VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education 2022

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER | | | | | | | | | Letter

FURTHER MATHEMATICS

Written examination 2

Friday 27 May 2022

Reading time: 10.00 am to 10.15 am (15 minutes)
Writing time: 10.15 am to 11.45 am (1 hour 30 minutes)

QUESTION AND ANSWER BOOK

Structure of book

Section A – Core	*Number of questions*	*Number of questions to be answered*	*Number of marks*
	8	8	36
Section B – Modules	*Number of modules*	*Number of modules to be answered*	*Number of marks*
	4	2	24
			Total 60

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 37 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A – Core

Instructions for Section A

Answer **all** questions in the spaces provided.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.

In 'Recursion and financial modelling', all answers should be rounded to the nearest cent unless otherwise instructed.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

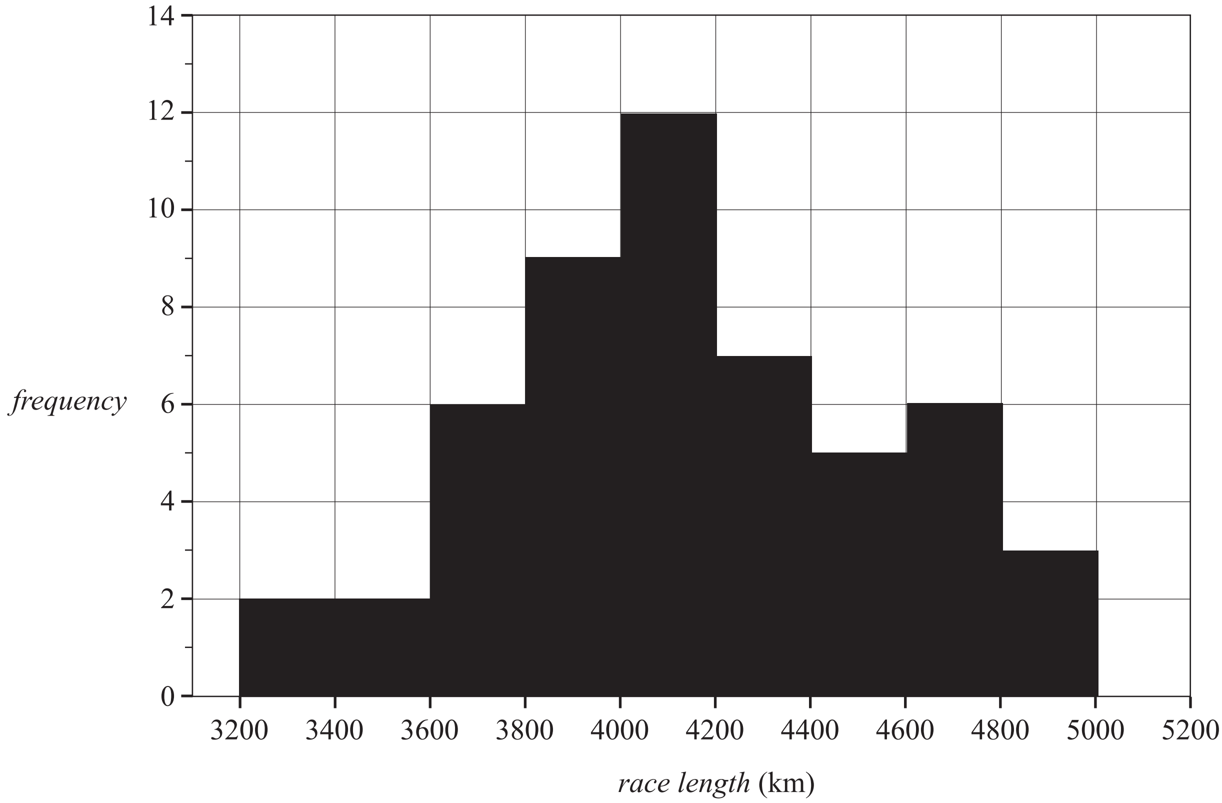
Data analysis

Question 1 (4 marks)

The Tour de France is the most famous road bike race in the world.

The *race length* of the Tour de France varies from year to year.

The histogram below shows the distribution of *race length*, in kilometres, for the years 1947–1998.



Data: 'Tour de France winners, podium, times', BikeRaceInfo website, <https://bikeraceinfo.com>

a. Calculate the number of years that the *race length* was between 4200 km and 4800 km. 1 mark

b. The distribution of *race length* can be approximated by a normal distribution with a mean of 4000 km and a standard deviation of 350 km.

Using the 68–95–99.7% rule, determine the percentage of years that the *race length* was less than 3300 km. 1 mark

c. The ages of the riders in the Tour de France are also approximately normally distributed.

A recent survey found that:

- 16% of the riders were under the age of 25.5 years
- 2.5% of the riders were over the age of 33 years.

Using the 68–95–99.7% rule, find the mean and the standard deviation of the normal distribution modelling this data. 2 marks

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Question 2 (8 marks)

The table below shows data for nine variables from the Tour de France for the years 1903–1912.

Year	*Winner*	*Country*	*Total time* (hours)	*Average speed* (km/h)	*Number of stages*	*Race length* (km)	*Number of starters*	*Number of finishers*
1903	Maurice Garin	France	94.550000	25.679	6	2428	60	21
1904	Henri Cornet	France	96.098889	24.849	6	2388	88	27
1905	Louis Trousselier	France	110.44944	27.107	11	2994	60	24
1906	René Pottier	France	189.56667	24.463	13	4637	82	14
1907	Lucien Petit-Breton	France	158.75139	28.470	14	4488	93	33
1908	Lucien Petit-Breton	France	156.89139	28.740	14	4488	112	36
1909	Francois Faber	Luxembourg	157.02278	28.658	14	4497	150	55
1910	Octave Lapize	France	162.69167	29.099	15	4737	110	41
1911	Gustave Garrigou	France	195.61667	27.322	15	5344	84	28
1912	Odile Defraye	Belgium	190.50778	27.763	15	5289	131	41

Data: 'Tour de France winners, podium, times', BikeRaceInfo website, <https://bikeraceinfo.com>

a. Write down the number of categorical variables in this data set. 1 mark

b. Determine, for the period of years 1903–1912

i. the median *number of stages* 1 mark

ii. the mean *race length*, in kilometres 1 mark

iii. the correlation coefficient (r) between the variables *number of starters* and *number of finishers*.
Round your answer to three decimal places. 1 mark

c. In 1904, Henri Cornet won the Tour de France with a total time of 96.098889 hours.

Write down this time rounded to five significant figures. 1 mark

d. A least squares line is to be fitted to the data so that the *number of finishers* can be predicted from the *number of starters*. The equation of this least squares line is

$$\textit{number of finishers} = -3.384 + 0.3648 \times \textit{number of starters}$$

i. Interpret the slope of this least squares line in terms of the *number of finishers* and the *number of starters* in the Tour de France. 1 mark

ii. In 1910, 110 riders started the race and 41 of these riders finished the race.

