

STUDENT NUMBER Letter

FURTHER MATHEMATICS

Written examination 2

Monday 16 November 2020

Reading time: 9.00 am to 9.15 am (15 minutes)

Writing time: 9.15 am to 10.45 am (1 hour 30 minutes)

QUESTION AND ANSWER BOOK

Structure of book

Section A – Core	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
	11	11	45
Section B – Modules	<i>Number of modules</i>	<i>Number of modules to be answered</i>	<i>Number of marks</i>
	4	1	15
			Total 60

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 45 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions

- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination

- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A – Core**Instructions for Section A**

Answer **all** questions in the spaces provided.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π , surds or fractions.

In ‘Recursion and financial modelling’, all answers should be rounded to the nearest cent unless otherwise instructed.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data analysis**Question 1** (3 marks)

Body mass index (BMI), in kilograms per square metre, was recorded for a sample of 32 men and displayed in the ordered stem plot below.

key: 21 | 6 = 21.6 $n = 32$

21		6	9	9				
22		1	2	5	6			
23		0	1	4	6	6	7	8
24		4	5	6	7	7	9	
25		6	8					
26		1	7	9				
27		3	7					
28		2						
29		1	8					
30		4						
31		1						

- a. Describe the shape of the distribution. 1 mark

- b. Determine the median *BMI* for this group of men. 1 mark

- c. People with a *BMI* of 25 or over are considered to be overweight.

What percentage of these men would be considered to be overweight?

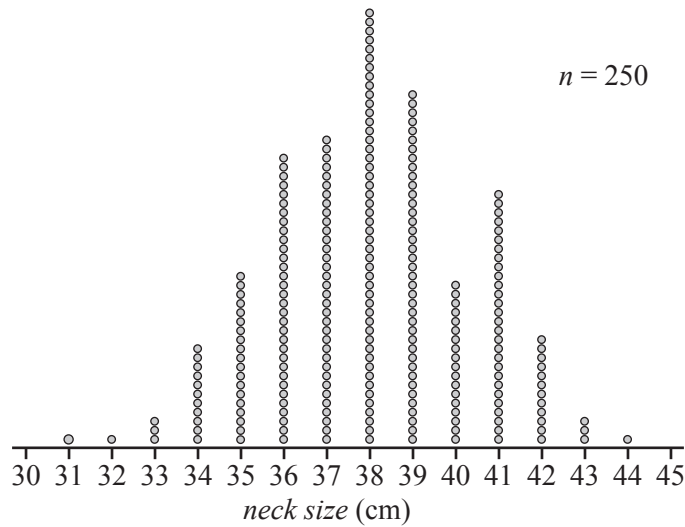
1 mark

DO NOT WRITE IN THIS AREA

SECTION A – continued
TURN OVER

Question 2 (5 marks)

The *neck size*, in centimetres, of 250 men was recorded and displayed in the dot plot below.



Data: RW Johnson, 'Fitting percentage of body fat to simple body measurements',
Journal of Statistics Education, 4:1, 1996,
<https://doi.org/10.1080/10691898.1996.11910505>

- a. Write down the modal *neck size*, in centimetres, for these 250 men. 1 mark

- b. Assume that this sample of 250 men has been drawn at random from a population of men whose *neck size* is normally distributed with a mean of 38 cm and a standard deviation of 2.3 cm.

- i. How many of these 250 men are **expected** to have a *neck size* that is more than three standard deviations above or below the mean?

Round your answer to the nearest whole number. 1 mark

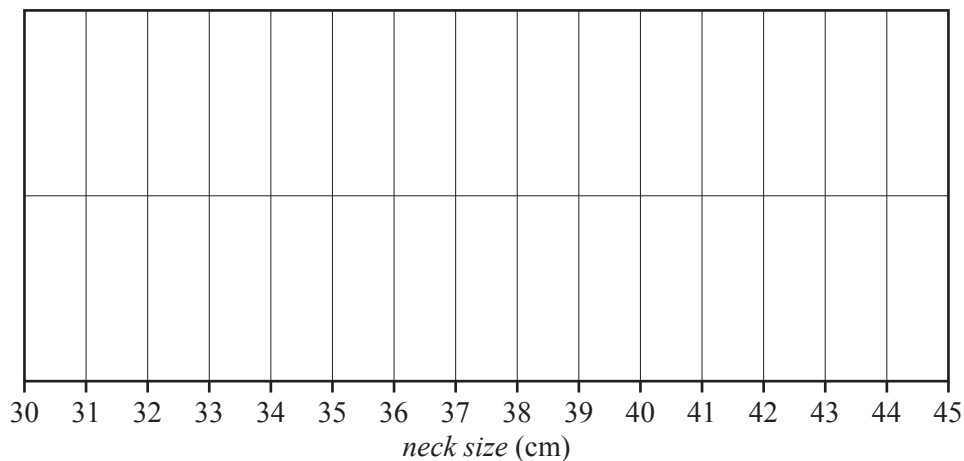
- ii. How many of these 250 men **actually** have a *neck size* that is more than three standard deviations above or below the mean? 1 mark

- c. The five-number summary for this sample of neck sizes, in centimetres, is given below.

Minimum	First quartile (Q_1)	Median	Third quartile (Q_3)	Maximum
31	36	38	39	44

Use the five-number summary to construct a boxplot, showing any outliers if appropriate, on the grid below.

2 marks



DO NOT WRITE IN THIS AREA

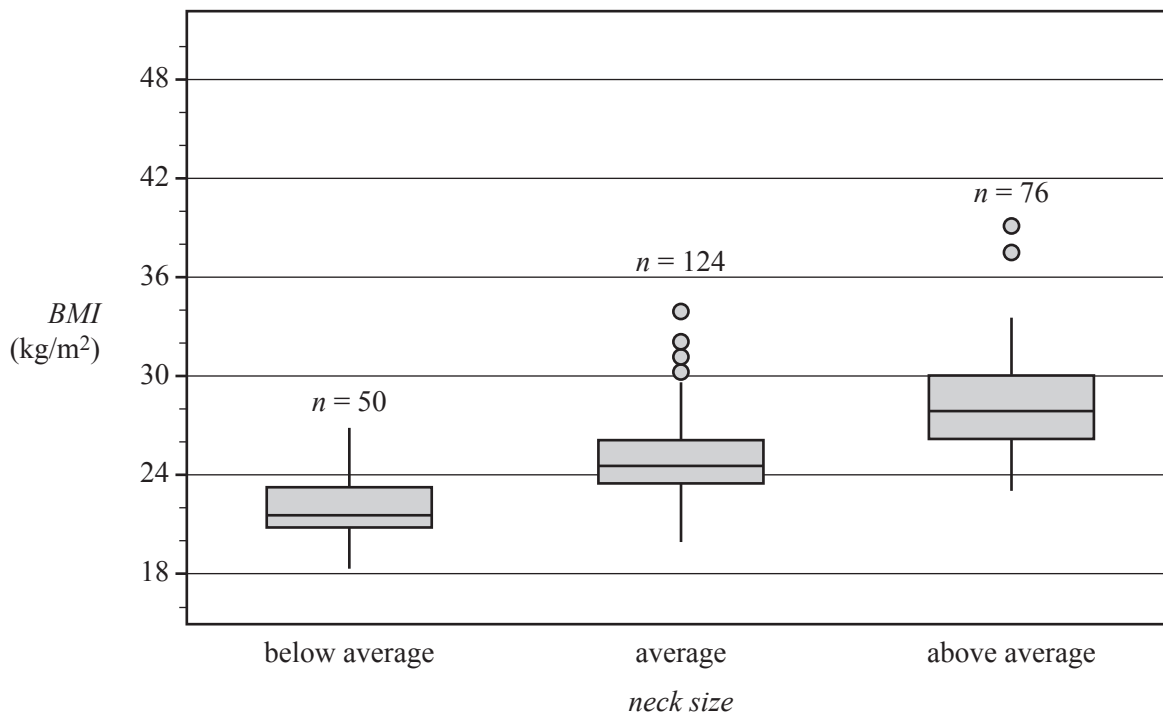
Question 3 (5 marks)

In a study of the association between *BMI* and *neck size*, 250 men were grouped by *neck size* (below average, average and above average) and their *BMI* recorded.

