VICTORIAN CURRICULUM
AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2022

FURTHER MATHEMATICS

Written examination 1

Thursday 26 May 2022

Reading time: 10.00 am to 10.15 am (15 minutes)
Writing time: 10.15 am to 11.45 am (1 hour 30 minutes)

MULTIPLE-CHOICE QUESTION BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of modules*	*Number of modules to be answered*	*Number of marks*
A – Core	24	24			24
B – Modules	32	16	4	2	16
					Total 40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question book of 38 pages
- Formula sheet
- Answer sheet for multiple-choice questions
- Working space is provided throughout the book.

Instructions
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

At the end of the examination
- You may keep this question book and the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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SECTION A – Core

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1; an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data analysis

Question 1

Data relating to the following four variables was recorded in a study of the properties of meteor craters:

- *location* (Northern Hemisphere, Southern Hemisphere)
- *country*
- *diameter* (in kilometres)
- *age* (in millions of years)

The number of these variables that are categorical variables is

A. 0

B. 1

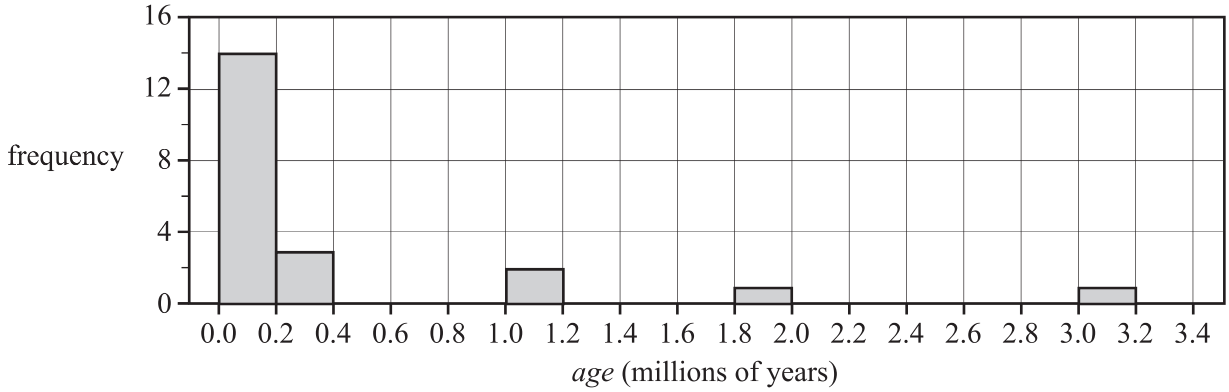
C. 2

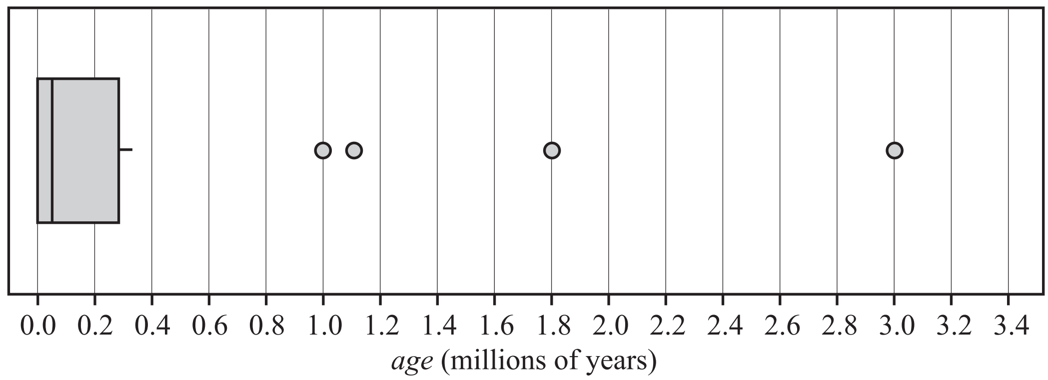
D. 3

E. 4

Question 2

The distribution of crater *age*, in millions of years, is displayed below as a histogram and a boxplot.





Data: Planetary and Space Science Centre, Earth Impact Database, <https://passc.net/EarthImpactDatabase/>

The shape of the distribution of the variable *age* is best described as

A. negatively skewed.

B. positively skewed.

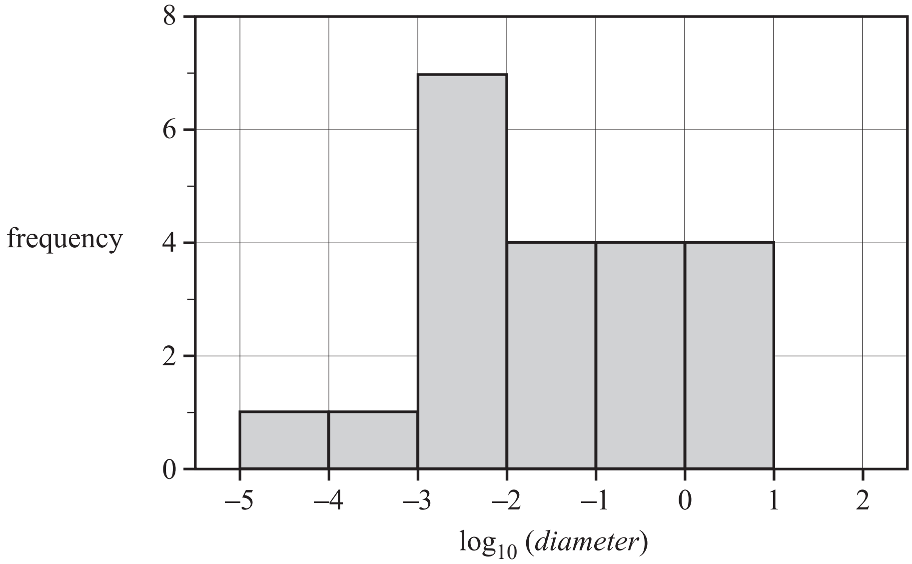
C. negatively skewed with four outliers.

D. positively skewed with four outliers.

E. approximately symmetric with four outliers.

Use the following information to answer Questions 3 and 4.

The histogram below shows the distribution of the *diameter* of craters, in kilometres, plotted on a $\log_{10}$ scale.



Data: Planetary and Space Science Centre, Earth Impact Database, <https://passc.net/EarthImpactDatabase/>

Question 3

The percentage of craters in this sample with a *diameter* less than 1 km is closest to

A. 4%
B. 19%
C. 81%
D. 85%
E. 100%

Question 4

The median *diameter*, in kilometres, of this sample of craters is between

A. 0.001 and 0.01
B. 0.01 and 0.1
C. 0.1 and 1
D. 1 and 10
E. 10 and 100

Use the following information to answer Questions 5 and 6.

The data in the table below was collected in a study of the association between the *delivery time* of parcels and the *delivery distance*, in kilometres.