Determine the prediction error (residual) if the least squares line had been used to predict the *number of finishers* in 1910.
Round your answer to the nearest whole number. 2 marks

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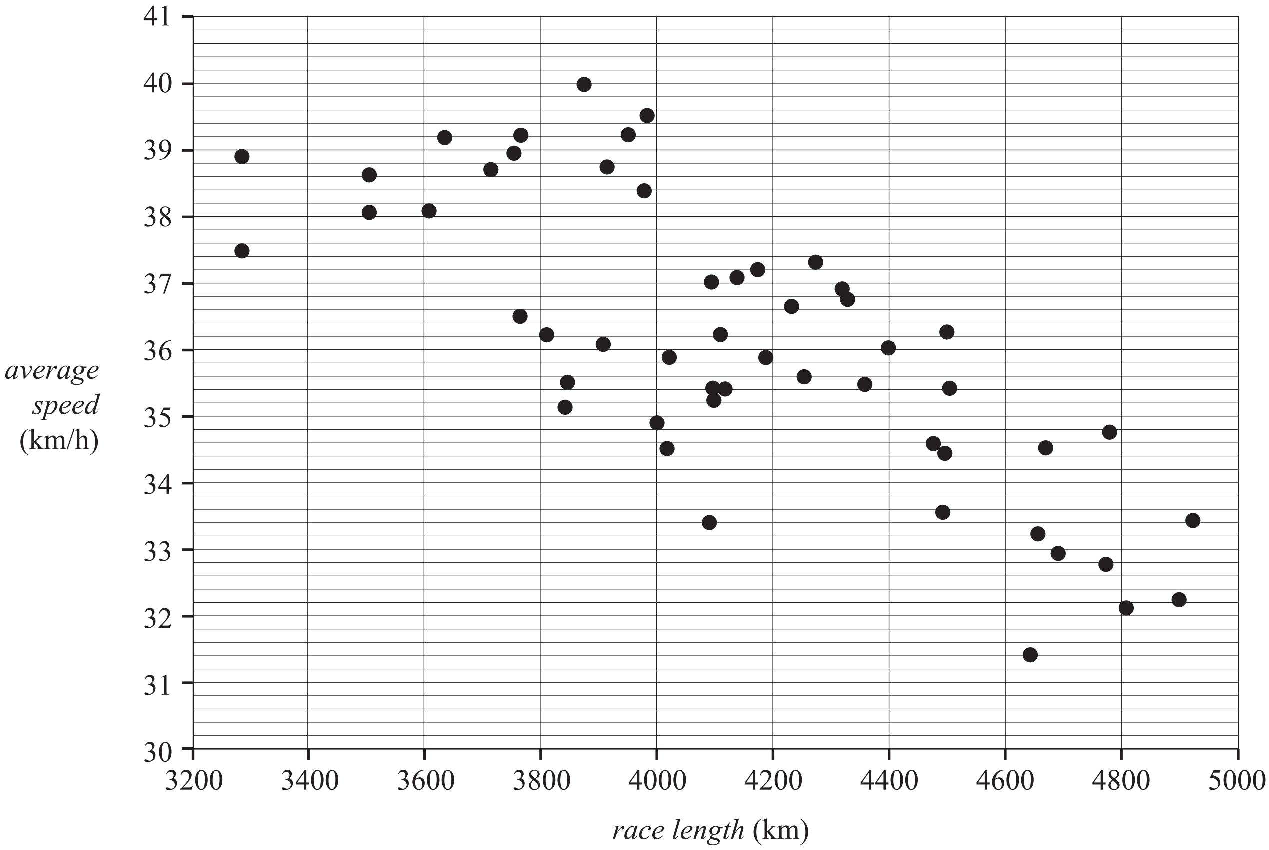
Question 3 (8 marks)

The scatterplot below shows the *average speed*, in kilometres per hour, plotted against the *race length*, in kilometres, for the winners of the Tour de France from 1947 to 1998.



Data: 'Tour de France winners, podium, times', BikeRaceInfo website, <https://bikeraceinfo.com>

A least squares line is to be fitted to the scatterplot. The equation of the least squares line is

average speed = 52.689 – 0.00400 × *race length*

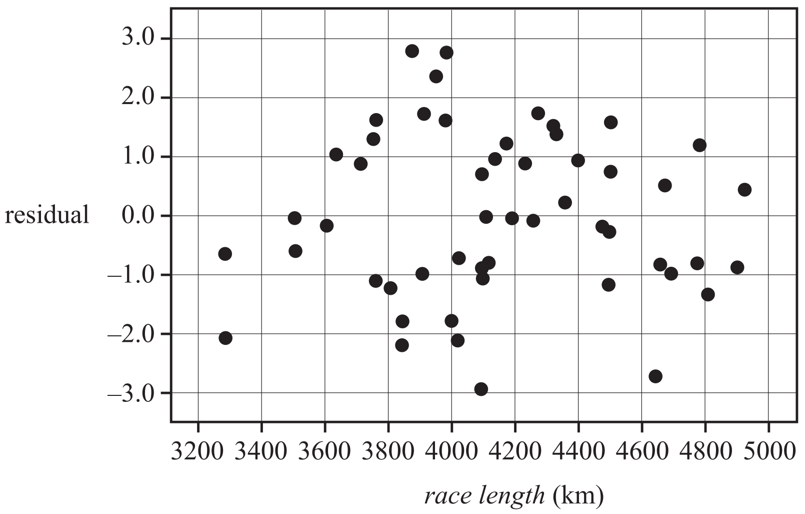
a. Draw the graph of the least squares line on the **scatterplot above**. 1 mark

(*Answer on the scatterplot above.*)

b. If the *race length* is 4400 km, predict the *average speed*, in kilometres per hour, of the winner of the race.
Round your answer to one decimal place. 1 mark

c. Is the prediction made in **part b.** an interpolation or an extrapolation? 1 mark

d. The residual plot associated with fitting a least squares line to this data is shown below.



i. The residual plot can be used to test one of the assumptions about the nature of the association between *average speed* and *race length*.

What is this assumption? 1 mark

ii. The residual plot supports this assumption.

Explain why. 1 mark

The mean and the standard deviation of both the *average speed* and the *race length* for the Tour de France from 1947 to 1998 were also calculated. This data is shown in the table below.

	Mean	**Standard deviation**
Average speed **(km/h)**	36.101	2.1389
Race length **(km)**	4144.9	406.08

e. In 1971, the *race length* of the Tour de France was 3608 km and the *average speed* of the winner was 38.084 km/h.

Determine the standardised value (z) of this average speed.

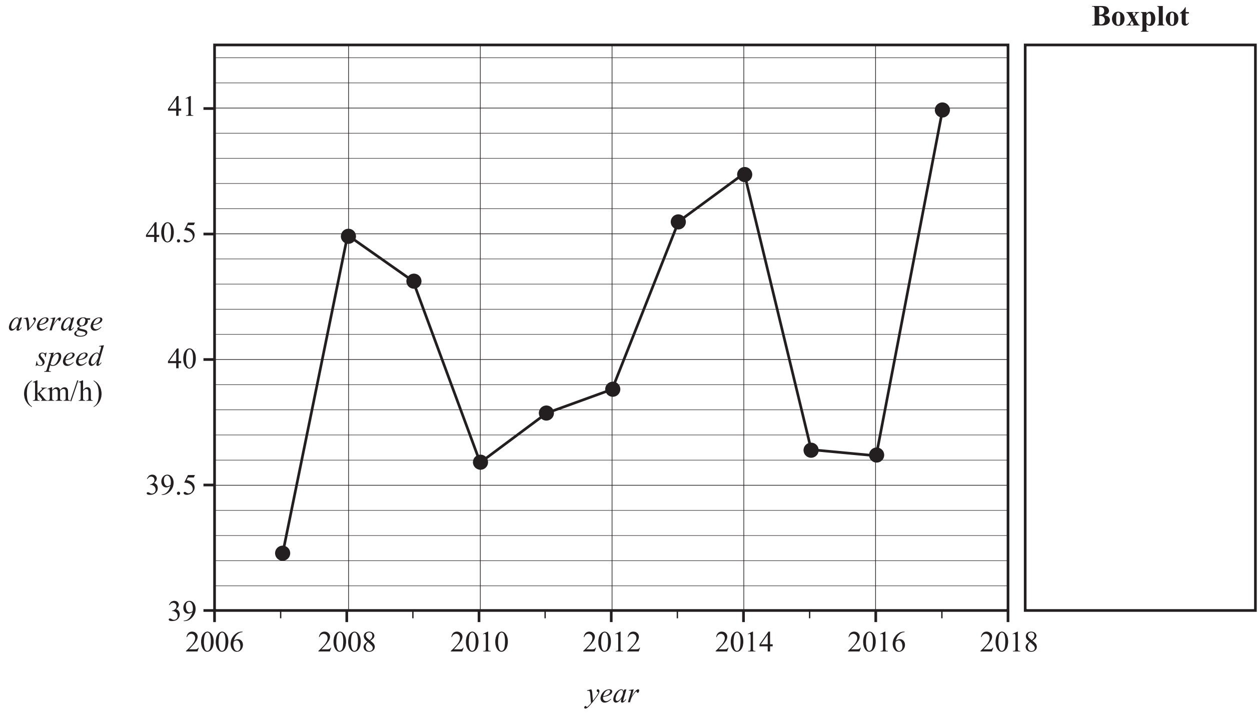
Round your answer to three decimal places. 1 mark

f. Determine the percentage of variation in *average speed* explained by the variation in *race length*.