Five-number summaries describing the distribution of *BMI* for each group are displayed in the table below along with the group size.

The associated boxplots are shown below the table.

<i>Neck size</i>	Group size	<i>BMI</i> (kg/m ²)				
		Min.	Q_1	Median	Q_3	Max.
below average	50	18.1	20.6	21.6	23.2	26.8
average	124	19.8	23.4	24.6	26.0	33.9
above average	76	23.1	26.25	28.1	29.95	39.1



Data: RW Johnson, 'Fitting percentage of body fat to simple body measurements', *Journal of Statistics Education*, 4:1, 1996, <<https://doi.org/10.1080/10691898.1996.11910505>>

- a. What percentage of these 250 men are classified as having a below average neck size? 1 mark

- b. What is the interquartile range (IQR) of *BMI* for the men with an average neck size? 1 mark

- c. People with a *BMI* of 30 or more are classified as being obese.

Using this criterion, how many of these 250 men would be classified as obese? Assume that the *BMI* values were all rounded to one decimal place.

1 mark

- d. Do the boxplots support the contention that *BMI* is associated with *neck size*? Refer to the values of an appropriate statistic in your response.

2 marks

DO NOT WRITE IN THIS AREA

SECTION A – continued
TURN OVER

Question 4 (5 marks)

The *age*, in years, *body density*, in kilograms per litre, and *weight*, in kilograms, of a sample of 12 men aged 23 to 25 years are shown in the table below.

<i>Age</i> (years)	<i>Body density</i> (kg/litre)	<i>Weight</i> (kg)
23	1.07	70.1
23	1.07	90.4
23	1.08	73.2
23	1.08	85.0
24	1.03	84.3
24	1.05	95.6
24	1.07	71.7
24	1.06	95.0
25	1.07	80.2
25	1.09	87.4
25	1.02	94.9
25	1.09	65.3

- a. For these 12 men, determine
- i. their median *age*, in years 1 mark

 - ii. the mean of their *body density*, in kilograms per litre. 1 mark

- b. A least squares line is to be fitted to the data with the aim of predicting *body density* from *weight*.
- i. Name the explanatory variable for this least squares line. 1 mark

 - ii. Determine the slope of this least squares line.
Round your answer to three significant figures. 1 mark

- c. What percentage of the variation in *body density* can be explained by the variation in *weight*?
Round your answer to the nearest percentage.

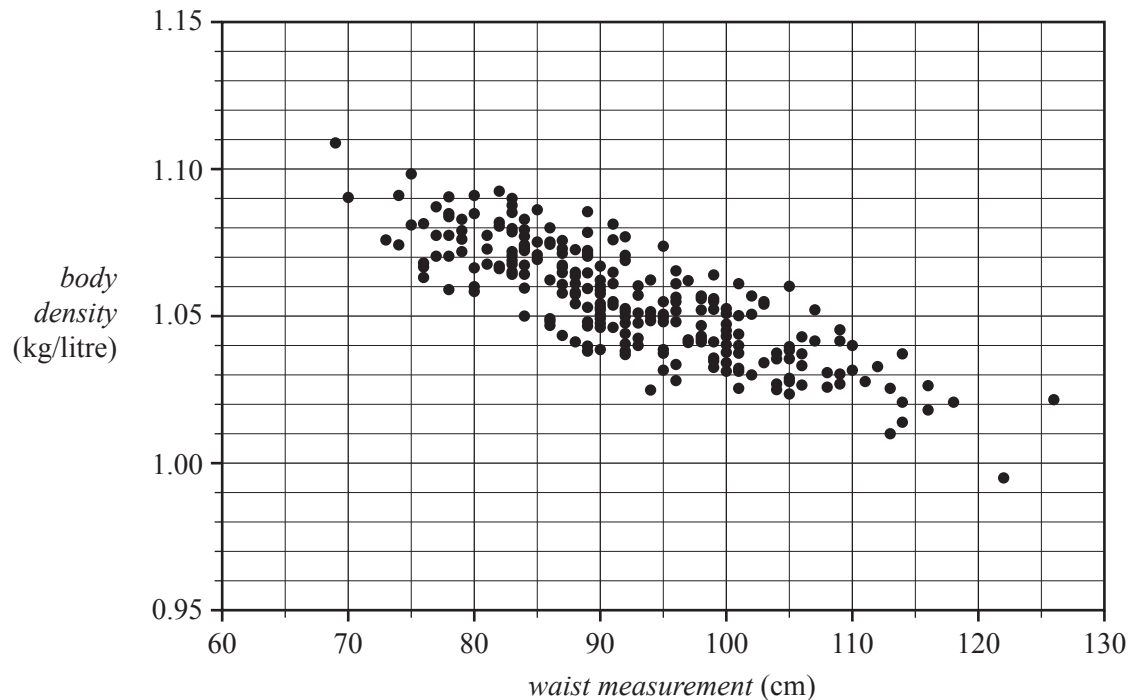
1 mark

DO NOT WRITE IN THIS AREA

SECTION A – continued
TURN OVER

Question 5 (7 marks)

The scatterplot below shows *body density*, in kilograms per litre, plotted against *waist measurement*, in centimetres, for 250 men.



Data: RW Johnson, 'Fitting percentage of body fat to simple body measurements', *Journal of Statistics Education*, 4:1, 1996, <<https://doi.org/10.1080/10691898.1996.11910505>>

When a least squares line is fitted to the scatterplot, the equation of this line is

$$\text{body density} = 1.195 - 0.001512 \times \text{waist measurement}$$

- a. Draw the graph of this least squares line on the **scatterplot above**. 1 mark

(Answer on the scatterplot above.)

- b. Use the equation of this least squares line to predict the *body density* of a man whose *waist measurement* is 65 cm.
Round your answer to two decimal places. 1 mark

- c. When using the equation of this least squares line to make the prediction in **part b.**, are you extrapolating or interpolating? 1 mark

- d. Interpret the slope of this least squares line in terms of a man's *body density* and *waist measurement*. 1 mark

- e. In this study, the body density of the man with a waist measurement of 122 cm was 0.995 kg/litre.

Show that, when this least squares line is fitted to the scatterplot, the residual, rounded to two decimal places, is -0.02

1 mark

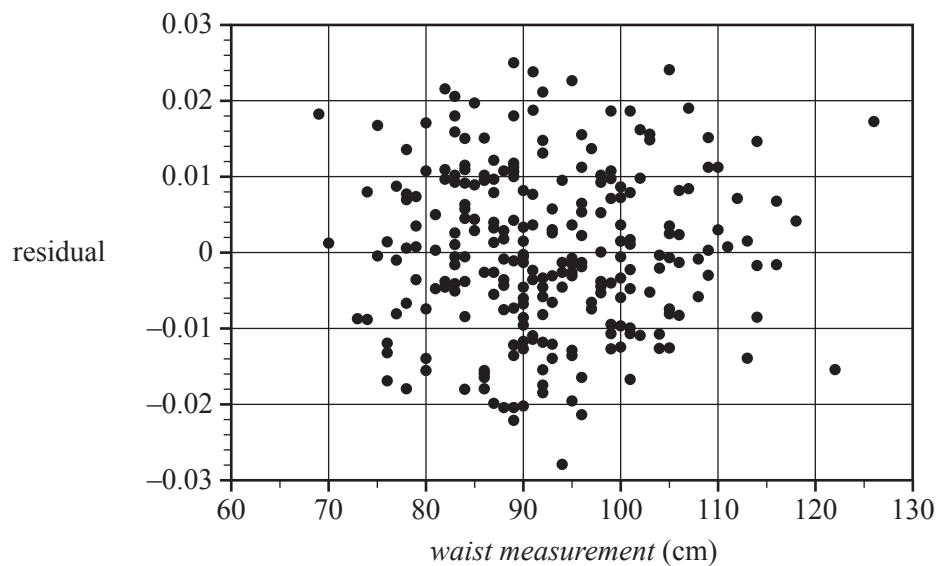
- f. The coefficient of determination for this data is 0.6783

Write down the value of the correlation coefficient r .

Round your answer to three decimal places.

1 mark

- g. The residual plot associated with fitting a least squares line to this data is shown below.