Delivery time	***Delivery distance***		
	less than 175 km	**175 to 250 km**	**more than 250 km**
one day	15	10	5
two days	10	10	10
three or more days	5	5	20

Question 5

In this study, the percentage of parcels that had a *delivery time* of three or more days was closest to

A. 30%
B. 33%
C. 40%
D. 44%
E. 90%

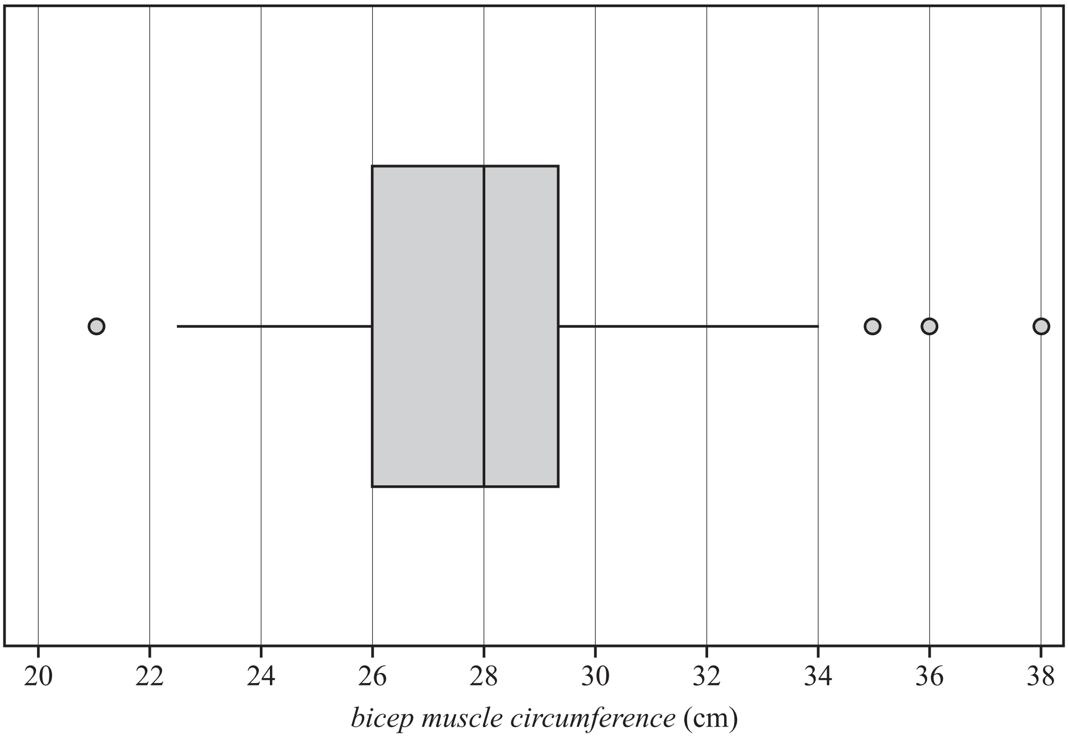
Question 6

To investigate the association between the *delivery time* of parcels and the *delivery distance*, the most appropriate way to graphically display the data in the table above is as a

A. histogram.
B. parallel boxplot.
C. time series plot.
D. segmented bar chart.
E. back-to-back stem plot.

Use the following information to answer Questions 7–10.

The boxplot below shows the distribution of *bicep muscle circumference*, in centimetres, for a sample of 44 people.



Question 7

The number of people from the sample with a *bicep muscle circumference* greater than 22 cm is

A. 1
B. 22
C. 33
D. 43
E. 44

Question 8

The percentage of people from the sample with a *bicep muscle circumference* between 26 cm and 34 cm is closest to

A. 25%
B. 36%
C. 50%
D. 68%
E. 75%

Question 9

The mean *bicep muscle circumference* for the 44 people in the sample is 28.16 cm.

The people whose *bicep muscle circumference* is an outlier are excluded and the mean is recalculated.

This new mean *bicep muscle circumference*, in centimetres, will be closest to

A. 24.68
B. 27.73
C. 27.95
D. 28.17
E. 28.27

Question 10

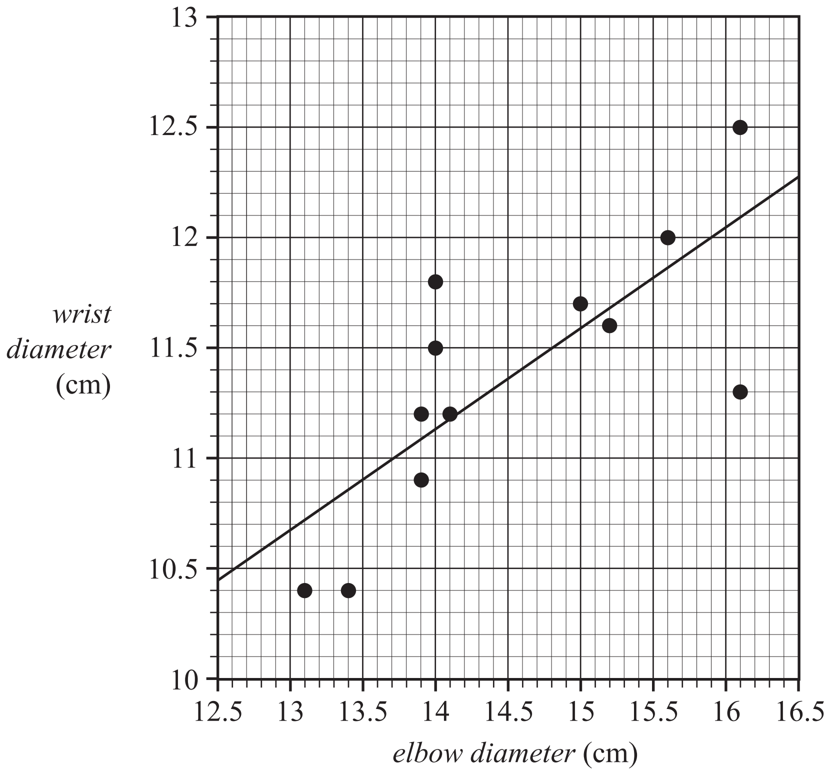
A final check of the data revealed that the value of an outlier of 20 cm was incorrectly recorded as 21 cm.

If the value of the outlier is corrected to 20 cm, the only statistic for which the value will change is the

A. median.
B. first quartile.
C. third quartile.
D. interquartile range.
E. range.

Use the following information to answer Questions 11 and 12.

Competitors in a sporting event measured their *wrist diameter* and *elbow diameter* in centimetres. The results are shown in the scatterplot below. A least squares line has been fitted to the scatterplot. *Wrist diameter* is the response variable.



Question 11

When the least squares line is used to predict the *wrist diameter*, in centimetres, of a competitor with an *elbow diameter* of 13.4 cm, the residual, in centimetres, is closest to

A. –4.5
B. –0.5
C. 0.5
D. 4.5
E. 10.4

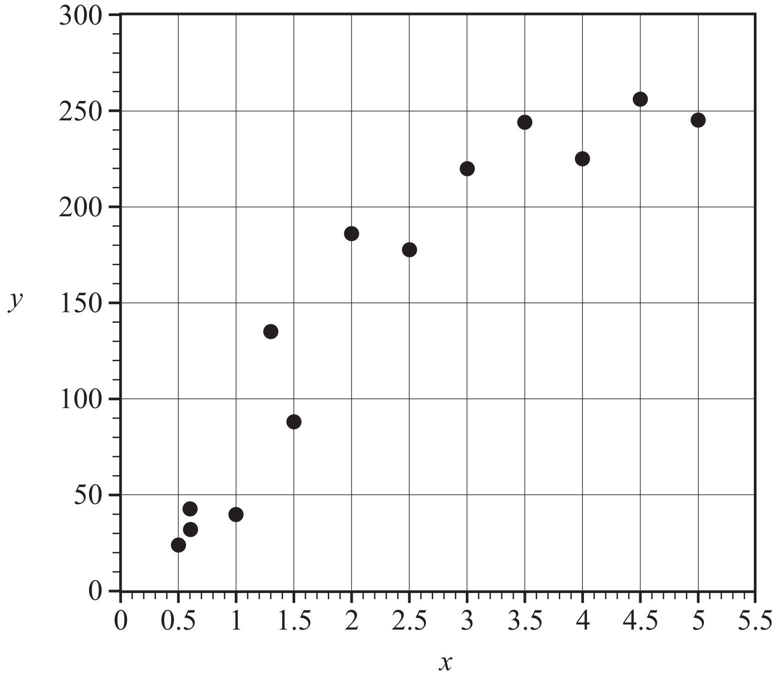
Question 12

The equation of the least squares line is closest to

A. *wrist diameter* = –10.4 + 2.19 × *elbow diameter*
B. *wrist diameter* = 3.89 + 0.46 × *elbow diameter*
C. *wrist diameter* = 4.76 + 0.46 × *elbow diameter*
D. *wrist diameter* = 10.5 + 0.46 × *elbow diameter*
E. *wrist diameter* = 10.4 + 2.19 × *elbow diameter*

Question 13

The scatterplot below is non-linear.