Round your answer to one decimal place. 2 marks

Question 4 (4 marks)

The *average speed* of the winners, in kilometres per hour, of the Tour de France for the years 2007–2017 is displayed in the time series plot below.



Data: 'Tour de France statistics: Dates, stages, average speed, length, number of entrants and finishers', BikeRaceInfo website, <https://bikeraceinfo.com>

a. Use the time series plot to construct a **vertical boxplot** to show the distribution of the *average speed* of these winners for the years 2007–2017. This boxplot has no outliers. Draw the vertical boxplot in the box provided to the right of the time series plot above. Use the same scale as the time series plot. 2 marks

(*Answer in the box above.*)

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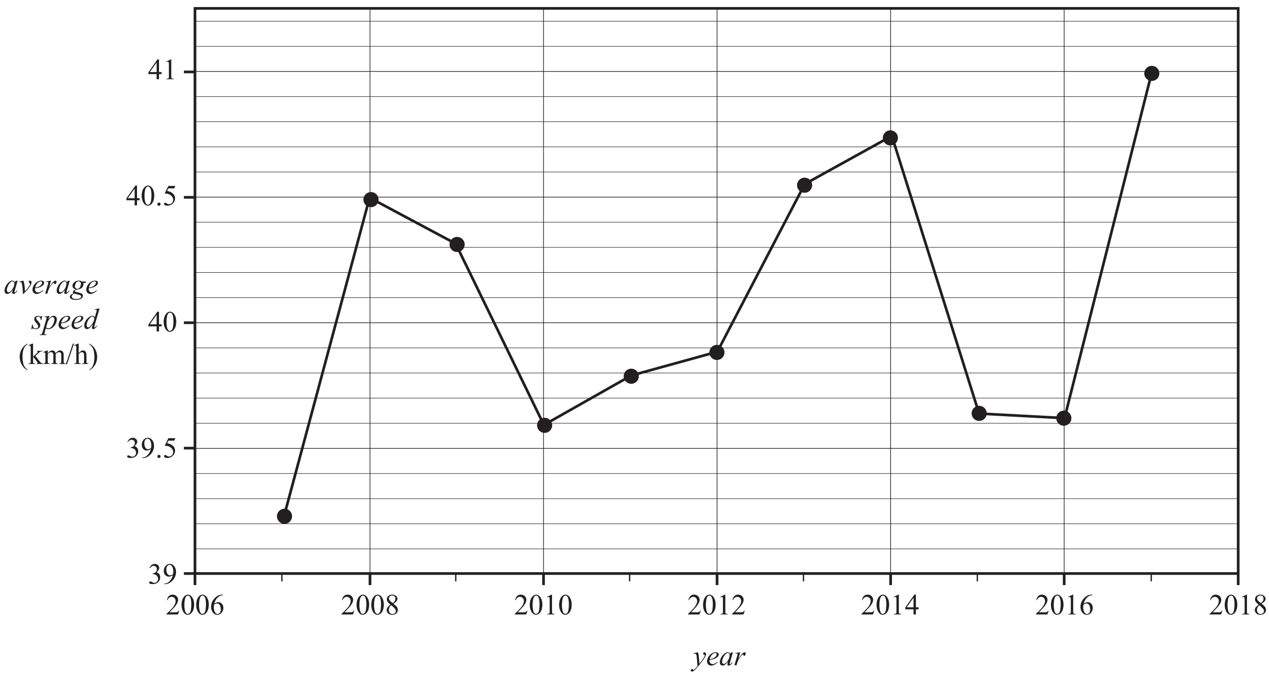
b. Use seven-median smoothing to smooth the time series plot below. Mark the smoothed values with crosses (╳) and connect them with a line. 2 marks



Recursion and financial modelling

Question 5 (3 marks)

Barb has recently become the owner of a gymnasium.

At this gymnasium, membership fees are paid monthly.

Let B_n be the total value of the membership fees, in dollars, received by Barb n months after she took ownership of the gymnasium.

The recurrence relation that models the value of the membership fees paid monthly, in dollars, is

$$B_0 = 4000, \qquad B_{n+1} = R \times B_n$$

a. The total value of the membership fees received after one month is $B_1 = 5800$.

Show that R is equal to 1.45 1 mark

b. Showing recursive calculations, determine the value of B_3, which is the total value of membership fees received after three months. 1 mark

c. After how many months will the total value of membership fees received first exceed \$50 000? 1 mark

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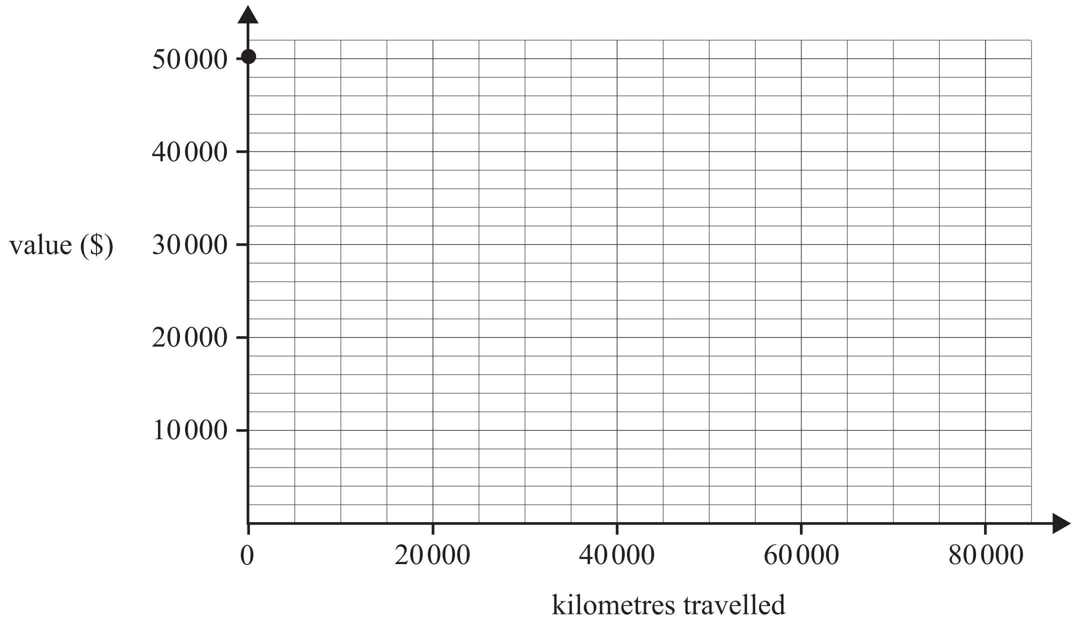
Question 6 (3 marks)

Barb drives a company car that she bought for \$50 000.

She depreciates the value of the car by \$0.20 for each kilometre that she travels in it.

a. On the graph below, plot the **four** points that represent the value of the car after Barb travels 20 000 km, 40 000 km, 60 000 km and 80 000 km. 1 mark



b. Complete the rule below that gives the value of the car, C_n, in dollars, after n kilometres have been travelled. Write your answers in the boxes provided. 1 mark

$$C_n = \boxed{} + \boxed{} \times n$$

c. Barb travels 20 000 km in the car each year.

The value of the car could also be considered to be depreciating at a flat rate.

