginning of F, the longest path after a crash has a length of 68 minutes. To match this length;

- A should be reduced by 12 at a cost of $60
- D should be reduced by 10 at a cost of $40
- K should be reduced by 12 at a cost of $24

From the end of F to the start of I, the longest path after the crash is 30. To match this length;

- G should be reduced by 20 at a cost of $80
- H should be reduced by 5 at a cost of $25

The total cost is therefore $229. 1M

MODULE 3: GEOMETRY AND MEASUREMENT

QUESTION 1

a. $Area = \frac{1}{2}(250 + 550) \times 400 = 160\,000\,\text{m}^2$ 1M

The camp shape is a trapezium, so the formula for area of a trapezium is used.

b. 1700 m 1M

The known lengths are 250 + 400 +550 = 1200 m.

The remaining fence can be obtained using Pythagoras' Theorem as shown:

$$\sqrt{300^2 + 400^2} = 500\,\text{m}$$

Therefore the total perimeter is 1200 + 500 = 1700 m.

c. 314.2 m^2 1M

Each shelter is formed from a rectangle of canvas with a length of 20 m and a width given by half the circumference of a circle of radius 5 m:

$$width = \frac{1}{2} \times (2 \times \pi \times 5) = 15.7079...\text{m}$$

Therefore the area is 20 × 15.7079…= 314.159…≈ 314.2 m^2

d. 393 m^3 1M

The volume is half of a cylinder of radius 5 m and length 20 m:

$$V = \frac{1}{2}(\pi \times 5^2 \times 20) = 758.398... \approx 758m^3$$

QUESTION 2

a. Johannesburg's angle from the equator is 26º which is less than Echuca's angle of 36º from the equator. 1M

Echuca is at 36ºS, 145ºE while Johannesburg is at 26ºS, 28ºE. The distance from the equator is given by the parallel of latitude which is the angle north or south of the equator. As Johannesburg's angle from the equator is 26º which is less than Echuca's angle from the equator, Johannesburg is closer.

b. 9.48 pm 1M

Johannesburg and Echuca are 145 – 28 = 117º apart.

117º represents a time difference of $\frac{117}{15} = 7.8$ hours which is the same as 7 hours and 48 minutes.

From 2.00 pm they need to go forward 7 hours and 48 minutes, so 9.48 pm.

c. 1340 km 1M

There is 36 – 24 = 12 º difference in longitude between the two locations.
As the difference is in longitude, the distance is along a great circle of radius 6400 km, so the distance is $\frac{12}{360}\times 2\times \pi\times 6400 = 1340.412... \approx 1340$ km.

QUESTION 3

a. 6.0º 1M

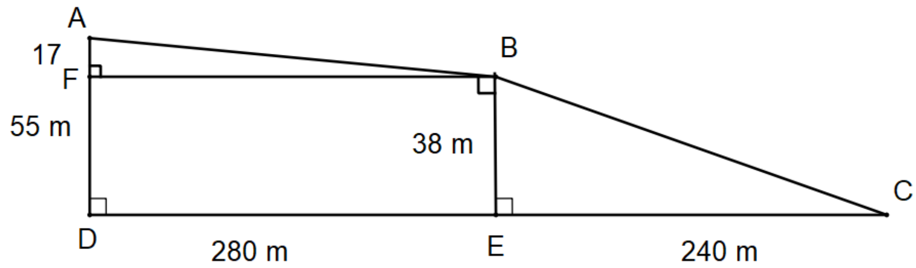
The angle of depression from A to C is the angle creating a fall of 55 m over a total distance of 520 m, so the angle of depression is:

$$\tan(\theta°) = \frac{55}{520}$$

$$\theta° = \tan^{-1}(\frac{55}{520}) = 6.037... \approx 6.0°$$

b. 174.5º 1M

Angle ABC is made up of the sum of angles ABF, FBE and EBC in the diagram below.
Angle FBE is a right angle:



Angle ABF:

$$\tan(\theta°) = \frac{17}{280}$$

$$\theta° = \tan^{-1}(\frac{17}{280}) = 3.4744...°$$

Angle EBC:

$$\tan(\theta°) = \frac{240}{38}$$

$$\theta° = \tan^{-1}(\frac{240}{38}) = 81.002...°$$

Angle ABC = 90 + 3.4744… + 81.002… ≈174.5º

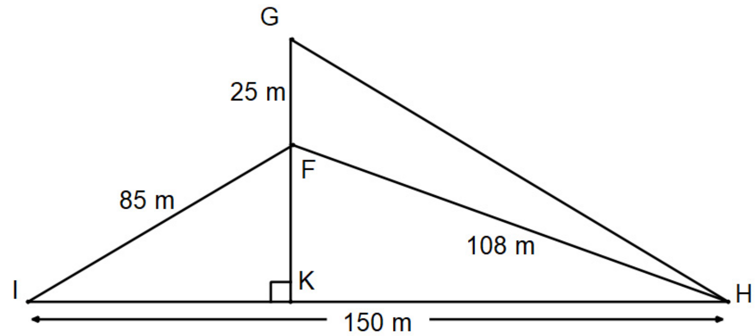
c. The calculation $\cos(FHI) = \dfrac{150^2 + 108^2 - 85^2}{2 \times 150 \times 108}$ must be shown. 1M

$FHI = 33.7519... \approx 33.8°$

The angle FHI is best found using the cosine rule as all sides of the triangle are known, but none of the angles. The calculation must be shown as this is a "show that" question.

d. 124 m 1M

This question could be approached using a geometry program on CAS, but the algebraic solution is shown here using the diagram below:



We know that angle FHI is 33.8º from part c.

Perpendicular height of triangle FK: $\sin(33.8°) = \dfrac{FK}{108}$ 1M

$FK = 60.0799...$

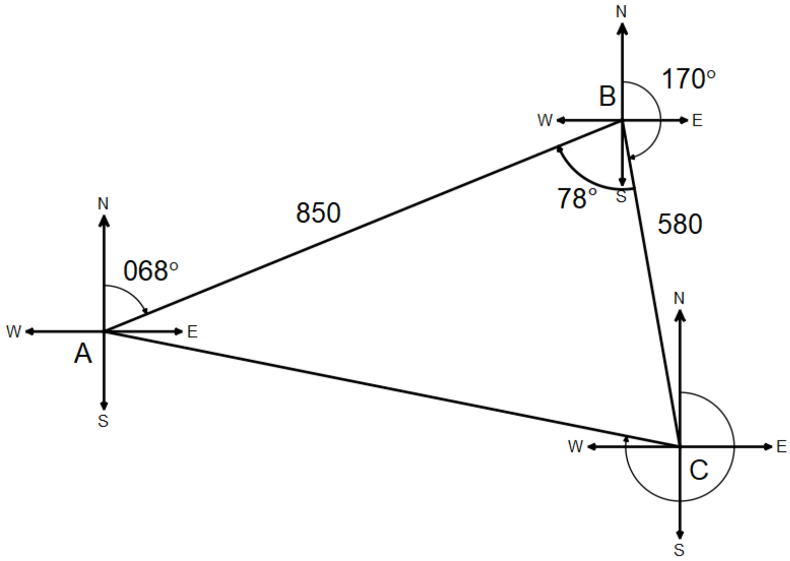
Length KH: $KH = \sqrt{108^2 - 60.0799^2} = 89.7463...$

Length GH: $GH = \sqrt{89.7463^2 + 85.0799^2} = 123.6648...$

This is 124 m correct to the nearest metre. 1M

QUESTION 4

A diagram of the running track is shown below:



a. 241 113 m² 1M

The angle between the first two legs of 850 and 580 metres is 68º +10º = 78º.

The area of the park is $\frac{1}{2} \times 850 \times 580 \times \sin(78°) = 241113.38... \approx 241\,113$ m².

b. 286º 2M

Length AC: $AC = \sqrt{850^2 + 580^2 - 2 \times 850 \times 580 \times \cos(78°)} = 924.066...\text{ m}$