To linearise this scatterplot, it would be best to plot

A. $\log_{10} y$ against x

B. $\dfrac{1}{y}$ against x

C. y against x

D. y against x^2

E. y against $\dfrac{1}{x}$

Question 14

The seasonal index for smart television sales in June is 1.25

This means that smart television sales in June are

A. 80% of the average monthly sales of smart televisions.

B. 80% less than the average monthly sales of smart televisions.

C. the same as the average monthly sales of smart televisions in other months.

D. 20% more than the average monthly sales of smart televisions.

E. 25% more than the average monthly sales of smart televisions.

Question 15

The table below shows the daily sales, in dollars, of a fast-food restaurant for the first week of May 2021.

Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Daily sales ($)	13 024	9897	11 234	12 678	14 978	12 354	8978

Using two-mean smoothing with centring, the smoothed sales value for Wednesday is closest to

A. $11 261

B. $11 461

C. $11 956

D. $12 179

E. $12 851

Question 16

The table below shows the long-term average visitor numbers for a regional art gallery for quarters 1–3 and the seasonal index for quarter 3.

The long-term average visitor numbers for quarter 4 and the seasonal index for each of quarters 1, 2 and 4 are missing.

	Quarter			
	1	**2**	**3**	**4**
Long-term average visitor numbers	12 623	10 815	11 467	
Seasonal index			0.9569	

The seasonal index for quarter 1 is closest to

A. 0.87

B. 0.96

C. 0.98

D. 1.02

E. 1.05

Recursion and financial modelling

Question 17

Steve invested $4000 in a savings account that pays 2% per annum simple interest.

After how many years will the savings account have earned a total of $400 in interest?

A. 4
B. 5
C. 6
D. 8
E. 10

Question 18

Alana has purchased a new camera.

Alana depreciates the value of her camera using the unit cost method of depreciation.

The recurrence relation below models the value of her camera, in dollars, V_n, after n hours of use.

$$V_0 = 9000, \qquad V_{n+1} = V_n - 30$$

Alana intends to replace the camera when its value is 10% of the purchase price.

If she does this, the total number of hours that Alana will use her camera is

A. 30
B. 180
C. 210
D. 240
E. 270

Question 19

Which one of the following recurrence relations generates a sequence in which values grow geometrically?

A. $M_0 = 4000, \quad M_{n+1} = 0.8\, M_n$
B. $M_0 = 4000, \quad M_{n+1} = 1.2\, M_n$
C. $M_0 = 4000, \quad M_{n+1} = 0.8\, M_n + 100$
D. $M_0 = 4000, \quad M_{n+1} = M_n + 100$
E. $M_0 = 4000, \quad M_{n+1} = 1.2\, M_n + 100$

Question 20

James invests \$30 000 in a savings account.

This account earns interest at the rate of 4.2% per annum, compounding monthly.

Let V_n represent the balance of this account after n months.

Which one of the following is a correct recursive calculation for this account?

A. $V_1 = 1.042 \times 30000$

B. $V_2 = 1.0042 \times 30105$

C. $V_3 = 1.035 \times 30210.37$

D. $V_4 = 1.0035 \times 30316.10$

E. $V_5 = 1.00035 \times 30422.21$

Question 21

The annual interest rate for Mustafa's reducing balance loan is 2.4%.

Interest is calculated immediately before each payment.

Four lines of the amortisation table for Mustafa's loan are shown below.

Payment number	Payment (\$)	Interest (\$)	Principal reduction (\$)	Balance (\$)
0	0.00	0.00	0.00	100 000.00
1	720.00	600.00	120.00	99 880.00
2	720.00	599.28	120.72	99 759.28
3	720.00	598.56	121.44	99 637.84

Mustafa made his first payment on 1 January 2021.

He made his second payment on

A. 8 January 2021

B. 15 January 2021

C. 1 February 2021

D. 1 April 2021

E. 1 January 2022

Question 22

The balance of a perpetuity, in dollars, from one time period to the next can be represented by a recurrence relation of the form

$$V_0 = a, \qquad V_{n+1} = 1.004V_n - b$$

where V_n is the balance of the perpetuity after n time periods.

Which of the following could **not** be values for a and b?

A. $a = 50\,000$, $b = 200$

B. $a = 60\,000$, $b = 240$

C. $a = 200\,000$, $b = 800$

D. $a = 350\,000$, $b = 1500$

E. $a = 1\,000\,000$, $b = 4000$

Question 23

Bibi took out a loan of $3000 from her bank.

Interest for this loan compounded monthly.

In the first year, Bibi made no repayments.

The effective rate of interest for the first year was 4.18%.

The balance of Bibi's loan after one year was closest to

A. $3000.00

B. $3123.00

C. $3125.40

D. $3127.38

E. $3127.83

Question 24

Yashita took out a loan of $200 000, which was to be repaid with equal weekly repayments over 20 years.

The interest rate for this loan is 3.8% per annum, compounding weekly.

Yashita repaid the scheduled amount each week for 10 years.

After 10 years, she decided to add an additional $20 to each repayment to allow her to fully repay the loan in a shorter time period.

As a result of this change, the number of weeks required to repay the loan in full will be reduced by

A. 42

B. 47

C. 54

D. 56

E. 70

SECTION B – Modules

Instructions for Section B

Select **two** modules and answer **all** questions within the selected modules in pencil on the answer sheet provided for multiple-choice questions.

Show the modules you are answering by shading the matching boxes on your multiple-choice answer sheet **and** writing the name of the module in the box provided.

Choose the response that is **correct** for the question.

A correct answer scores 1; an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

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Contents **Page**

Module 1 – Matrices

Before answering these questions, you must **shade** the 'Matrices' box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

Consider the following matrices, M and N.

$$M = \begin{bmatrix} 47 & 96 & 52 & 75 \\ 29 & 63 & 25 & 82 \\ 77 & 97 & 17 & 34 \\ 66 & 89 & 57 & 19 \end{bmatrix} \qquad N = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Calculating the matrix product MN will produce a matrix that

A. sums the rows of matrix M.

B. is equal to matrix M.

C. is the transpose of matrix M.

D. sums the columns of matrix M.

E. adds 1 to every element of matrix M.

Question 2

If $H = \begin{bmatrix} 1 & 4 & 7 & 5 & 9 \\ 3 & 8 & 2 & 6 & 2 \\ 1 & 6 & 5 & 4 & 7 \\ 4 & 9 & 8 & 5 & 3 \end{bmatrix}$ and the matrix product $GH = \begin{bmatrix} 4 & 3 & 9 & 0 & 1 \\ 2 & 7 & 3 & 6 & 8 \\ 6 & 8 & 5 & 4 & 2 \end{bmatrix}$, then the order of matrix G is

A. 3×3

B. 3×4

C. 3×5

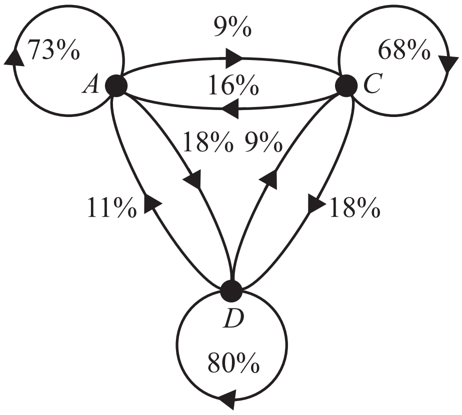
D. 4×3

E. 4×5

Question 3

The three most popular teas served in a cafe are Assam (A), Chinese (C) and Darjeeling (D).