What annual flat rate percentage would be used? 1 mark

Question 7 (4 marks)

Barb took out a reducing balance loan of $800 000 to pay for renovations to the gymnasium.
Interest is calculated quarterly and Barb makes quarterly repayments.

Four lines of the amortisation table for Barb's loan are shown below.

Payment number	Payment ($)	Interest ($)	Principal reduction ($)	Balance ($)
0	0.00	0.00	0.00	800 000.00
1	12 148.80	6680.00	5468.80	794 531.20
2	12 148.80	6634.34	5514.46	789 016.74
3	12 148.80	6588.29	5560.51	783 456.23

a. Show that the annual interest rate of the loan is 3.34% per annum. 1 mark

b. How many years will it take Barb to fully repay the loan? 1 mark

c. Let V_n be the balance of the loan after n quarters.

Write a recurrence relation in terms of V_0, V_{n+1} and V_n that can model the balance of Barb's loan from quarter to quarter. 2 marks

Question 8 (2 marks)

Barb invested \$225 000 in an annuity, from which she receives a regular monthly payment.

For this annuity investment, interest compounds monthly.

The recurrence relation that models the value of the annuity investment, A_n, in dollars, after n months is

$$A_0 = 225\,000, \qquad A_{n+1} = 1.0045A_n - 1500$$

After 10 years of receiving regular payments from the annuity, Barb lowered the monthly payment figure to \$1200 in order to extend the length of the annuity.

The final monthly payment that Barb received was less than \$1200.

Determine how much money Barb received in the final monthly payment.

Round your answer to the nearest dollar.

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SECTION B – Modules

Instructions for Section B

Select **two** modules and answer **all** questions within the selected modules.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Contents **Page**

Module 1 – Matrices

Question 1 (3 marks)

A retirement village has a cinema onsite for its residents.

The cost of one cinema ticket depends on what time of day a movie is playing: morning (M), afternoon (A) or evening (E).

Matrix C represents the cost of one ticket, in dollars, for each ticket type.

$$C = \begin{array}{c} \begin{bmatrix} 4.50 \\ 5.50 \\ 7.00 \end{bmatrix} \end{array} \begin{array}{l} M \\ A \\ E \end{array}$$

a. Interpret the value of element c_{31}. 1 mark

On one day, the cinema sells 20 morning tickets, 45 afternoon tickets and 62 evening tickets.

b. Write a matrix calculation, using matrix C and a row matrix, to show that the total value of the three ticket types sold on this day was \$771.50 1 mark

c. The total value of each ticket type sold on this day could be determined by multiplying a 3 × 3 matrix called matrix R by matrix C.

Complete matrix R below. 1 mark

$$R = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$$

Question 2 (3 marks)

There are six locations in the retirement village: administration (A), the cinema (C), the fitness centre (F), the library (L), the pool (P) and the residential block (R).

These locations are connected by a number of paths.

Matrix W shows the locations that are directly connected by paths.

$$W = \begin{array}{c} \begin{array}{cccccc} A & C & F & L & P & R \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ A \\ C \\ F \\ L \\ P \\ R \end{array}$$

In this matrix:

- the '1' in row F, column A indicates that a path directly connects the fitness centre to administration
- the '0' in row F, column P indicates that the fitness centre and the pool are not directly connected by a path.

a. Which location is directly connected by paths to more locations than any other? 1 mark

b. When matrix W^2 is calculated, the element in row L, column R is equal to 2.

Write down the paths represented by this element. 1 mark

c. In matrix $W + W^2$, the values in row F, column R and row R, column F are both zero.

$$W + W^2 = \begin{array}{c} \begin{array}{cccccc} A & C & F & L & P & R \end{array} \\ \begin{bmatrix} 3 & 2 & 2 & 3 & 1 & 1 \\ 2 & 3 & 2 & 2 & 2 & 1 \\ 2 & 2 & 2 & 2 & 1 & 0 \\ 3 & 2 & 2 & 4 & 1 & 2 \\ 1 & 2 & 1 & 1 & 2 & 1 \\ 1 & 1 & 0 & 2 & 1 & 2 \end{bmatrix} \end{array} \begin{array}{c} \\ A \\ C \\ F \\ L \\ P \\ R \end{array}$$

Explain the significance of this. 1 mark

Question 3 (6 marks)

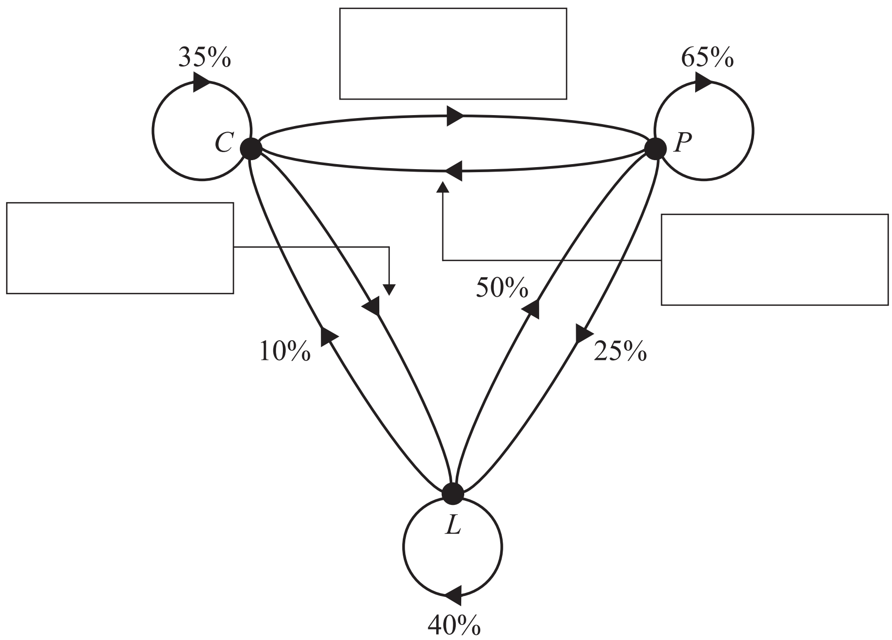
Every day, each of the 450 residents of the retirement village choose between daily activities in one of three locations: the cinema (C), the pool (P) or the library (L).

The regular transition matrix T shows how the residents' choices change from day to day.

$$\begin{array}{cc} & \textit{today} \\ & \begin{array}{ccc} C & P & L \end{array} \\ T = & \begin{bmatrix} 0.35 & 0.10 & 0.10 \\ 0.50 & 0.65 & 0.50 \\ 0.15 & 0.25 & 0.40 \end{bmatrix} \begin{array}{l} C \\ P \\ L \end{array} \; \textit{tomorrow} \end{array}$$

a. An equivalent transition diagram is shown below but is incomplete.

Complete the transition diagram by writing the three missing percentages in the boxes provided. 1 mark



Let S_n represent the state matrix that shows the number of residents who participated in activities at each location n days after 1 January.

The matrix recurrence relation that is used to predict daily participation at each location is

$$S_0 = \begin{bmatrix} 100 \\ 260 \\ 90 \end{bmatrix} \begin{array}{l} C \\ P \\ L \end{array}, \quad S_{n+1} = T \times S_n, \quad \text{where } T = \begin{array}{c} \textit{today} \\ \begin{array}{ccc} C & P & L \end{array} \\ \begin{bmatrix} 0.35 & 0.10 & 0.10 \\ 0.50 & 0.65 & 0.50 \\ 0.15 & 0.25 & 0.40 \end{bmatrix} \end{array} \begin{array}{l} C \\ P \\ L \end{array} \; \textit{tomorrow}$$

b. Complete matrix S_1 to show the number of residents participating in activities at each location on 2 January. 1 mark

$$S_1 = \begin{array}{c} \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \end{bmatrix} \end{array} \begin{array}{l} C \\ P \\ L \end{array}$$

c. How many of the 450 residents participated in a different activity on 2 January than the activity they participated in on 1 January? 1 mark

d. How many residents are expected to choose to participate in activities at the library on 5 January?