Does this residual plot support the assumption of linearity that was made when fitting this line to this data? Briefly explain your answer.

1 mark

Question 6 (5 marks)

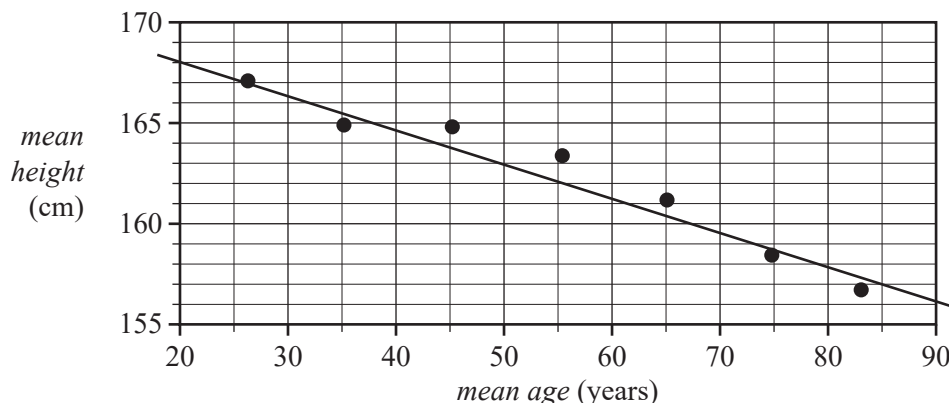
The table below shows the *mean age*, in years, and the *mean height*, in centimetres, of 648 women from seven different age groups.

	Age group						
	Twenties	Thirties	Forties	Fifties	Sixties	Seventies	Eighties
<i>Mean age (years)</i>	26.3	35.2	45.2	55.3	65.1	74.8	83.1
<i>Mean height (cm)</i>	167.1	164.9	164.8	163.4	161.2	158.4	156.7

Data: J Sorkin et al., ‘Longitudinal change in height of men and women: Implications for interpretation of the body mass index’, *American Journal of Epidemiology*, vol. 150, no. 9, 1999, p. 971

- a. What was the difference, in centimetres, between the *mean height* of the women in their twenties and the *mean height* of the women in their eighties? 1 mark

A scatterplot displaying this data shows an association between the *mean height* and the *mean age* of these women. In an initial analysis of the data, a line is fitted to the data by eye, as shown.



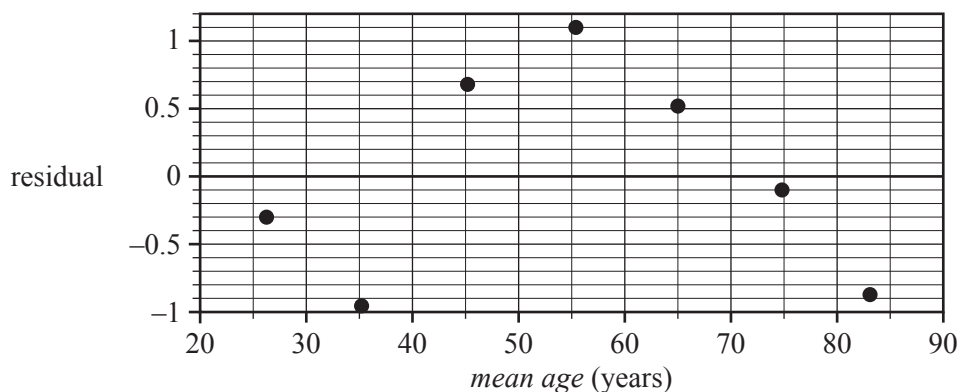
- b. Describe this association in terms of strength and direction. 1 mark

- c. The line on the scatterplot passes through the points (20, 168) and (85, 157).
 Using these two points, determine the equation of this line. Write the values of the intercept and the slope in the appropriate boxes below.
 Round your answers to three significant figures. 1 mark

mean height = + × *mean age*

DO NOT WRITE IN THIS AREA

- d. In a further analysis of the data, a least squares line was fitted.
The associated residual plot that was generated is shown below.



The residual plot indicates that the association between the *mean height* and the *mean age* of women is non-linear.

The data presented in the table on page 12 is repeated below. It can be linearised by applying an appropriate transformation to the variable *mean age*.

<i>Mean age (years)</i>	26.3	35.2	45.2	55.3	65.1	74.8	83.1
<i>Mean height (cm)</i>	167.1	164.9	164.8	163.4	161.2	158.4	156.7

Apply an appropriate transformation to the variable *mean age* to linearise the data. Fit a least squares line to the transformed data and write its equation below.

Round the values of the intercept and the slope to four significant figures.

2 marks

Recursion and financial modelling

Question 7 (4 marks)

Samuel owns a printing machine.

The printing machine is depreciated in value by Samuel using flat rate depreciation.

The value of the machine, in dollars, after n years, V_n , can be modelled by the recurrence relation

$$V_0 = 120\,000, \quad V_{n+1} = V_n - 15\,000$$

- a. By what amount, in dollars, does the value of the machine decrease each year? 1 mark

- b. Showing recursive calculations, determine the value of the machine, in dollars, after two years. 1 mark

- c. What annual flat rate percentage of depreciation is used by Samuel? 1 mark

- d. The value of the machine, in dollars, after n years, V_n , could also be determined using a rule of the form $V_n = a + bn$.

Write down this rule for V_n .

1 mark

DO NOT WRITE IN THIS AREA

Question 8 (3 marks)

Samuel has a reducing balance loan.

The first five lines of the amortisation table for Samuel's loan are shown below.

Payment number	Payment (\$)	Interest (\$)	Principal reduction (\$)	Balance (\$)
0	0.00	0.00	0.00	320 000.00
1	1600.00	960.00	640.00	319 360.00
2	1600.00	958.08	641.92	318 718.08
3	1600.00	956.15		318 074.23
4	1600.00			

Interest is calculated monthly and Samuel makes monthly payments of \$1600.

Interest is charged on this loan at the rate of 3.6% per annum.

a. Using the values in the amortisation table

i. calculate the principal reduction associated with payment number 3

1 mark

ii. calculate the balance of the loan after payment number 4 is made.

Round your answer to the nearest cent.

1 mark

b. Let S_n be the balance of Samuel's loan after n months.

Write down a recurrence relation, in terms of S_0 , S_{n+1} and S_n , that could be used to model the month-to-month balance of the loan.

1 mark

DO NOT WRITE IN THIS AREA

Question 9 (3 marks)

Samuel opens a savings account.

Let B_n be the balance of this savings account, in dollars, n months after it was opened.

The month-to-month value of B_n can be determined using the recurrence relation shown below.

$$B_0 = 5000, \quad B_{n+1} = 1.003B_n$$

- a. Write down the value of B_4 , the balance of the savings account after four months.

Round your answer to the nearest cent.

1 mark

- b. Calculate the monthly interest rate percentage for Samuel's savings account.

1 mark

- c. After one year, the balance of Samuel's savings account, to the nearest dollar, is \$5183.

If Samuel had deposited an additional \$50 at the end of each month immediately after the interest was added, how much extra money would be in the savings account after one year?

Round your answer to the nearest dollar.

1 mark

DO NOT WRITE IN THIS AREA

Question 10 (3 marks)

Samuel now invests \$500 000 in an annuity from which he receives a regular monthly payment.

The balance of the annuity, in dollars, after n months, A_n , can be modelled by a recurrence relation of the form

$$A_0 = 500\,000, \quad A_{n+1} = kA_n - 2000$$

- a. Calculate the balance of this annuity after two months if $k = 1.0024$ 1 mark

- b. Calculate the annual compound interest rate percentage for this annuity if $k = 1.0024$ 1 mark

- c. For what value of k would this investment act as a simple perpetuity? 1 mark

DO NOT WRITE IN THIS AREA

Question 11 (2 marks)

Later, Samuel took out a new reducing balance loan.

The interest rate for this loan was 4.1% per annum, compounding monthly.

The balance of the loan after four years of monthly repayments was \$329 587.25

The balance of the loan after seven years of monthly repayments was \$280 875.15

Samuel will continue to make the same monthly repayment.

To ensure the loan is fully repaid, to the nearest cent, the required final repayment will be lower.

In the first seven years, Samuel made 84 monthly repayments.