Regular patrons each buy one pot of tea per day and choose the type of tea they will buy the next day according to the transition diagram below.



Which one of the following transition matrices correctly represents this transition diagram?

A.
$$\begin{array}{c} \quad today \\ \begin{array}{ccc} A & C & D \end{array} \\ T = \begin{bmatrix} 0.73 & 0.18 & 0.09 \\ 0.18 & 0.68 & 0.11 \\ 0.09 & 0.16 & 0.80 \end{bmatrix} \begin{array}{c} A \\ C \\ D \end{array} \quad tomorrow \end{array}$$

B.
$$\begin{array}{c} \quad today \\ \begin{array}{ccc} A & C & D \end{array} \\ T = \begin{bmatrix} 0.73 & 0.16 & 0.11 \\ 0.09 & 0.68 & 0.09 \\ 0.18 & 0.18 & 0.80 \end{bmatrix} \begin{array}{c} A \\ C \\ D \end{array} \quad tomorrow \end{array}$$

C.
$$\begin{array}{c} \quad today \\ \begin{array}{ccc} A & C & D \end{array} \\ T = \begin{bmatrix} 0.73 & 0.18 & 0.09 \\ 0.09 & 0.68 & 0.11 \\ 0.18 & 0.16 & 0.80 \end{bmatrix} \begin{array}{c} A \\ C \\ D \end{array} \quad tomorrow \end{array}$$

D.
$$\begin{array}{c} \quad today \\ \begin{array}{ccc} A & C & D \end{array} \\ T = \begin{bmatrix} 0.80 & 0.11 & 0.16 \\ 0.09 & 0.73 & 0.09 \\ 0.18 & 0.18 & 0.68 \end{bmatrix} \begin{array}{c} A \\ C \\ D \end{array} \quad tomorrow \end{array}$$

E.
$$\begin{array}{c} \quad today \\ \begin{array}{ccc} A & C & D \end{array} \\ T = \begin{bmatrix} 0.68 & 0.16 & 0.11 \\ 0.09 & 0.80 & 0.18 \\ 0.09 & 0.18 & 0.73 \end{bmatrix} \begin{array}{c} A \\ C \\ D \end{array} \quad tomorrow \end{array}$$

Question 4

Hailey will interview one international fashion model every day of the week from Monday to Friday.

The models are Johanna (*J*), Lian (*L*), Ngo (*N*), Phillipe (*P*) and Regan (*R*).

On Monday, Hailey will interview Regan.

Hailey will use the permutation matrix below to choose the model to be interviewed on each of the next four days.

$$\begin{array}{c} \textit{today} \\ \begin{array}{ccccc} J & L & N & P & R \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{l} \\ \\ J \\ L \\ N \quad \textit{tomorrow} \\ P \\ R \end{array}$$

The order in which Hailey will interview the remaining models for the next four days is

A. Johanna, Lian, Ngo, Phillipe.

B. Lian, Ngo, Phillipe, Johanna.

C. Lian, Ngo, Johanna, Phillipe.

D. Ngo, Johanna, Phillipe, Lian.

E. Phillipe, Johanna, Ngo, Lian.

Question 5

$$qx + 1.2y = 17$$
$$2.25x + 1.5y = 11$$

The set of simultaneous linear equations above does **not** have a unique solution if q has the value

A. -1.80

B. -0.80

C. 0

D. 0.80

E. 1.80

Question 6

Five friends, Bhavi (B), Kai (K), Oscar (O), Sian (S) and Xavier (X), played a round-robin table tennis tournament. Each friend played each of the others once. Every game had a winner and a loser.

The one-step dominance matrix constructed from the tournament's results is shown below.

$$\begin{array}{cc} & \textit{loser} \\ & \begin{array}{ccccc} B & K & O & S & X \end{array} \\ \textit{winner} \begin{array}{c} B \\ K \\ O \\ S \\ X \end{array} & \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

A '1' in this matrix shows that the player named in that row defeated the player named in that column.

For example, the '1' in row 3, column 4 shows that Oscar defeated Sian.

Which one of the following tables shows the number of one-step and two-step dominances accumulated by each player in the tournament?

A.

Player	One-step dominances	Two-step dominances
Bhavi	2	2
Kai	3	3
Oscar	1	4
Sian	1	2
Xavier	3	6

B.

Player	One-step dominances	Two-step dominances
Bhavi	2	2
Kai	1	3
Oscar	1	1
Sian	3	4
Xavier	3	6

C.

Player	One-step dominances	Two-step dominances
Bhavi	2	2
Kai	1	3
Oscar	3	4
Sian	3	6
Xavier	1	2

D.

Player	One-step dominances	Two-step dominances
Bhavi	2	2
Kai	1	3
Oscar	3	4
Sian	1	1
Xavier	3	5

E.

Player	One-step dominances	Two-step dominances
Bhavi	2	2
Kai	1	3
Oscar	3	4
Sian	1	1
Xavier	3	6

Question 7

A population of nocturnal rodents was resettled on an isolated island and their behaviour was monitored for one week.

During the day, the rodents sheltered at one of four locations named *A*, *B*, *C* and *D*.

On the first day, 40% of the rodents were observed sheltering at location *A* and 20% were observed sheltering at each of the other locations.

The rodents were expected to change the location at which they sheltered each day according to transition matrix *T*.

$$\begin{array}{cc} & \textit{this day} \\ & \begin{array}{cccc} A & B & C & D \end{array} \\ T = & \begin{bmatrix} 1 & 0.15 & 0 & 0.35 \\ 0 & 0.55 & 0 & 0.15 \\ 0 & 0.10 & 1 & 0.10 \\ 0 & 0.20 & 0 & 0.40 \end{bmatrix} \begin{array}{c} A \\ B \\ C \\ D \end{array} \quad \textit{next day} \end{array}$$

On the second day, 1000 rodents were observed sheltering at location *A*.

The total number of rodents that were resettled on the isolated island on day one was

A. 500
B. 1000
C. 2000
D. 4000
E. 5000

Question 8

Devi, Eric and Fong work for a company that services security equipment for shopping centres.

The matrix M shows the time it takes, in minutes, for Devi (D), Eric (E) and Fong (F) to each service an alarm system (A), a break-in sensor (B) and a camera (C).

$$M = \begin{array}{c} \begin{array}{ccc} D & E & F \end{array} \\ \begin{bmatrix} 14 & 18 & 16 \\ 11 & 12 & 14 \\ 19 & 16 & 13 \end{bmatrix} \end{array} \begin{array}{c} \\ A \\ B \\ C \end{array}$$

The matrix N displays the number of alarm systems, break-in sensors and cameras to be serviced in the small suburban shopping centres of Tindale (T), Ulbridge (U) and Verten (V).

$$N = \begin{array}{c} \begin{array}{ccc} T & U & V \end{array} \\ \begin{bmatrix} 5 & 4 & 6 \\ 9 & 8 & 11 \\ 11 & 10 & 12 \end{bmatrix} \end{array} \begin{array}{c} \\ A \\ B \\ C \end{array}$$

Which one of the following matrices displays the time it would take each of Devi, Eric and Fong, working individually, to service all the alarm systems, break-in sensors and cameras in each of the three shopping centres?