Angle BCA:

$$\frac{\sin(BCA)}{850} = \frac{\sin(78°)}{924.066...}$$ 1M

$BCA = 64.12...°$

Bearing is 360º – 64.12º – 10º = 285.88 ≈ 286º. 1M

MODULE 4: GRAPHS AND RELATIONS

QUESTION 1

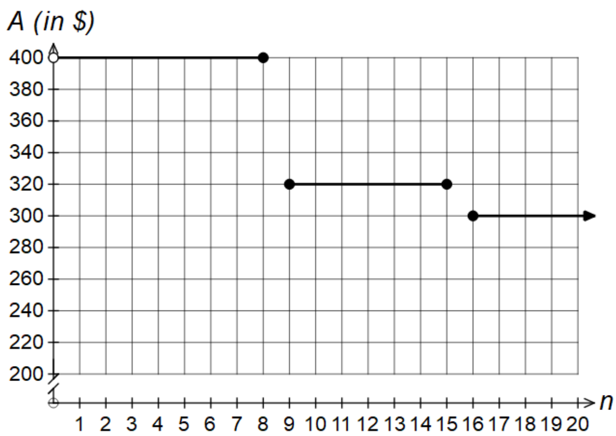
a. \$2000 — 1M

It costs \$500 per person for up to and including seven people to go on a tour, therefore it would cost $5 \times 400 = \$2000$.

b. ≤ 15 — 1M

The line where the amount per person is \$320 extends from 9 to 15. Both 9 and 15 are included because they are represented by closed ends. Therefore the sign $\leq$ and the value 15 are both required.

c. The line shown below including a closed end at (16, 300). — 1M

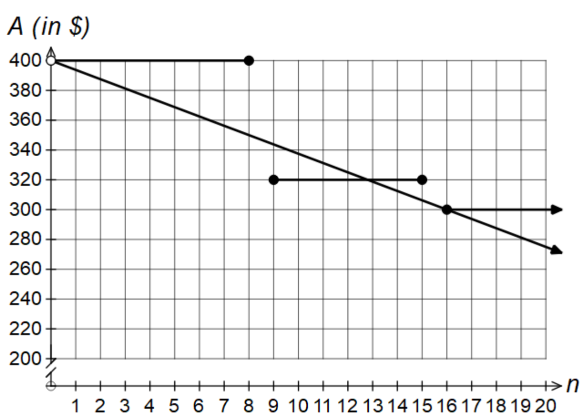


d. Eight people — 1M

If eight people go on the tour, the cost would be $8 \times 400 = \$3200$

If ten people go on the tour, the price drops to \$320 per person, so it also costs $10 \times 320 = \$3200$.

e. The line shown as below: — 1M



The line has a vertical intercept of (0, 400). It passes through (16, 300) as shown:

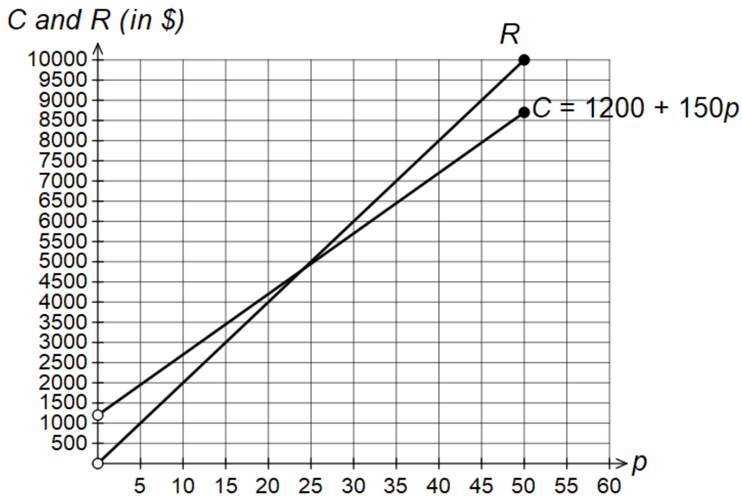
$A = 400 - 6.25 \times 16 = 300$

f. 9, 10, 11 and 12 people 1M

It can be seen that the line $A = 400 - 6.25n$ is below all points on the step graph except 9, 10, 11 and 12 people. The points (0, 400) and (16, 300) are shared by both graphs, so at these points the amount per person is the same rather than more or less.

QUESTION 2

a. Line as shown below. Line must start at (0, 1200) and terminate at (50, 8700): 1M



b. $R = 200p$ 1M

The line has a vertical intercept of (0, 0), so the equation is of the form $R = mp$ where m is the gradient.