From this point on, how many more monthly repayments will Samuel make to fully repay the loan?

DO NOT WRITE IN THIS AREA**END OF SECTION A**

SECTION B – Modules**Instructions for Section B**

Select **one** module and answer **all** questions within the selected module.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π , surds or fractions.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Contents	Page
Module 1 – Matrices	20
Module 2 – Networks and decision mathematics	26
Module 3 – Geometry and measurement	33
Module 4 – Graphs and relations	40

DO NOT WRITE IN THIS AREA

Module 1 – Matrices

Question 1 (6 marks)

The three major shopping centres in a large city, Eastmall (E), Grandmall (G) and Westmall (W), are owned by the same company.

The total number of shoppers at each of the centres at 1.00 pm on a typical day is shown in matrix V .

$$V = \begin{bmatrix} E & G & W \\ 2300 & 2700 & 2200 \end{bmatrix}$$

- a. Write down the order of matrix V .

1 mark

Each of these centres has three major shopping areas: food (F), clothing (C) and merchandise (M).

The proportion of shoppers in each of these three areas at 1.00 pm on a typical day is the same at all three centres and is given in matrix P below.

$$P = \begin{bmatrix} 0.48 \\ 0.27 \\ 0.25 \end{bmatrix} \begin{matrix} F \\ C \\ M \end{matrix}$$

- b. Grandmall's management would like to see 700 shoppers in its merchandise area at 1.00 pm.

If this were to happen, how many shoppers, in total, would be at Grandmall at this time?

1 mark

- c. The matrix $Q = P \times V$ is shown below. Two of the elements of this matrix are missing.

$$Q = \begin{bmatrix} E & G & W \\ 1104 & \text{---} & 1056 \\ 621 & \text{---} & 594 \\ 575 & 675 & 550 \end{bmatrix} \begin{matrix} F \\ C \\ M \end{matrix}$$

- i. Complete **matrix Q above** by filling in the missing elements.

1 mark

(Answer on matrix Q above.)

- ii. The element in row i and column j of matrix Q is q_{ij} .

What does the element q_{23} represent?

1 mark

The average daily amount spent, in dollars, by each shopper in each of the three areas at Grandmall in 2019 is shown in matrix A_{2019} below.

$$A_{2019} = \begin{bmatrix} 21.30 \\ 34.00 \\ 14.70 \end{bmatrix} \begin{matrix} F \\ C \\ M \end{matrix}$$

On one particular day, 135 shoppers spent the average daily amount on food, 143 shoppers spent the average daily amount on clothing and 131 shoppers spent the average daily amount on merchandise.

- d. Write a matrix calculation, using matrix A_{2019} , showing that the total amount spent by all these shoppers is \$9663.20

1 mark

- e. In 2020, the average daily amount spent by each shopper was expected to change by the percentage shown in the table below.

Area	food	clothing	merchandise
Expected change	increase by 5%	decrease by 15%	decrease by 1%

The average daily amount, in dollars, expected to be spent in each area in 2020 can be determined by forming the matrix product

$$A_{2020} = K \times A_{2019}$$

Write down matrix K .

1 mark

$$K =$$

DO NOT WRITE IN THIS AREA

Question 2 (3 marks)

The preferred number of cafes (x) and sandwich bars (y) in Grandmall's food court can be determined by solving the following equations written in matrix form.

$$\begin{bmatrix} 5 & -9 \\ 4 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$$

- a. The value of the determinant of the 2×2 matrix is 1.

Use this information to explain why this matrix has an inverse.

1 mark

- b. Write the three missing values of the inverse matrix that can be used to solve these equations.

1 mark

$$\begin{bmatrix} \underline{\hspace{2cm}} & 9 \\ \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \end{bmatrix}$$

- c. Determine the preferred number of sandwich bars for Grandmall's food court.

1 mark

Question 3 (4 marks)

An offer to buy the Westmall shopping centre was made by a competitor.

One market research project suggested that if the Westmall shopping centre were sold, each of the three centres (Westmall, Grandmall and Eastmall) would continue to have regular shoppers but would attract and lose shoppers on a weekly basis.

Let S_n be the state matrix that shows the expected number of shoppers at each of the three centres n weeks after Westmall is sold.

A matrix recurrence relation that generates values of S_n is

$$S_{n+1} = T \times S_n$$

$$\text{where } T = \begin{array}{c} \text{this week} \\ \begin{array}{ccc} W & G & E \end{array} \\ \begin{bmatrix} 0.80 & 0.09 & 0.10 \\ 0.12 & 0.79 & 0.10 \\ 0.08 & 0.12 & 0.80 \end{bmatrix} \begin{array}{l} W \\ G \\ E \end{array} \text{ next week,} \end{array} \quad S_0 = \begin{bmatrix} 250\,000 \\ 230\,000 \\ 200\,000 \end{bmatrix} \begin{array}{l} W \\ G \\ E \end{array}$$

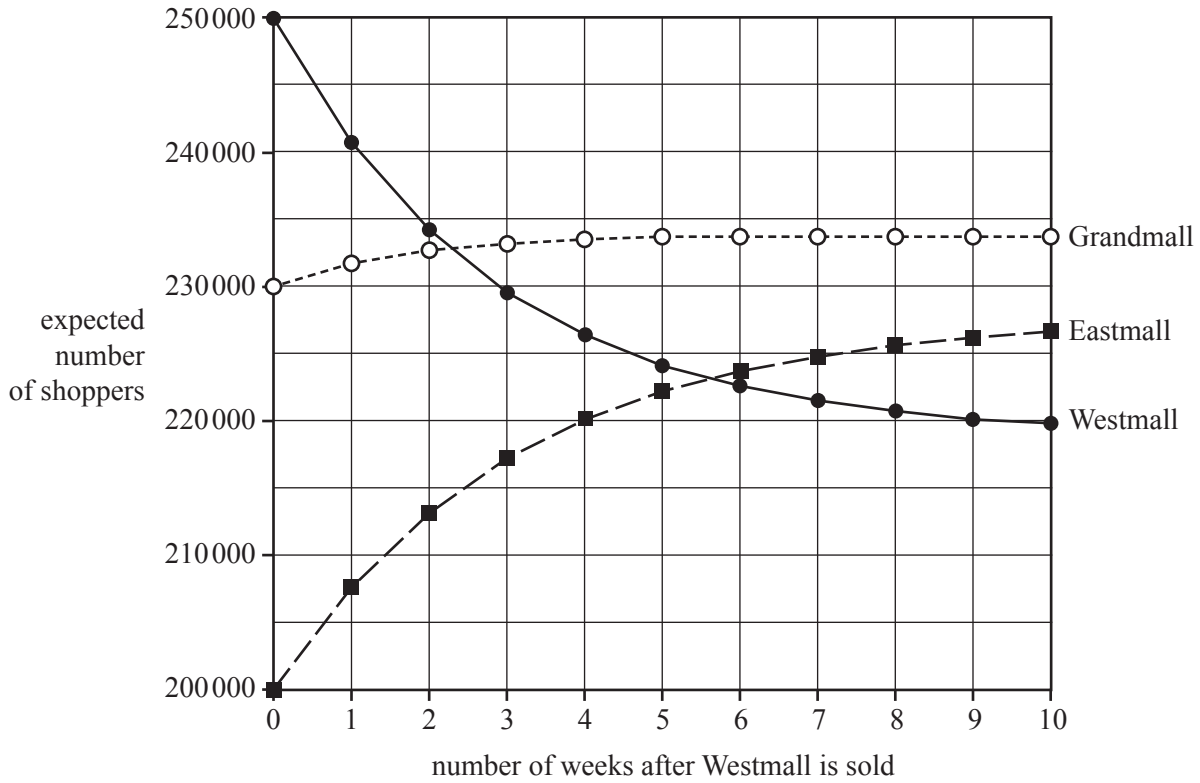
- a. Calculate the state matrix, S_1 , to show the expected number of shoppers at each of the three centres one week after Westmall is sold.

1 mark

$$S_1 = \begin{bmatrix} \text{_____} \\ \text{_____} \\ \text{_____} \end{bmatrix} \begin{array}{l} W \\ G \\ E \end{array}$$

DO NOT WRITE IN THIS AREA

Using values from the recurrence relation on page 23, the graph below shows the expected number of shoppers at Westmall, Grandmall and Eastmall for each of the 10 weeks after Westmall is sold.