A.
$$\begin{array}{c} \begin{array}{ccc} D & E & F \end{array} \\ \begin{bmatrix} 70 & 72 & 96 \\ 99 & 96 & 154 \\ 209 & 160 & 156 \end{bmatrix} \end{array} \begin{array}{c} \\ T \\ U \\ V \end{array}$$

B.
$$\begin{array}{c} \begin{array}{ccc} D & E & F \end{array} \\ \begin{bmatrix} 228 & 234 & 214 \\ 423 & 434 & 399 \\ 492 & 510 & 472 \end{bmatrix} \end{array} \begin{array}{c} \\ T \\ U \\ V \end{array}$$

C.
$$\begin{array}{c} \begin{array}{ccc} D & E & F \end{array} \\ \begin{bmatrix} 378 & 334 & 433 \\ 374 & 328 & 432 \\ 349 & 306 & 406 \end{bmatrix} \end{array} \begin{array}{c} \\ T \\ U \\ V \end{array}$$

D.
$$\begin{array}{c} \begin{array}{ccc} D & E & F \end{array} \\ \begin{bmatrix} 378 & 374 & 349 \\ 334 & 328 & 306 \\ 433 & 432 & 406 \end{bmatrix} \end{array} \begin{array}{c} \\ T \\ U \\ V \end{array}$$

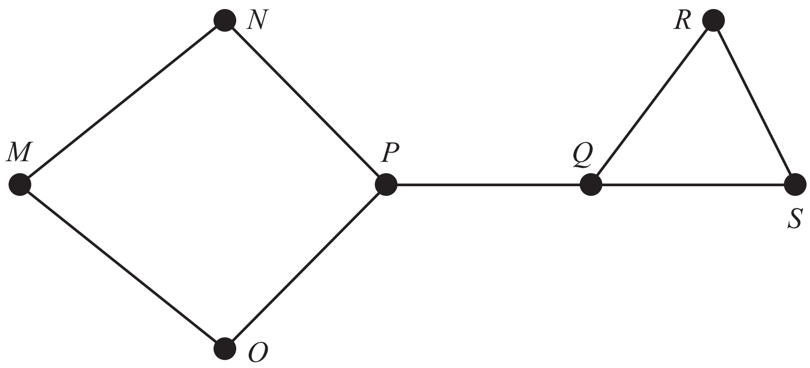
E.
$$\begin{array}{c} \begin{array}{ccc} D & E & F \end{array} \\ \begin{bmatrix} 408 & 360 & 474 \\ 317 & 280 & 366 \\ 382 & 334 & 446 \end{bmatrix} \end{array} \begin{array}{c} \\ T \\ U \\ V \end{array}$$

Module 2 – Networks and decision mathematics

Before answering these questions, you must **shade** the 'Networks and decision mathematics' box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

The graph below has seven vertices labelled M, N, O, P, Q, R and S, and eight edges.



Which one of the following edges is a bridge?

A. MN
B. NP
C. OP
D. PQ
E. QS

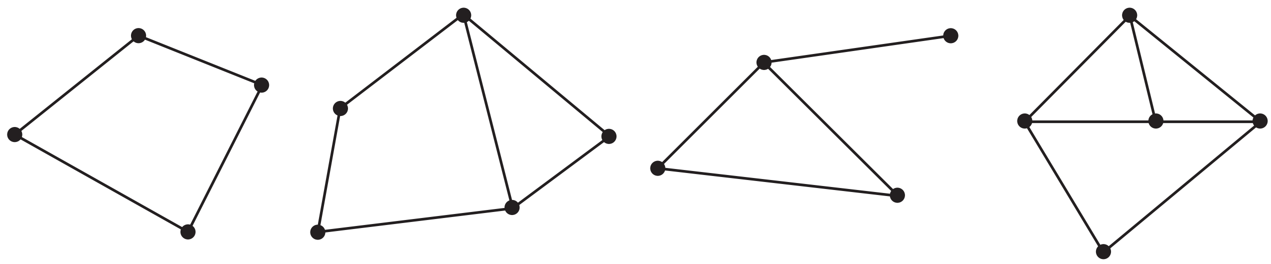
Question 2

A manufacturing business employs six different drivers to deliver their products to 10 different stores.

This delivery structure could be represented graphically by

A. a spanning tree.
B. an Eulerian trail.
C. a Hamiltonian path.
D. a complete graph.
E. a bipartite graph.

Question 3



How many of the graphs above have an Eulerian trail?

A. 0

B. 1

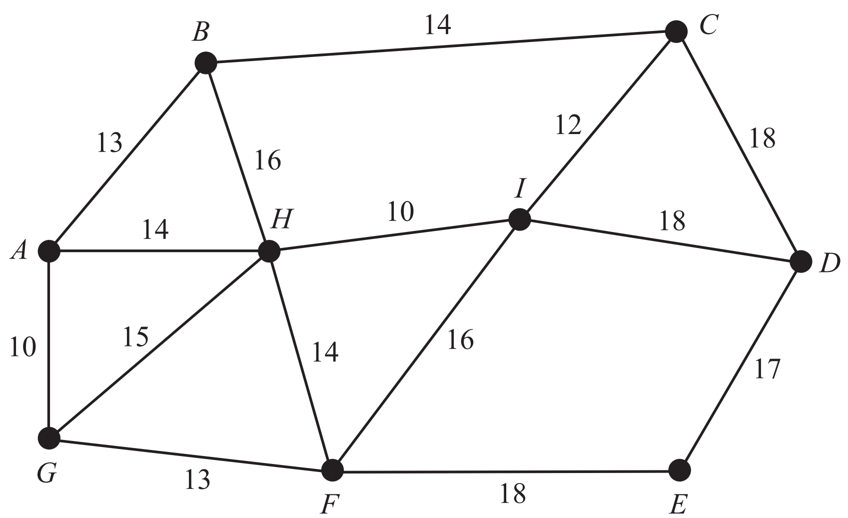
C. 2

D. 3

E. 4

Question 4

The diagram below shows the lengths, in metres, of cables connecting nine internet ports in the computer network of a business. The internet ports are labelled *A* to *I*.



The business is planning to replace the cables in the computer network.

The minimum length of cable, in metres, required to ensure that all nine internet ports stay connected is

A. 103

B. 107

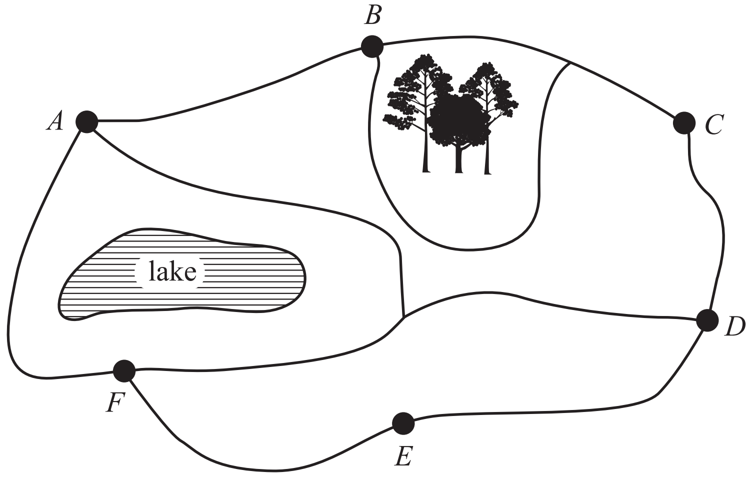
C. 109

D. 123

E. 125

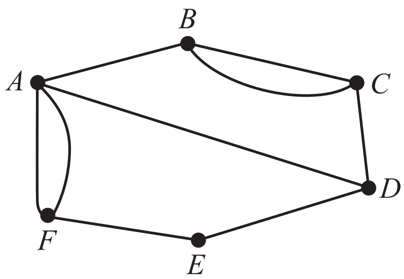
Question 5

The map below shows six camp sites, labelled A, B, C, D, E and F, which are joined by paths.