The gradient is $\frac{10000 - 0}{50 - 0} = 200$ so the equation is $R = 200p$.

c. At break even point:

$R = C$

$200p = 1200 + 150p$

$50p = 1200$ 1M

$p = \frac{1200}{50} = 24$

Must show equating $200p = 1200 + 150p$ and then algebraic steps to reach 24.

d. \$242.50 (per passenger) 1M

$$P = R - C$$
$$2500 = 40x - (1200 + 150 \times 40)$$
$$40x = 9700$$
$$x = \frac{9700}{40} = \$242.50$$

QUESTION 3

a. $x + y \leq 50$ 1M

The bus can only hold 50 people, so the total number of guests and the total number of staff must be less than or equal to 50.

b. There must be at least one staff member for every five guests. 1M

The following table can help explain the inequality:

Number of Guests (x)	**Number of Staff (y)**
5	At least $\frac{5}{5} = 1$
10	At least $\frac{10}{5} = 2$
15	At least $\frac{15}{5} = 3$
20	At least $\frac{20}{5} = 4$

c. $P = 550x - 1000y$ 1M

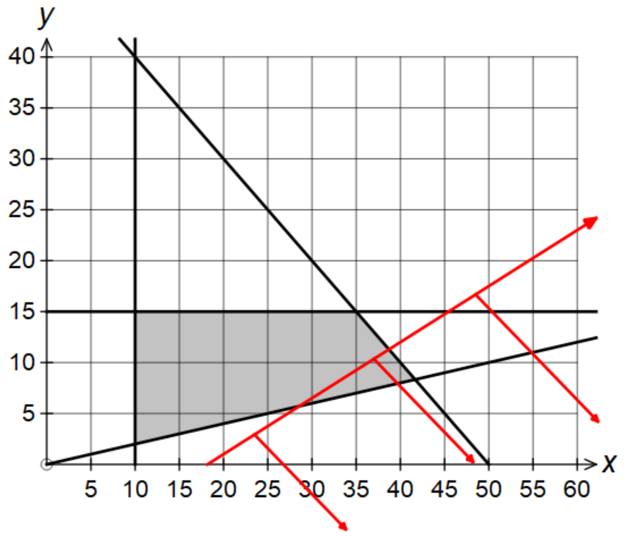
The tour company charge \$550 per guest and have to pay \$1000 for every staff member per tour, so the amount of \$1000 is subtracted from the revenue of \$550 per guest.

d. \$14 000 2M

The objective function can be transposed as follows:

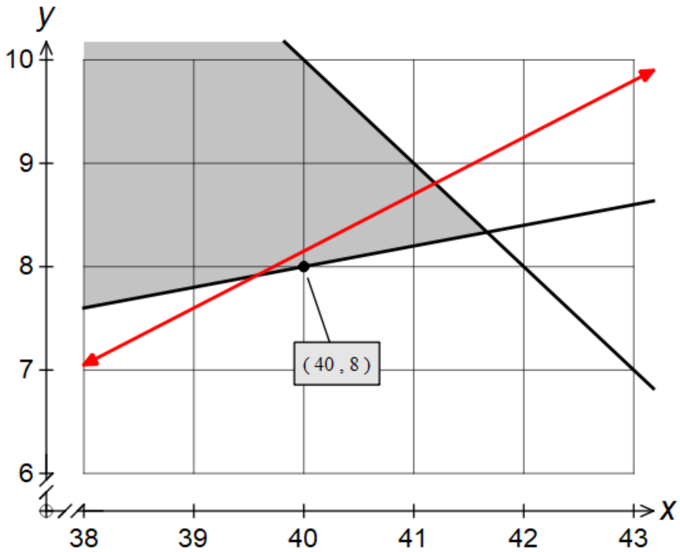
$$P = 550x - 1000y$$
$$1000y = 550x - P$$
$$y = \frac{550}{1000}x - \frac{P}{1000}$$
$$y = 0.55x - \frac{P}{1000}$$

The objective function has been drawn on the feasible region below with arrows showing the direction to maximise:



The corner point last touched by the objective function would be $(41.\dot{6}, 8.\dot{3})$ but as the guests and staff are people, they must be integers.

Zooming in on this corner point gives the graph below:



Therefore the last integer point touched is at (40,8) with a Profit of
$40 \times 550 - 8 \times 1000 = \$14\ 000$.

Finding maximum profit of $14 000 2M

OR

Finding integer point in region of $(41.\dot{6}, 8.\dot{3})$ in region as maximum, 1M
eg (41, 9), (40, 8), (40, 9) or (40,10). Must be testing for integer points near that location.