- b. What is the difference in the expected weekly number of shoppers at Westmall from the time Westmall is sold to 10 weeks after Westmall is sold?

Give your answer correct to the nearest thousand.

1 mark

- c. Grandmall is expected to achieve its maximum number of shoppers sometime between the fourth and the tenth week after Westmall is sold.

Write down the week number in which this is expected to occur.

1 mark

- d. In the long term, what is the expected weekly number of shoppers at Westmall?

Round your answer to the nearest whole number.

1 mark

DO NOT WRITE IN THIS AREA

Question 4 (2 marks)

A second market research project also suggested that if the Westmall shopping centre were sold, each of the three centres (Westmall, Grandmall and Eastmall) would continue to have regular shoppers but would attract and lose shoppers on a weekly basis.

Let R_n be the state matrix that shows the expected number of shoppers at each of the three centres n weeks after Westmall is sold.

A matrix recurrence relation that generates values of R_n is

$$R_{n+1} = TR_n + B$$

$$\text{where } T = \begin{array}{c} \text{this week} \\ \begin{array}{ccc} W & G & E \\ \begin{bmatrix} 0.78 & 0.13 & 0.10 \end{bmatrix} W \\ \begin{bmatrix} 0.12 & 0.82 & 0.10 \end{bmatrix} G \\ \begin{bmatrix} 0.10 & 0.05 & 0.80 \end{bmatrix} E \end{array} \end{array} \text{ next week,} \quad B = \begin{array}{c} \begin{bmatrix} -400 \\ 700 \\ 500 \end{bmatrix} W \\ G \\ E \end{array}$$

The matrix R_2 is the state matrix that shows the expected number of shoppers at each of the three centres in the second week after Westmall is sold.

$$R_2 = \begin{array}{c} \begin{bmatrix} 239\,060 \\ 250\,840 \\ 192\,900 \end{bmatrix} W \\ G \\ E \end{array}$$

- a. Determine the expected number of shoppers at Westmall in the third week after it is sold. 1 mark

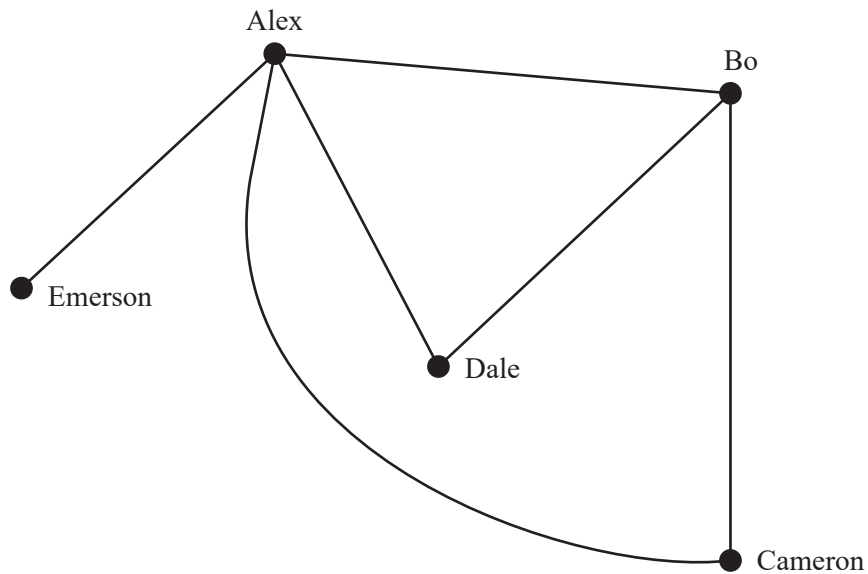
- b. Determine the expected number of shoppers at Westmall in the first week after it is sold. 1 mark

DO NOT WRITE IN THIS AREA

Module 2 – Networks and decision mathematics

Question 1 (3 marks)

The Sunny Coast Cricket Club has five new players join its team: Alex, Bo, Cameron, Dale and Emerson. The graph below shows the players who have played cricket together before joining the team. For example, the edge between Alex and Bo shows that they have previously played cricket together.



- a. How many of these players had Emerson played cricket with before joining the team? 1 mark
-
- b. Who had played cricket with both Alex and Bo before joining the team? 1 mark
-
-
- c. During the season, another new player, Finn, joined the team. Finn had not played cricket with any of these players before. Represent this information on the **graph above**. 1 mark

(Answer on the graph above.)

Question 2 (1 mark)

A cricket team has 11 players who are each assigned to a batting position.

Three of the new players, Alex, Bo and Cameron, can bat in position 1, 2 or 3.

The table below shows the average scores, in runs, for each player for the batting positions 1, 2 and 3.

		Batting position		
		1	2	3
Player	Alex	22	24	24
	Bo	25	25	21
	Cameron	24	25	19

Each player will be assigned to one batting position.

To which position should each player be assigned to **maximise** the team's score? Write your answer in the table below.

Player	Batting position
Alex	
Bo	
Cameron	

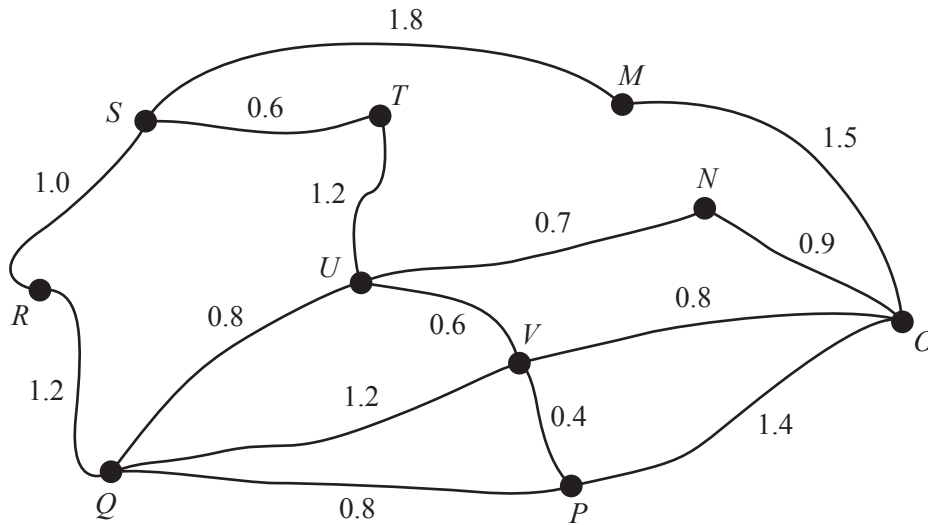
DO NOT WRITE IN THIS AREA

Question 3 (4 marks)

A local fitness park has 10 exercise stations: M to V .

The edges on the graph below represent the tracks between the exercise stations.

The number on each edge represents the length, in kilometres, of each track.



The Sunny Coast cricket coach designs three different training programs, **all starting at exercise station S** .

Training program number	Training details
1	The team must run to exercise station O .
2	The team must run along all tracks just once.
3	The team must visit each exercise station and return to exercise station S .

a. What is the shortest distance, in kilometres, covered in training program 1? 1 mark

b. i. What mathematical term is used to describe training program 2? 1 mark

ii. At which exercise station would training program 2 finish? 1 mark

DO NOT WRITE IN THIS AREA

- c. To complete training program 3 in the minimum distance, one track will need to be repeated.

Complete the following sentence by filling in the boxes provided.