Round your answer to the nearest whole number. 1 mark

e. In the long run, how many of the 450 residents are expected to change their activity location from the cinema to the library each day? 1 mark

f. From 1 March, the retirement village allowed each of the 450 residents to invite family members to the cinema.

The number of residents and family members who attended the cinema on 1 March was 100.

The number of residents at the pool on 1 March was 265.

The number of residents at the library on 1 March was 125.

The number of family members attending the cinema each day remained constant.

How many people are expected to attend the cinema on 3 March? 1 mark

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Module 2 – Networks and decision mathematics

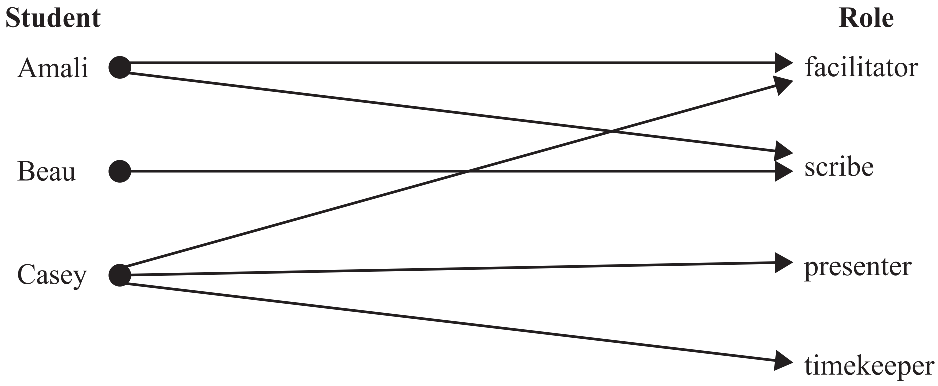
Question 1 (3 marks)

Three students, Amali, Beau and Casey, are working on a group project at school.

There are four roles to be allocated: facilitator, scribe, presenter and timekeeper.

The bipartite graph below illustrates the role that each student in the group could take.

Each student will have at least one role.



a. Which student must be allocated the role of scribe? 1 mark

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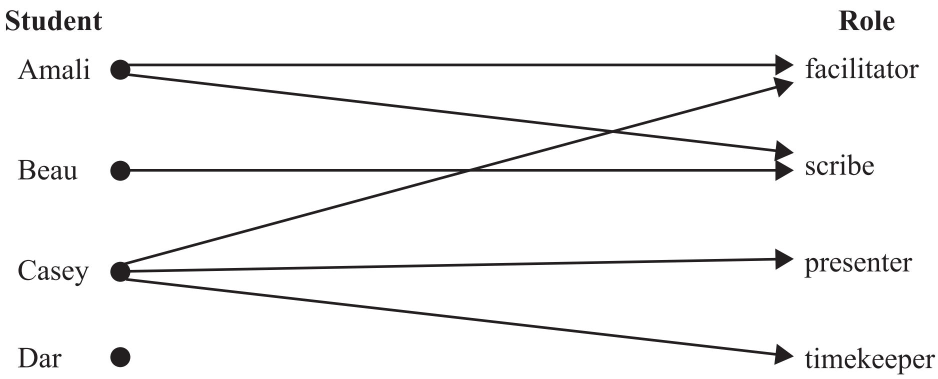
The teacher asks a fourth student, Dar, to join the group.

Dar says she can perform any role except facilitator.

b. Add Dar's potential roles to the bipartite graph below. 1 mark



c. Each of the four students will be given one role.

Dar is given the role of presenter.

What role must Casey be given? 1 mark

Question 2 (1 mark)

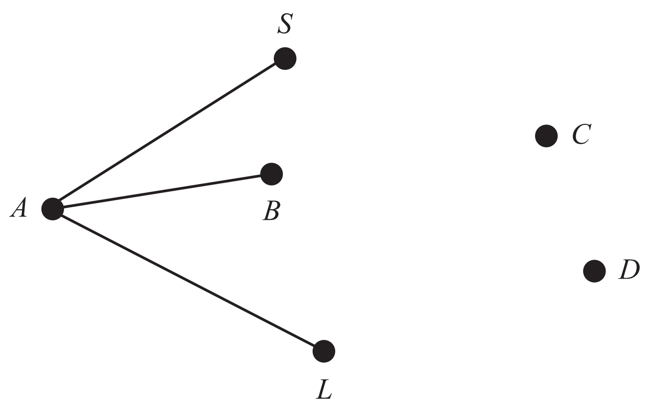
The four students, Amali (A), Beau (B), Casey (C) and Dar (D), live near the local library (L) and their school (S).

The adjacency matrix below shows the road connections between their homes, the local library and their school.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ L \\ S \end{array} \begin{array}{c} \begin{array}{cccccc} A & B & C & D & L & S \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

The incomplete network diagram below shows the road connections between Amali's, Beau's, Casey's and Dar's homes, and the local library and their school.

The edges represent the roads.



Use the adjacency matrix to complete the **network diagram above**.

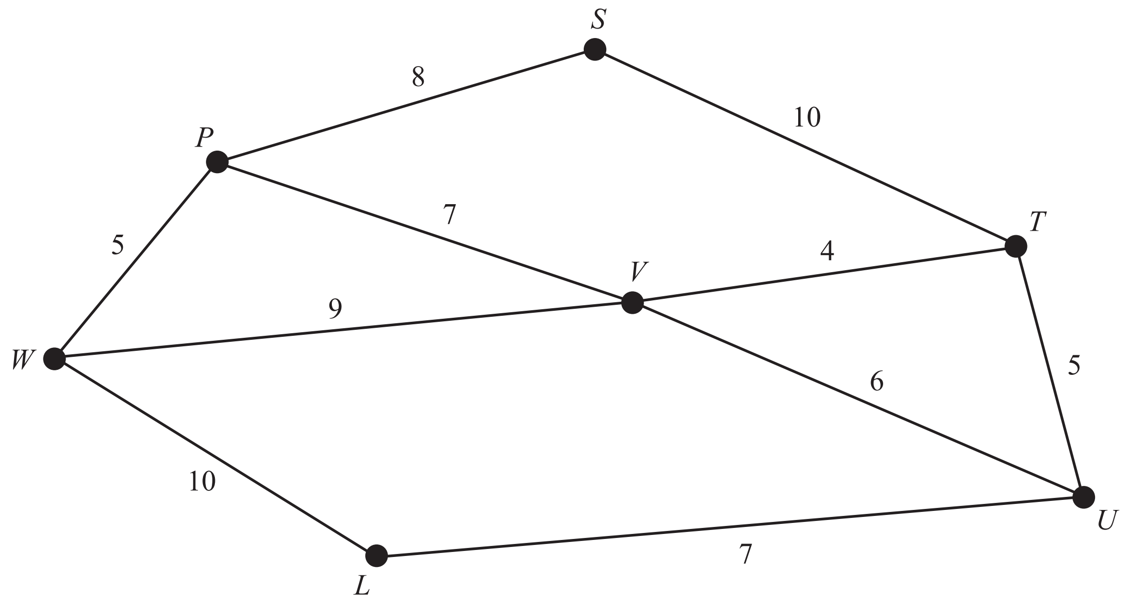
(*Answer on the network diagram above.*)

Question 3 (3 marks)

A paper business (P) supplies paper to the school (S), the local library (L) and four other nearby businesses (T, U, V and W).

The network diagram below shows the connections between these locations.

The edges represent the roads connecting the locations. The numbers on the edges indicate the distances, in kilometres, between the locations.



Last Monday, a delivery driver left from the paper business (P) and delivered paper to the school (S), the local library (L) and the four other businesses (T, U, V and W).

He returned to the paper business when all his deliveries were complete.