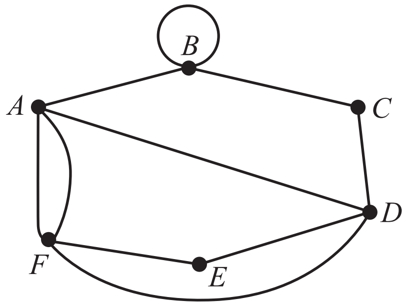


Which one of the following graphs could represent this map?

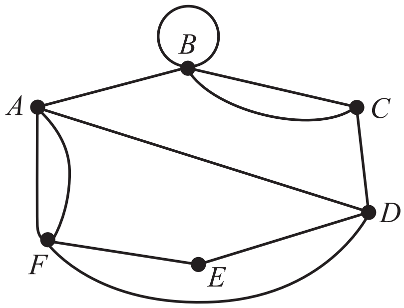
A.



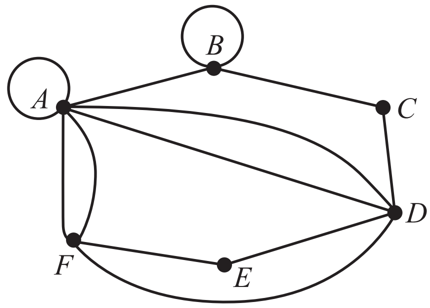
B.



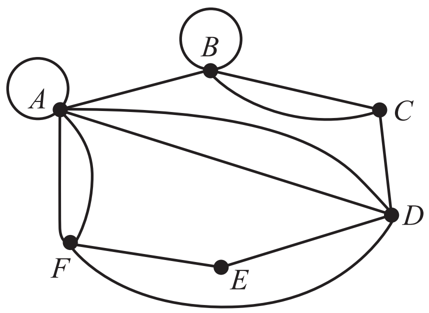
C.



D.



E.



Question 6

Five builders, Amida, Boris, Clea, Drew and Enzo, are working on a construction.

The construction has five components that must all be completed.

Each builder will be allocated only one component.

The table below shows the time, in hours, it would take each builder to complete each component of the construction.

	Amida	**Boris**	**Clea**	**Drew**	**Enzo**
1st component	6	6	5	7	7
2nd component	5	7	6	7	4
3rd component	8	5	7	5	9
4th component	7	7	8	6	6
5th component	4	4	4	5	5

The components of this construction must be completed in numerical order: 1st, 2nd, 3rd, 4th and 5th.

Each builder will be assigned to one component to ensure that construction is completed in the minimum time possible.

Which one of the following statements is **not** true?

A. Amida will complete her allocated component before Boris.

B. Boris will complete his allocated component after Clea.

C. Clea will complete her allocated component before Drew.

D. Drew will complete his allocated component after Enzo.

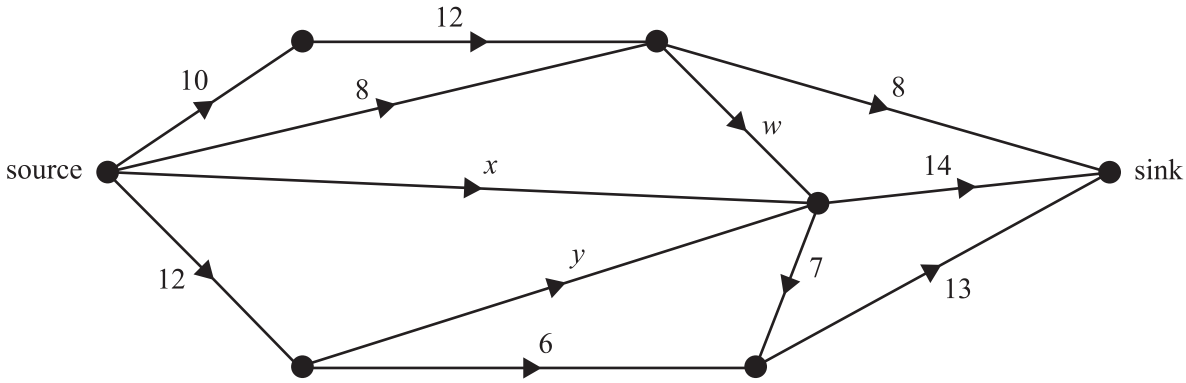
E. Enzo will complete his allocated component before Amida.

Question 7

The flow of oil through a series of pipes is shown in the network below.

The numbers on the edges show the capacity of each pipe in litres per minute.

Three edges have the capacities of w, x and y litres per minute.



The maximum flow through this directed network is 35 litres per minute.

This maximum flow can be achieved when

A. $w = 8$, $x = 6$, $y = 6$

B. $w = 10$, $x = 4$, $y = 7$

C. $w = 10$, $x = 6$, $y = 6$

D. $w = 12$, $x = 4$, $y = 7$

E. $w = 12$, $x = 4$, $y = 6$

Question 8

A project requires 11 activities, *A* to *K*.

The immediate predecessor(s) of each activity is shown in the table below.

Activity	**Immediate predecessor(s)**
A	–
B	–
C	–
D	*A*
E	*B*, *F*
F	*C*
G	*C*
H	*D*, *E*
I	*F*
J	*E*, *G*, *I*
K	*H*, *J*

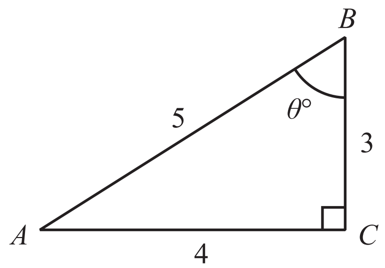
When a directed network for this project is drawn, the number of dummy activities required is

A. 1
B. 2
C. 3
D. 4
E. 5

Module 3 – Geometry and measurement

Before answering these questions, you must **shade** the 'Geometry and measurement' box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1



In triangle ABC, the value of $\tan(\theta°)$ is

A. $\frac{3}{4}$

B. $\frac{3}{5}$

C. $\frac{4}{3}$

D. $\frac{4}{5}$

E. $\frac{5}{3}$

Question 2

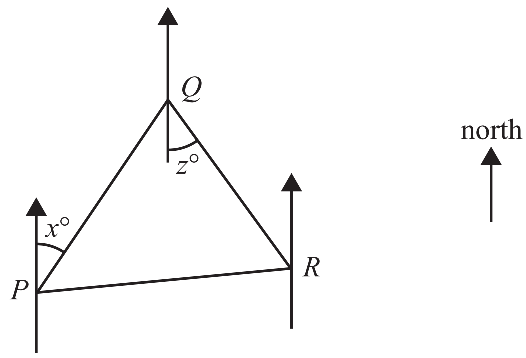
Null Island is a fictional location on Earth's surface at latitude 0° N and longitude 0° E.

Which one of the following locations is closest to Null Island?

A. Le Havre (49° N, 0° E)
B. Lleida (42° N, 1° E)
C. London (52° N, 0° W)
D. Tamale (9° N, 1° W)
E. Valencia (39° N, 0° W)

Question 3

The diagram below shows three locations: P, Q and R.



The bearing of P from Q can be found by calculating

A. $90 + z°$

B. $180 + x°$

C. $180 - x°$

D. $270 - z°$

E. $360 - x°$

Question 4

The town of Rotorua in New Zealand has latitude 38° S and longitude 176° E.

Assume that the radius of Earth is 6400 km.