1 mark

This track is between exercise station

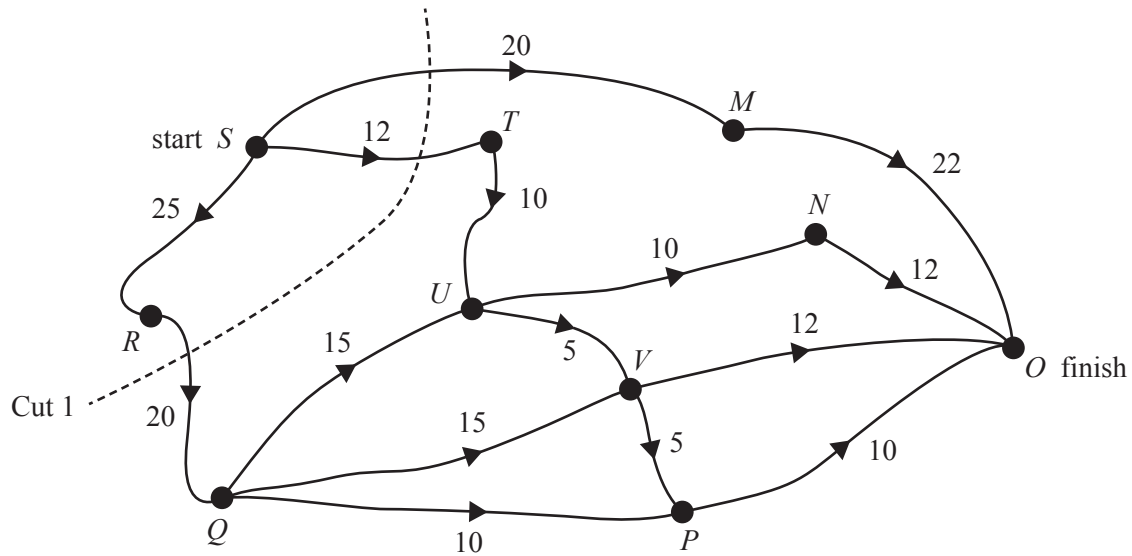
and exercise station

DO NOT WRITE IN THIS AREA

Question 4 (3 marks)

Training program 1 has the cricket team starting from exercise station S and running to exercise station O . For safety reasons, the cricket coach has placed a restriction on the maximum number of people who can use the tracks in the fitness park.

The directed graph below shows the capacity of the tracks, in number of people per minute.



- a. How many different routes from S to O are possible? 1 mark

When considering the possible flow of people through this network, many different cuts can be made.

- b. Determine the capacity of Cut 1, shown above. 1 mark

- c. What is the maximum flow from S to O , in number of people per minute? 1 mark

DO NOT WRITE IN THIS AREA

Question 5 (4 marks)

The Sunny Coast cricket clubroom is undergoing a major works project.

This project involves nine activities: A to I .

The table below shows the earliest start time (EST) and duration, in months, for each activity.

The immediate predecessor(s) is also shown.

The duration for activity C is missing.

Activity	EST	Duration	Immediate predecessor(s)
A	0	2	–
B	0	5	–
C	5		A, B
D	7	7	C
E	7	9	C
F	5	3	B
G	14	4	D
H	8	9	F
I	18	2	E, G, H

The information in the table above can be used to complete a directed network.

This network will require a dummy activity.

- a. Complete the following sentence by filling in the boxes provided. 1 mark

This dummy activity could be drawn as a directed edge from the end of activity to
the start of activity .

- b. What is the duration, in months, of activity C ? 1 mark

- c. Name the four activities that have a float time. 1 mark

- d. The project is to be crashed by reducing the completion time of one activity only.

What is the minimum time, in months, that the project can be completed in? 1 mark

DO NOT WRITE IN THIS AREA

THIS PAGE IS BLANK

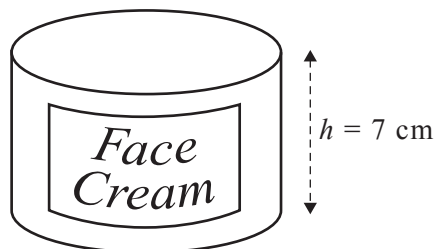
DO NOT WRITE IN THIS AREA

Module 3 – Geometry and measurement

Question 1 (6 marks)

Khaleda manufactures a face cream. The cream comes in a cylindrical container.

The area of the circular base is 43 cm^2 . The container has a height of 7 cm, as shown in the diagram below.



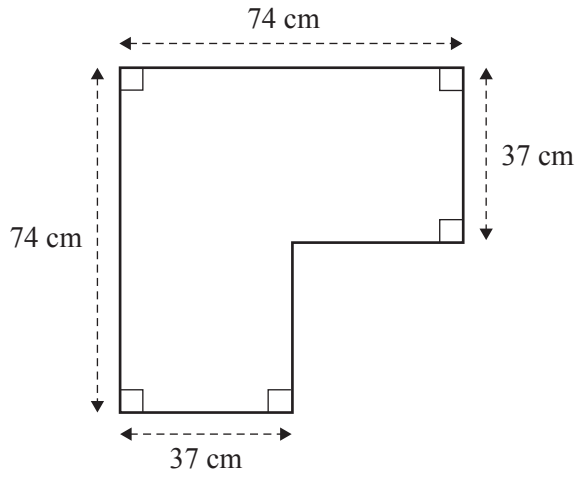
- a. What is the volume of the container, in cubic centimetres? 1 mark

- b. Write a calculation that shows that the radius of the cylindrical container, rounded to one decimal place, is 3.7 cm. 1 mark

- c. What is the total surface area of the container, in square centimetres, including the base and the lid? Round your answer to the nearest square centimetre. 1 mark

DO NOT WRITE IN THIS AREA

The diagram below shows the dimensions of a shelf that will display the containers.

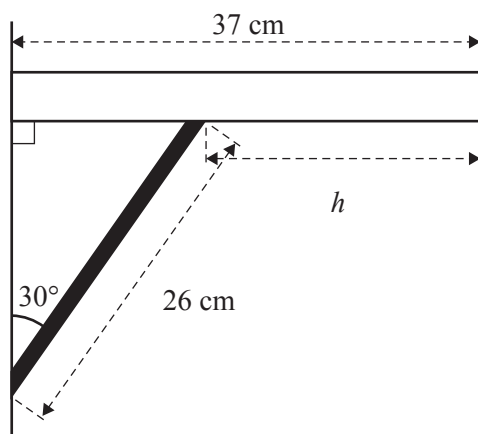


- d. What is the perimeter of the shelf, in centimetres? 1 mark

- e. The shelf will display the containers in a single layer. Each container will stand upright on the shelf.
 What is the maximum number of containers that can fit on the shelf? 1 mark

DO NOT WRITE IN THIS AREA

- f. The shelf sits against a wall at a 90° angle.
The shelf is supported by a 26 cm bracket that forms a 30° angle with the wall, as shown in the diagram below.



Find the value of h , the distance between the edge of the shelf and the bracket, in centimetres.

1 mark

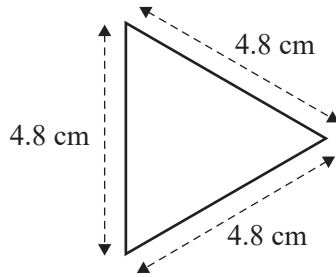
DO NOT WRITE IN THIS AREA

Question 2 (4 marks)

Khaleda has designed a logo for her business.

The logo contains two identical equilateral triangles.

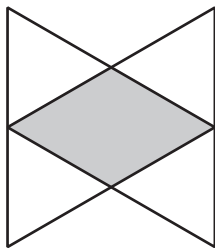
The side length of each triangle is 4.8 cm, as shown in the diagram below.



- a. Write a calculation to show that the area of one of the triangles, rounded to the nearest centimetre, is 10 cm^2 .

1 mark

In the logo, the two triangles overlap, as shown below. Part of the logo is shaded and part of the logo is not shaded.



- b. What is the area of the entire logo?
Round your answer to the nearest square centimetre.

1 mark

- c. What is the ratio of the area of the shaded region to the area of the non-shaded region of the logo?

1 mark

- d. The logo is enlarged and printed on boxes for shipping.
The enlarged logo and the original logo are similar in shape.
The area of the enlarged logo is four times the area of the original logo.

What is the height, in centimetres, of the enlarged logo?

1 mark

DO NOT WRITE IN THIS AREA

Question 3 (5 marks)

Khaleda manufactures the face cream in Dhaka, Bangladesh.

Dhaka is located at latitude 24° N and longitude 90° E.

Assume that the radius of Earth is 6400 km.

- a.** Write a calculation that shows that the radius of the small circle of Earth at latitude 24° N is 5847 km, rounded to the nearest kilometre. 1 mark

Khaleda receives an order from Abu Dhabi, United Arab Emirates (24° N, 54° E).