The distance that he travelled last Monday was 50 km.

a. What is a route that the driver could have travelled to make his deliveries? 1 mark

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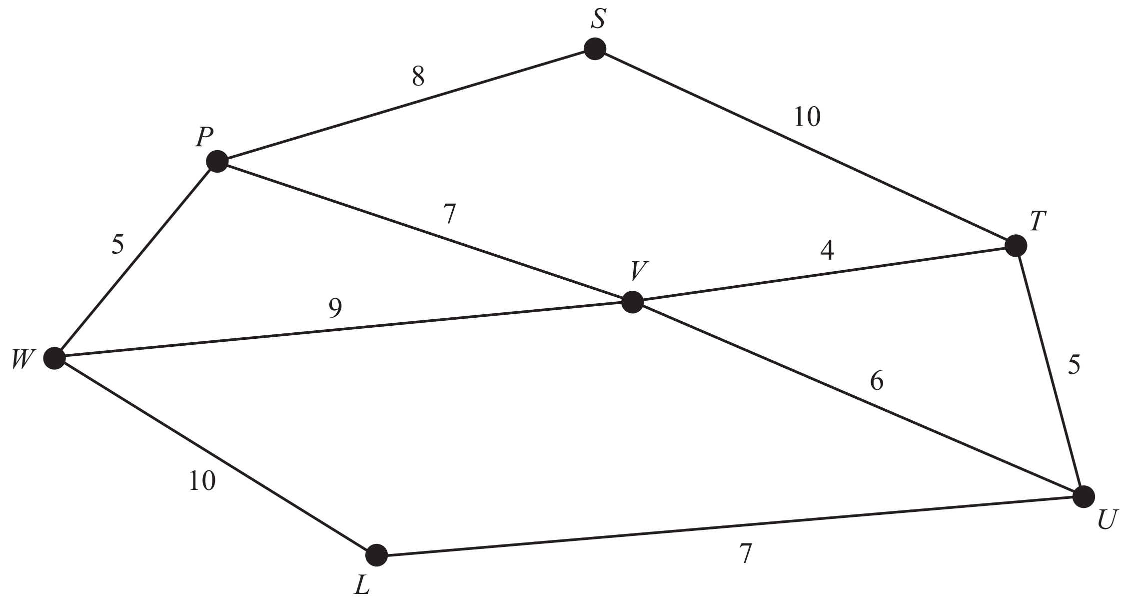
A new business (X) is added to the delivery driver's route.

Business X is connected to business W by a road that has a distance of 5 km.

A second road, which has a distance of 3 km, connects business X to business V.

A third road, which has a distance of 2 km, connects business X to the local library L.

b. Complete the network diagram below by adding the new business (X) and the three new roads. Label the new vertex 'X' and write the distances on the new edges. 1 mark



c. This Monday, the delivery driver will leave from the paper business (P) and deliver paper to the school (S), the local library (L) and the five other businesses (T, U, V, W and X).

He will return to the paper business when all his deliveries are complete.

He plans to travel the minimum distance possible.

What is the minimum distance the delivery driver can travel on this Monday? 1 mark

Question 4 (5 marks)

The staff at the local library are planning to upgrade their facilities.

This project will involve 10 activities: *A* to *J*.

Table 1 shows the duration of each activity, in weeks, and the immediate predecessor(s) for each activity.

Table 1

Activity	Duration	Immediate predecessor(s)
A	2	–
B	1	*A*
C	4	*B*
D	7	*B*
E	9	*B*
F	3	*C*
G	8	*D*, *F*
H	3	*D*, *F*
I	7	*E*, *H*
J	3	*G*, *I*

a. Table 2 shows the earliest start time (EST) and the latest start time (LST) for activities *A*, *B* and *C*. The EST for activity *C* and the LST for activity *B* are missing.

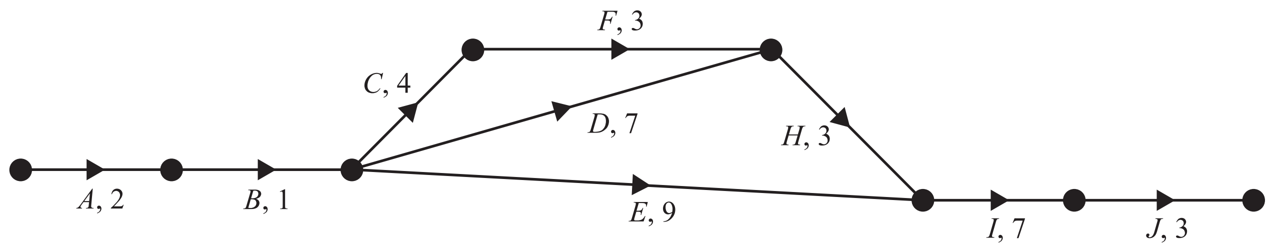
Use the information in Table 1 to complete Table 2. 1 mark

Table 2

Activity	EST	LST
A	0	0
B	2	
C		3

The directed network below shows the activities for the library upgrade and their duration, in weeks.

Activity G is missing from this network.



b. Draw activity G on the **directed network above**. 1 mark

(*Answer on the directed network above.*)

c. What is the minimum completion time, in weeks, for the library upgrade? 1 mark

d. A change has been made to two activities in the project.

One activity has had its duration increased by three weeks.

A second activity, which has a duration greater than three weeks, has had its duration decreased by three weeks.

The overall completion time for the project has not changed.

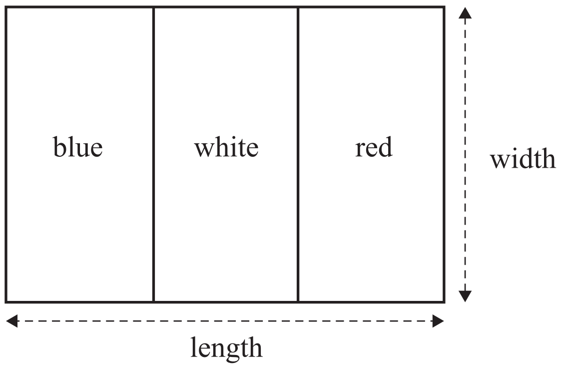
Identify all pairs of activities that could have had their completion times changed. For each pair identified, specify which activity will have an increase in completion time and which activity will have a decrease in completion time. 2 marks

Module 3 – Geometry and measurement

Question 1 (3 marks)

The French national flag consists of three colours, blue, white and red, in vertical stripes.

The area of each stripe is one-third of the total area of the flag, as shown in the diagram below.



a. One French flag, Flag A, has a width of 100 cm and a length of 150 cm.

Calculate the area, in square centimetres, of the **blue section** of Flag A. 1 mark

The width and length of the French flag are in the ratio 2:3.

b. A second French flag, Flag B, has a length of 60 cm.

Calculate the width, in centimetres, of Flag B. 1 mark

c. Flag A and Flag B are similar figures. Flag A is an enlargement of Flag B.
The length of Flag A is 150 cm and the length of Flag B is 60 cm.

Calculate the linear scale factor used to enlarge the length of Flag B to the length of Flag A. 1 mark

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Question 2 (2 marks)

The French island of Saint Pierre is located at latitude 47° N and longitude 56° W.

Assume that the radius of Earth is 6400 km.

a. Write a calculation that shows that the radius of the small circle of Earth at latitude 47° N is 4365 km.
Round your answer to the nearest kilometre. 1 mark

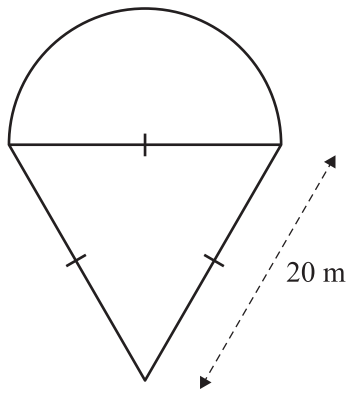
b. The French city of Bourges is located at latitude 47° N and longitude 2° E.

Find the shortest small circle distance between Saint Pierre and Bourges.
Round your answer to the nearest kilometre. 1 mark

Question 3 (5 marks)

To celebrate France's national day, Saint Pierre is hosting a carnival.