The shortest distance along the surface of Earth between Rotorua and the South Pole, in kilometres, is closest to

A. 4245

B. 5808

C. 9606

D. 14 298

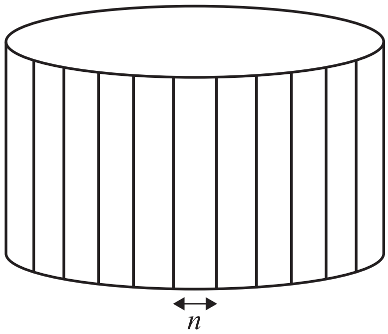
E. 19 659

Question 5

A cylindrical wooden barrel has a diameter of 78 cm.

The barrel is made from 49 planks of wood.

Each plank is the same size and is n centimetres wide, as shown in the diagram below.



The value of n, in centimetres, is closest to

A. 2

B. 4

C. 5

D. 8

E. 10

Question 6

A boat leaves a harbour and travels 3 km due east to Dangerous Reef.

It then travels 4 km to Green Reef on a bearing of 256°.

The boat then leaves Green Reef and returns to the harbour.

The direct distance between Green Reef and the harbour, in kilometres, is closest to

A. 1.31

B. 2.02

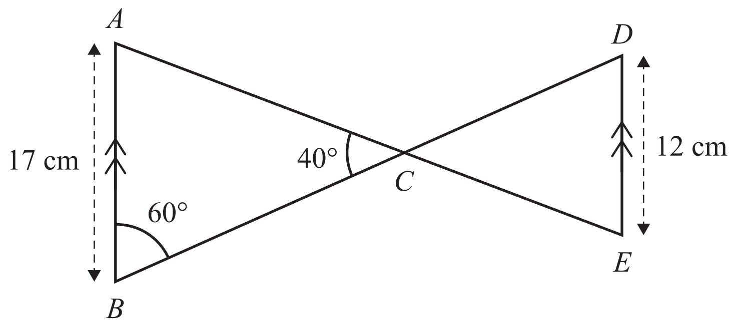
C. 4.38

D. 5.55

E. 6.95

Question 7

Triangle *ABC* is similar to triangle *EDC*, as shown in the diagram below.

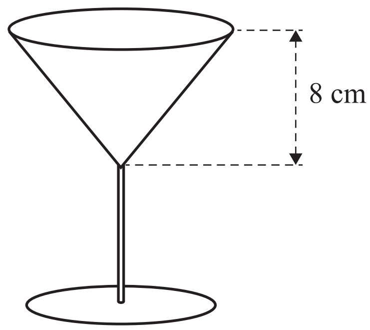


The length of *CD*, in centimetres, is closest to

A. 18.39
B. 22.68
C. 26.05
D. 32.44
E. 36.90

Question 8

The top of a glass is shaped like a cone.
The height of the cone is 8 cm, as shown in the diagram below.



Liquid is poured into the glass and fills two-thirds of the volume of the cone.
The height of the liquid inside the cone, in centimetres, is closest to

A. 2.37
B. 4.67
C. 5.33
D. 7.00
E. 8.00

CONTINUES OVER PAGE

Module 4 – Graphs and relations

Before answering these questions, you must **shade** the 'Graphs and relations' box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

The equation of a line that passes through the points (0, 5) and (2, 5) is

A. $x = 5$

B. $y = 5$

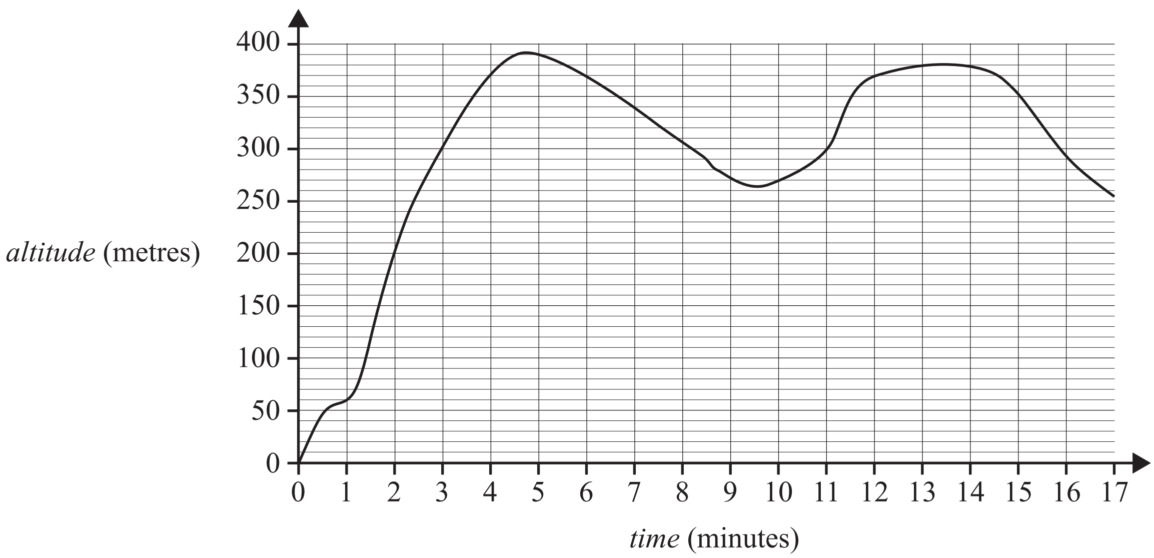
C. $y = 5x$

D. $y = 5x + 2$

E. $y = 2x + 5$

Use the following information to answer Questions 2 and 3.

The graph below shows the *altitude* of a hot air balloon over a 17-minute period.



Question 2

The total *time*, in minutes, that the hot air balloon has an *altitude* of at least 300 m is closest to

A. 3

B. 5

C. 9

D. 10

E. 16

Question 3

The average speed, in metres per minute, at which the hot air balloon descends during the period from five minutes to nine minutes is closest to

A. 25
B. 30
C. 35
D. 40
E. 50

Question 4

A bag shop has different prices for its small and large handbags.

The total cost for one small handbag and two large handbags is $385.

The cost of a large handbag is three times the cost of a small handbag.

The cost of a large handbag is

A. $55
B. $77
C. $165
D. $220
E. $231

Question 5

Anna makes and sells fruit smoothies.

The revenue, in dollars, made from selling 100 smoothies is $750.

The cost, C, in dollars, to make n smoothies each week is given by

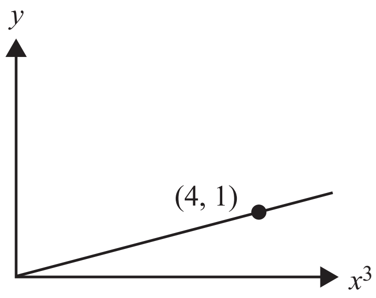
$$C = 800 + 3n$$

To make a profit of at least $2000, the minimum number of smoothies that Anna must sell each week is

A. 267
B. 400
C. 622
D. 623
E. 667

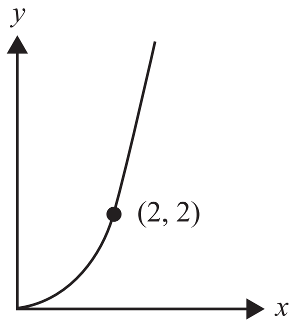
Question 6

The graph below shows a relationship between y and x^3.

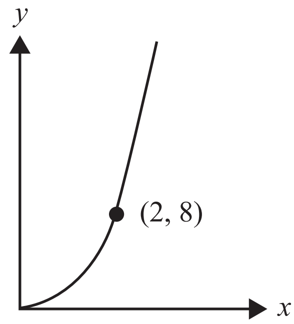


Which one of the following graphs shows this relationship between y and x?