- b.** Find the shortest small circle distance between Dhaka and Abu Dhabi. Round your answer to the nearest kilometre. 1 mark

Khaleda sends the order by plane from Dhaka (24° N, 90° E) to Abu Dhabi (24° N, 54° E).

The flight departs Dhaka at 1.00 pm and arrives in Abu Dhabi 11 hours later.

The time difference between Dhaka and Abu Dhabi is two hours.

- c.** What time did the flight arrive in Abu Dhabi? 1 mark

A helicopter takes the order from the airport to the customer's hotel.

The hotel is 27 km south and 109 km east of the airport.

- d.** Show that the bearing of the hotel from the airport is 104° , correct to the nearest degree. 1 mark

- e. After the delivery to the hotel, the helicopter returns to its hangar.
The hangar is located due south of the airport.
The helicopter flies directly from the hotel to the hangar on a bearing of 282° .

How far south of the airport is the hangar?

Round your answer to the nearest kilometre.

1 mark

DO NOT WRITE IN THIS AREA

Module 4 – Graphs and relations

Question 1 (3 marks)

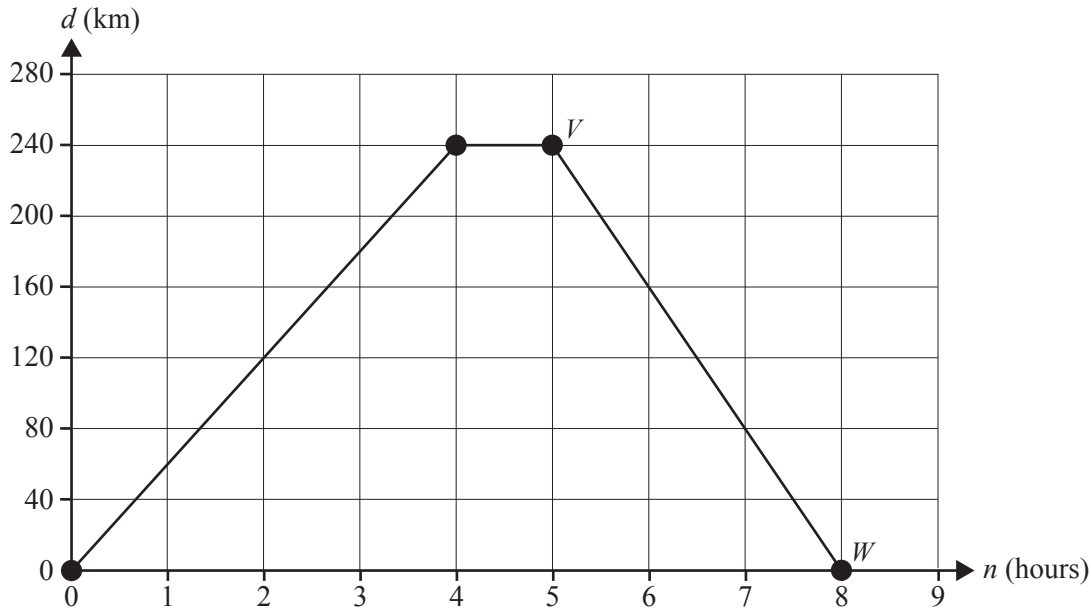
Kyla owns and manages a truck and car care business.

After a major repair on a truck, one of the mechanics took it on a long test drive.

The test drive started at 12 noon.

After four hours, the mechanic stopped to rest for one hour and then returned to the workshop.

The graph below shows the truck's distance from the workshop, d , in kilometres, and the number of hours of test driving, n , after 12 noon.



- a. At what time of the day did the mechanic arrive back at the workshop? 1 mark

- b. Find the average speed, in kilometres per hour, of the truck during the first four hours of the test drive. 1 mark

- c. On the return trip, the truck travelled at an average speed of 80 km/h.
The equation of the line segment VW that represents this part of the test drive is of the form

$$d = k - 80n$$

Find the value of k .

1 mark

Question 2 (3 marks)

Kyla organises fundraiser car shows at the business.

- a. Fifteen stallholders pay a set-up fee of \$120 each.
Twenty-six competitors pay \$30 each to enter their cars to win prizes.

How much money in total is raised from stallholders and competitors at each car show? 1 mark

- b. Admission fees for the show are \$5 per adult and \$2 per child.
\$1644 was raised from the 537 people who attended the most recent car show.

How many children attended this car show? 1 mark

- c. The table below displays the *speed of the car*, s , in kilometres per hour, and the *fuel consumption*, F , in kilometres per litre, of one of the cars at two particular speeds.

<i>speed of the car</i> (s)	60 km/h	100 km/h
<i>fuel consumption</i> (F)	10 km/litre	6 km/litre

The equation $F = \frac{k}{s}$, where k is the constant of proportionality, is used to model the relationship between *fuel consumption* and *speed of the car*.

Determine the fuel consumption of the car, in kilometres per litre, when it is travelling at 80 km/h. 1 mark

DO NOT WRITE IN THIS AREA

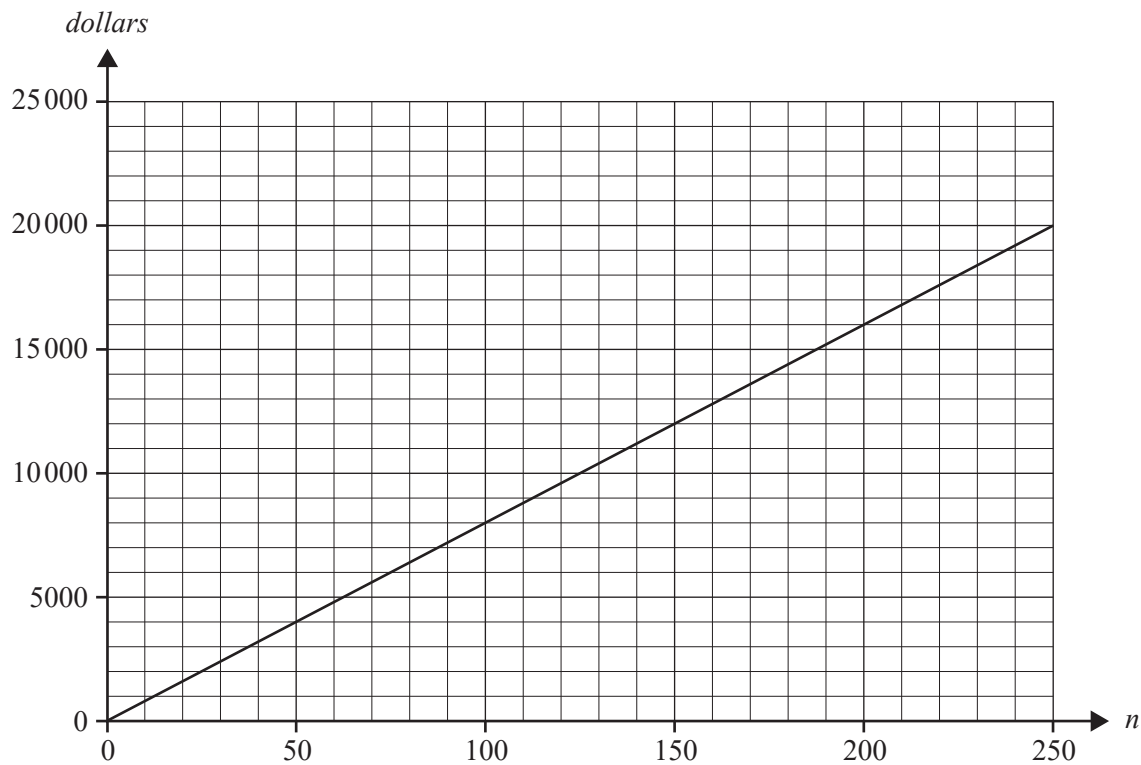
Question 3 (4 marks)

Kyla's business also manufactures car seat covers.

The *monthly revenue*, R , in dollars, from selling n seat covers is given by

$$R = 80n$$

This relationship is shown on the graph below.



The *monthly cost*, C , in dollars, of making n seat covers is given by

$$C = 36n + 5000$$

- a. Sketch the *monthly cost*, C , of making n seat covers on the **graph above**. 1 mark

(Answer on the graph above.)