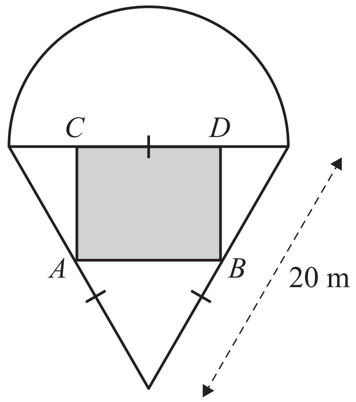
The carnival is held in a field. The shape of the field is composed of a semicircle and an equilateral triangle with side lengths of 20 m, as shown in the diagram below.



a. Calculate the perimeter, in metres, of the outside of the field.
Round your answer to two decimal places. 1 mark

b. Calculate the total area, in square metres, of the field.
Round your answer to two decimal places. 2 marks

c. A rectangular stage, shown shaded in the diagram below, is set up on the field.



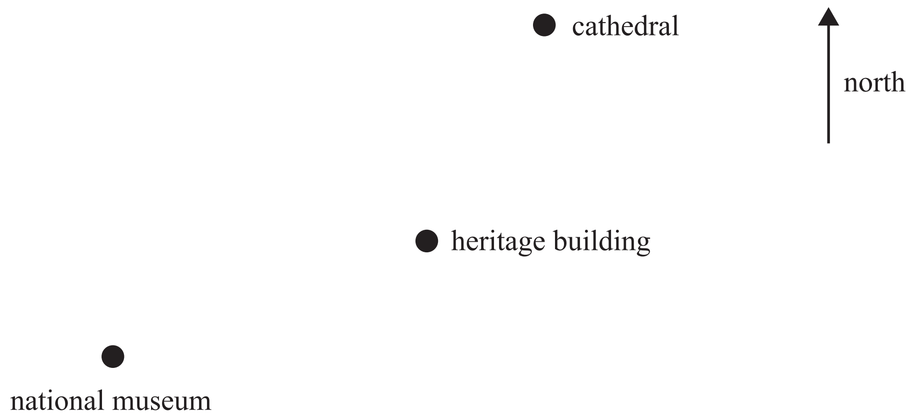
Points A and B are at the midpoint of two sides of the triangle.
Points C and D are on the line separating the semicircle and the triangle.

Calculate the area, in square metres, of the rectangular stage.
Round your answer to one decimal place. 2 marks

Question 4 (2 marks)

Celebrations for France's national day are being held at three locations in Saint Pierre: the national museum, the heritage building and the cathedral.

The relative locations of the national museum, the heritage building and the cathedral are shown below.



The distance between the national museum and the heritage building is 453 m.

The distance between the national museum and the cathedral is 630 m.

The heritage building is on a bearing of 052° from the national museum.

The cathedral is on a bearing of 042° from the national museum.

a. Calculate the distance, in metres, between the heritage building and the cathedral.
Round your answer to the nearest whole number. 1 mark

b. Calculate the bearing of the cathedral from the heritage building.
Round your answer to the nearest degree. 1 mark

CONTINUES OVER PAGE

Module 4 – Graphs and relations

Question 1 (3 marks)

Saints Airlines offers air travel between destinations in Australia.

The price of each flight includes a service fee of \$60, plus \$0.40 per kilometre travelled.

The *price* of a flight, P, in dollars, to travel n kilometres, for *distances* up to 400 km is given by

$$P = 60 + 0.4n$$

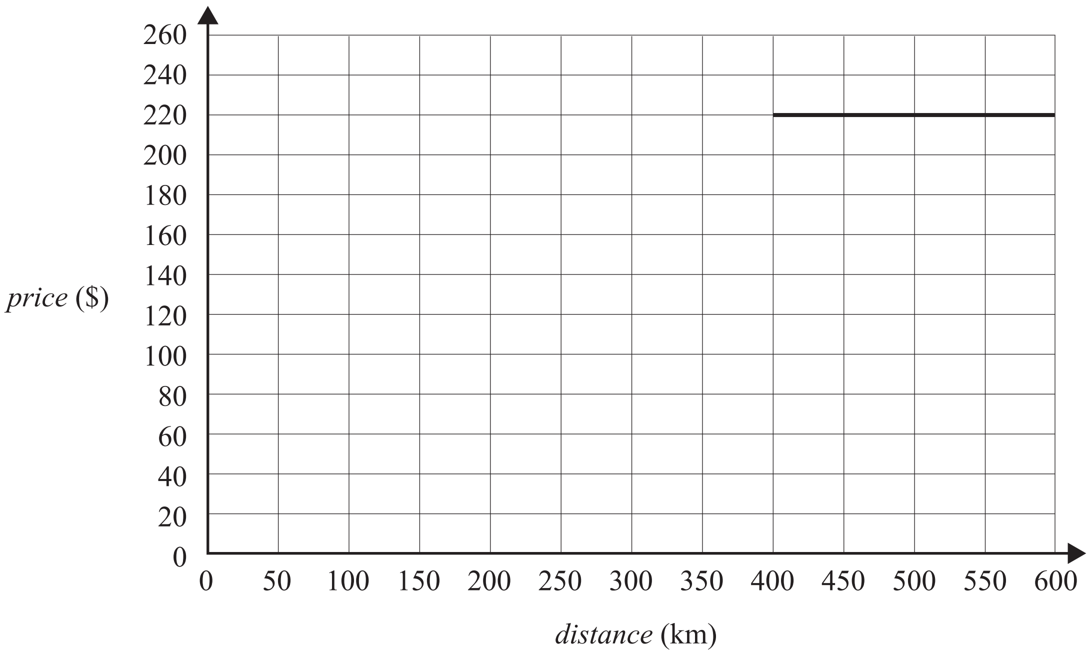
a. What is the price, in dollars, of a 250 km flight? 1 mark

The *price* of a flight for *distances* between 400 km and 600 km is capped at \$220 and therefore

$$P = \begin{cases} 60 + 0.4n & 0 \le n \le 400 \\ 220 & 400 < n \le 600 \end{cases}$$

b. The graph below shows the *price* of a flight against *distance* travelled for $400 < n \le 600$.

Complete the graph by adding the *price* of a flight against *distance* travelled for $0 \le n \le 400$. 1 mark



c. Due to an increase in fuel costs, Saints Airlines will restructure its airfare pricing for all flights up to 600 km.

It will now charge \$159 for a 200 km flight and \$285 for a 500 km flight.

The new *price*, C, to travel n kilometres is of the form

$$C = a + b \times n$$

Find the values of a and b. 1 mark

Question 2 (3 marks)

Staff at Saints Airlines regularly check the noise levels of their aircraft engines.

The sound intensity of an engine from one of the planes is measured.

Table 1 displays the relationship between *sound intensity*, in watts per square metre, and *distance* from the engine, in metres.