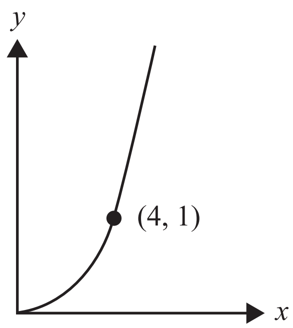
A.



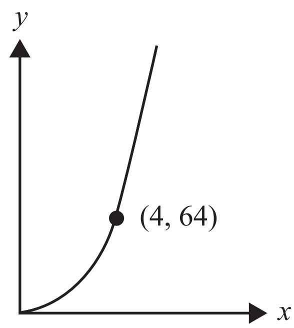
B.



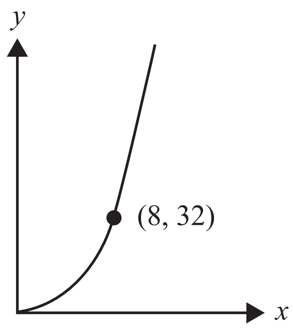
C.



D.



E.

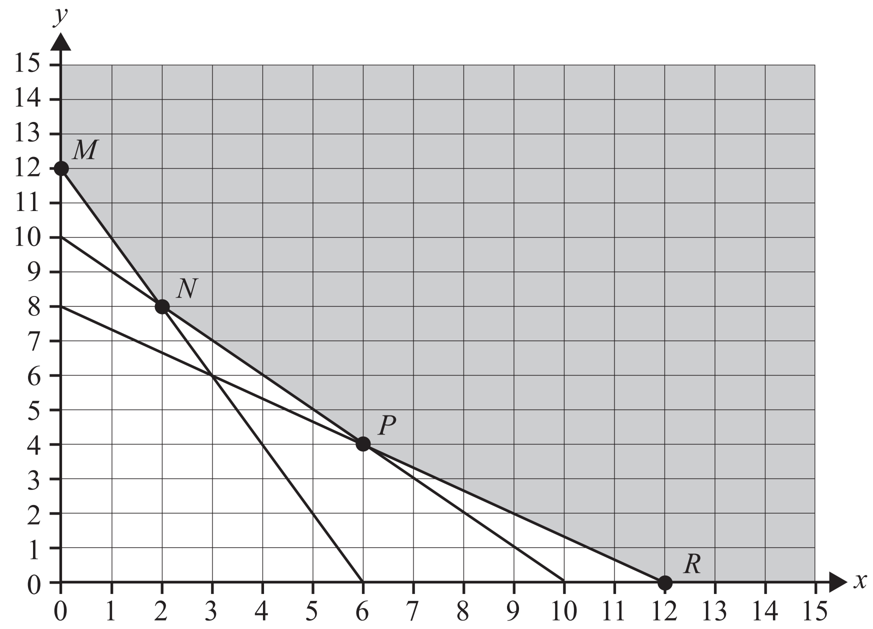


Question 7

The constraints of a linear programming problem are represented by the following set of inequalities.

$$x \geq 0$$
$$y \geq 0$$
$$x + y \geq 10$$
$$2x + y \geq 12$$
$$2x + 3y \geq 24$$

The feasible region for this problem is shaded in the diagram below.



The objective function for this problem is $Z = 2.4x + 3.6y$.

The minimum value of the objective function will occur at

A. point N only.

B. point P only.

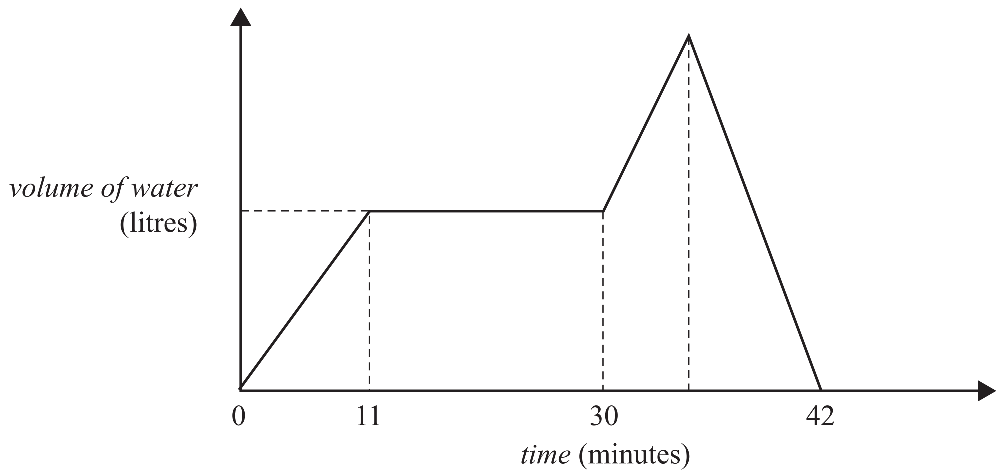
C. points M, N, P and R.

D. any point along line segment $\overline{NP}$.

E. any point along line segment $\overline{PR}$.

Question 8

The graph below shows the *volume of water*, in litres, in a tank over a period of *time*, in minutes.



Initially, the *volume of water* in the tank increased at a rate of 4 litres per minute for 11 minutes.

The volume did not change for the next 19 minutes.

The volume then increased at a rate of 8 litres per minute for some time.

Finally, the *volume of water* in the tank decreased at a rate of 12 litres per minute until the tank was empty.

The whole process took 42 minutes.

The maximum volume of water, in litres, in the tank during the 42 minutes was

A. 84
B. 88
C. 92
D. 96
E. 108

END OF MULTIPLE-CHOICE QUESTION BOOK

VICTORIAN CURRICULUM
AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2022

FURTHER MATHEMATICS

Written examination 1

FORMULA SHEET

Instructions

This formula sheet is provided for your reference.
A multiple-choice question book is provided with this formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Further Mathematics formulas

Core – Data analysis

standardised score	$z = \dfrac{x - \bar{x}}{s_x}$
lower and upper fence in a boxplot	lower $Q_1 - 1.5 \times IQR$ upper $Q_3 + 1.5 \times IQR$
least squares line of best fit	$y = a + bx$, where $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$
residual value	residual value = actual value – predicted value
seasonal index	seasonal index = $\dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

Core – Recursion and financial modelling

first-order linear recurrence relation	$u_0 = a$, $u_{n+1} = bu_n + c$
effective rate of interest for a compound interest loan or investment	$r_{effective} = \left[\left(1 + \dfrac{r}{100n}\right)^n - 1\right] \times 100\%$

Module 1 – Matrices

determinant of a 2 × 2 matrix	$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
inverse of a 2 × 2 matrix	$A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $\det A \neq 0$
recurrence relation	S_0 = initial state, $S_{n+1} = T\,S_n + B$

Module 2 – Networks and decision mathematics

Euler's formula	$v + f = e + 2$

Module 3 – Geometry and measurement

area of a triangle	$A = \frac{1}{2}bc\sin(\theta^\circ)$
Heron's formula	$A = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{1}{2}(a+b+c)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$a^2 = b^2 + c^2 - 2bc\cos(A)$
circumference of a circle	$2\pi r$
length of an arc	$r \times \frac{\pi}{180} \times \theta^\circ$
area of a circle	πr^2
area of a sector	$\pi r^2 \times \frac{\theta^\circ}{360}$
volume of a sphere	$\frac{4}{3}\pi r^3$
surface area of a sphere	$4\pi r^2$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a prism	area of base × height
volume of a pyramid	$\frac{1}{3}$ × area of base × height

Module 4 – Graphs and relations

gradient (slope) of a straight line	$m = \frac{y_2 - y_1}{x_2 - x_1}$
equation of a straight line	$y = mx + c$

END OF FORMULA SHEET