- b. Find the least number of seat covers that need to be sold in order to make a profit. 1 mark

Last month, 180 seat covers were sold and the profit was \$2920.

Kyla believes that if she lowers the selling price of the seat covers she will sell 60 more seat covers this month.

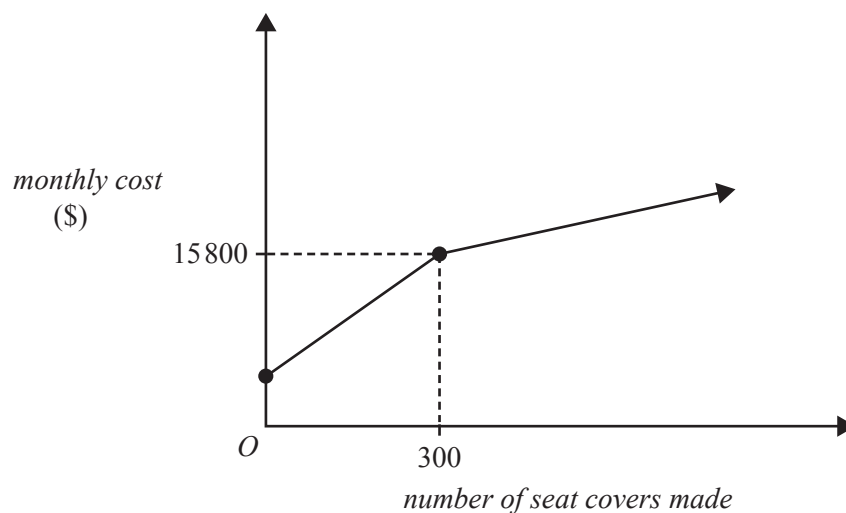
- c. If Kyla maintains the same profit of \$2920 this month, find the new selling price of the seat covers. 1 mark

- d. Kyla finds that the *monthly cost*, C , of making n seat covers changes when more than 300 seat covers are made.

The *monthly cost*, C , of making n seat covers is now given by

$$C = \begin{cases} 36n + 5000 & 0 \leq n \leq 300 \\ mn + p & n > 300 \end{cases}$$

This is represented on the graph below.



If 350 seat covers cost \$17 100 to make, find the value of m .

1 mark

DO NOT WRITE IN THIS AREA

Question 4 (5 marks)

Another section of Kyla's business services and details cars and trucks.

Every vehicle is both serviced and detailed.

Each car takes two hours to service and one hour to detail.

Each truck takes three hours to service and three hours to detail.

Let x represent the number of cars that are serviced and detailed each day.

Let y represent the number of trucks that are serviced and detailed each day.

Past records suggest there are constraints on the servicing and detailing of vehicles each day.

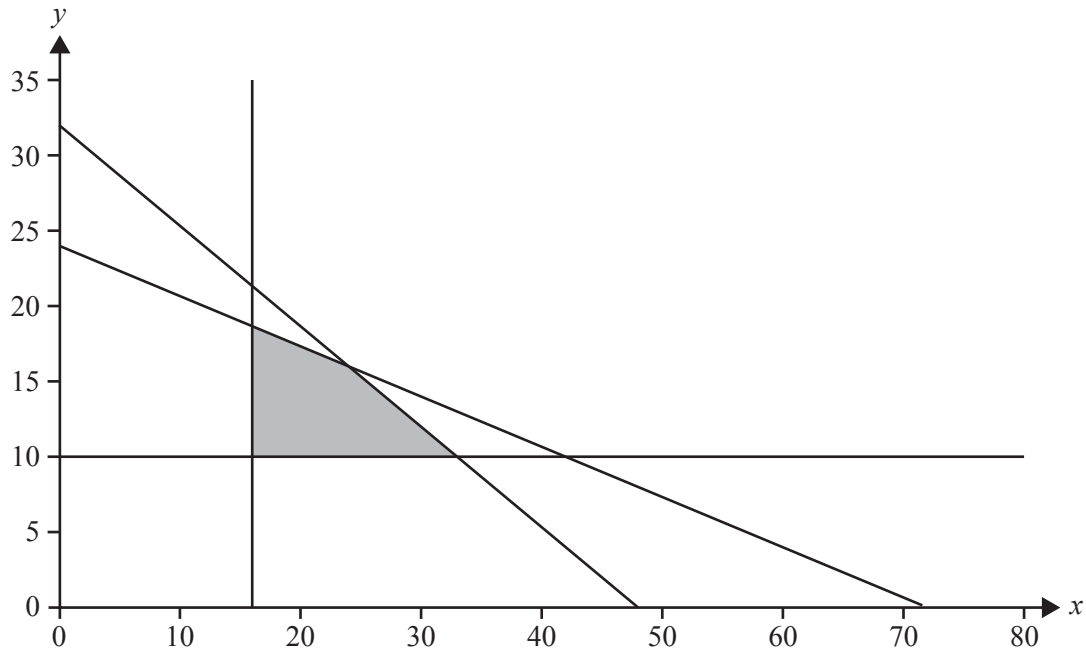
These constraints are represented by Inequalities 1 to 4 below.

Inequality 1	$x \geq 16$	
Inequality 2	$y \geq 10$	
Inequality 3	$2x + 3y \leq 96$	(servicing department)
Inequality 4	$x + 3y \leq 72$	(detailing department)

- a. Explain the meaning of Inequality 1 in the context of this problem. 1 mark

- b. Each employee at the business works eight hours per day.
What is the maximum number of employees who can work in the servicing department each day? 1 mark

The graph below shows the feasible region (shaded) that satisfies Inequalities 1 to 4.



- c. On a day when 20 cars are serviced and detailed, what is the maximum number of trucks that can be serviced and detailed?

1 mark

- d. When servicing and detailing, the business makes a profit of \$150 per car and \$225 per truck.

List the points within the feasible region that will result in a maximum profit for the day.

2 marks

DO NOT WRITE IN THIS AREA

END OF QUESTION AND ANSWER BOOK

**Victorian Certificate of Education
2020**

FURTHER MATHEMATICS

Written examination 2

FORMULA SHEET

Instructions

This formula sheet is provided for your reference.
A question and answer book is provided with this formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Further Mathematics formulas

Core – Data analysis

standardised score	$z = \frac{x - \bar{x}}{s_x}$
lower and upper fence in a boxplot	lower $Q_1 - 1.5 \times IQR$ upper $Q_3 + 1.5 \times IQR$
least squares line of best fit	$y = a + bx$, where $b = r \frac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$
residual value	residual value = actual value – predicted value
seasonal index	seasonal index = $\frac{\text{actual figure}}{\text{deseasonalised figure}}$

Core – Recursion and financial modelling

first-order linear recurrence relation	$u_0 = a, \quad u_{n+1} = bu_n + c$
effective rate of interest for a compound interest loan or investment	$r_{\text{effective}} = \left[\left(1 + \frac{r}{100n} \right)^n - 1 \right] \times 100\%$

Module 1 – Matrices

determinant of a 2×2 matrix	$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \quad \det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
inverse of a 2×2 matrix	$A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}, \quad \text{where } \det A \neq 0$
recurrence relation	$S_0 = \text{initial state}, \quad S_{n+1} = TS_n + B$

Module 2 – Networks and decision mathematics

Euler's formula	$v + f = e + 2$
-----------------	-----------------

Module 3 – Geometry and measurement

area of a triangle	$A = \frac{1}{2}bc \sin(\theta^\circ)$
Heron's formula	$A = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{1}{2}(a+b+c)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$a^2 = b^2 + c^2 - 2bc \cos(A)$
circumference of a circle	$2\pi r$
length of an arc	$r \times \frac{\pi}{180} \times \theta^\circ$
area of a circle	πr^2
area of a sector	$\pi r^2 \times \frac{\theta^\circ}{360}$
volume of a sphere	$\frac{4}{3}\pi r^3$
surface area of a sphere	$4\pi r^2$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a prism	area of base \times height
volume of a pyramid	$\frac{1}{3} \times$ area of base \times height

Module 4 – Graphs and relations

gradient (slope) of a straight line	$m = \frac{y_2 - y_1}{x_2 - x_1}$
equation of a straight line	$y = mx + c$