Table 1

Distance **(m)**	20	25	40	50
Sound intensity **(W/m^2)**	225	144	56.25	36

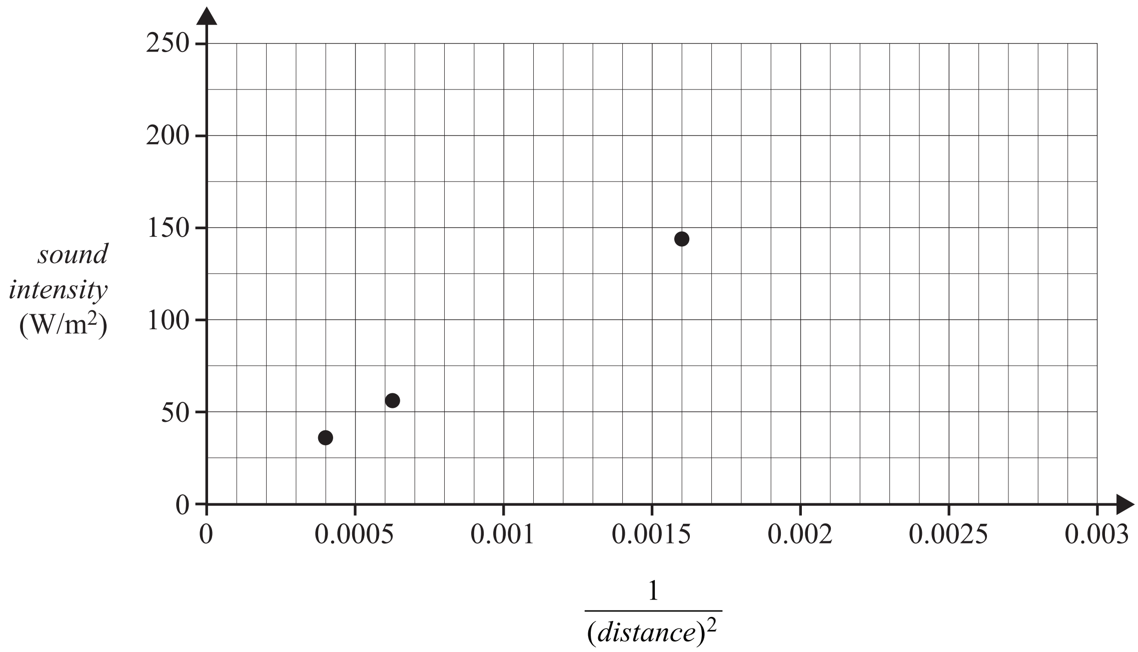
a. Complete the calculation below to show that the relationship between *sound intensity* and *distance* from the engine is not linear. 1 mark

$$\frac{144-225}{25-20} \neq \_\_\_\_\_\_\_\_\_\_$$

Table 2 is used to construct a graph of *sound intensity* against $\frac{1}{(distance)^2}$, as shown below.

Table 2

Distance **(m)**	20	25	40	50
$\frac{1}{(distance)^2}$		0.0016	0.000625	0.0004
Sound intensity **(W/m^2)**	225	144	56.25	36



DO NOT WRITE IN THIS AREA

b. On the **graph on page 34**, plot the point that relates to the *sound intensity* at a *distance* of 20 m from the engine. 1 mark

(*Answer on the graph on page 34.*)

c. The equation $\text{sound intensity} = \dfrac{k}{(\text{distance})^2}$, where k is the constant of proportionality, describes the relationship between *sound intensity* and *distance* from the engine.

Determine the *sound intensity*, in watts per square metre, at a *distance* of 75 m from the engine. 1 mark

Question 3 (2 marks)

Saints Airlines sells meals onboard its flights.

The cost, C, in dollars, of producing n meals is given by

$$C = 3.5n + 900$$

On one flight, the airline sold 200 meals at the regular price. It also sold 100 meals at half the regular price to passengers who had a discount voucher.

The airline was able to break even on the sale of these 300 meals.

Determine the regular price of a meal served onboard a Saints Airlines flight.

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Question 4 (4 marks)

Saints Airlines transports cargo in large and small containers.

Small containers have a base area of 2 m^2 and large containers have a base area of 3 m^2.

There is 100 m^2 of floor space available in each plane for the containers.

The number of large containers transported on each plane is no more than twice the number of small containers transported.

Each plane must transport at least five large containers.

Let x represent the number of small containers transported per plane.

Let y represent the number of large containers transported per plane.

These constraints are represented by the following three inequalities.

Inequality 1	$2x + 3y \leq 100$
Inequality 2	$y \leq 2x$
Inequality 3	$y \geq 5$

A fourth constraint is represented by the inequality $x \leq 40$.

a. Explain the meaning of the fourth inequality in the context of Saints Airlines transporting cargo. 1 mark

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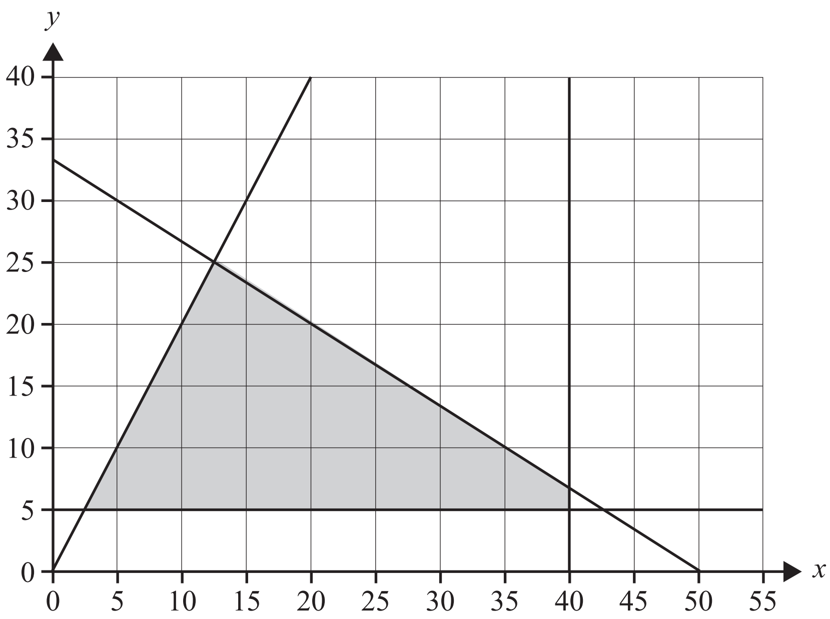
b. The graph below shows the feasible region (shaded) that satisfies these four inequalities.



If 15 large containers are transported, find the minimum and maximum number of small containers that could be transported with them. 1 mark

minimum number =

maximum number =

c. Saints Airlines charges $150 to transport each small container and $200 to transport each large container.

Determine the maximum revenue, per flight, that Saints Airlines can make from transporting cargo. 2 marks

END OF QUESTION AND ANSWER BOOK

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education 2022

FURTHER MATHEMATICS

Written examination 2

FORMULA SHEET

Instructions

This formula sheet is provided for your reference.
A question and answer book is provided with this formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Further Mathematics formulas

Core – Data analysis

standardised score	$z = \dfrac{x - \bar{x}}{s_x}$
lower and upper fence in a boxplot	lower $Q_1 - 1.5 \times IQR$ upper $Q_3 + 1.5 \times IQR$
least squares line of best fit	$y = a + bx$, where $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$
residual value	residual value = actual value – predicted value
seasonal index	seasonal index = $\dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

Core – Recursion and financial modelling

first-order linear recurrence relation	$u_0 = a$, $u_{n+1} = bu_n + c$
effective rate of interest for a compound interest loan or investment	$r_{effective} = \left[\left(1 + \dfrac{r}{100n}\right)^n - 1\right] \times 100\%$

Module 1 – Matrices

determinant of a 2 × 2 matrix	$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
inverse of a 2 × 2 matrix	$A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $\det A \neq 0$
recurrence relation	S_0 = initial state, $S_{n+1} = T\,S_n + B$

Module 2 – Networks and decision mathematics

Euler's formula	$v + f = e + 2$

Module 3 – Geometry and measurement

area of a triangle	$A=\frac{1}{2}bc\sin(\theta^\circ)$
Heron's formula	$A=\sqrt{s(s-a)(s-b)(s-c)}$, where $s=\frac{1}{2}(a+b+c)$
sine rule	$\frac{a}{\sin(A)}=\frac{b}{\sin(B)}=\frac{c}{\sin(C)}$
cosine rule	$a^2 = b^2 + c^2 - 2bc\cos(A)$
circumference of a circle	$2\pi r$
length of an arc	$r\times\frac{\pi}{180}\times\theta^\circ$
area of a circle	πr^2
area of a sector	$\pi r^2\times\frac{\theta^\circ}{360}$
volume of a sphere	$\frac{4}{3}\pi r^3$
surface area of a sphere	$4\pi r^2$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a prism	area of base × height
volume of a pyramid	$\frac{1}{3}$ × area of base × height

Module 4 – Graphs and relations

gradient (slope) of a straight line	$m=\frac{y_2-y_1}{x_2-x_1}$
equation of a straight line	$y = mx + c$

END OF FORMULA SHEET