

VCE General Mathematics Units 1&2

Written Examination 1

Suggested Solutions

1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E
16	A	B	C	D	E
17	A	B	C	D	E
18	A	B	C	D	E
19	A	B	C	D	E
20	A	B	C	D	E
21	A	B	C	D	E
22	A	B	C	D	E
23	A	B	C	D	E
24	A	B	C	D	E
25	A	B	C	D	E
26	A	B	C	D	E
27	A	B	C	D	E
28	A	B	C	D	E
29	A	B	C	D	E
30	A	B	C	D	E
31	A	B	C	D	E
32	A	B	C	D	E
33	A	B	C	D	E
34	A	B	C	D	E
35	A	B	C	D	E
36	A	B	C	D	E
37	A	B	C	D	E
38	A	B	C	D	E
39	A	B	C	D	E
40	A	B	C	D	E

AREA OF STUDY 1

- Data analysis, probability and statistics
-

Question 1 D

D is correct. Hair dye and shampoo brands are both examples of categorical nominal data.

A is incorrect. Height is an example of numerical continuous data and postcodes are an example of categorical ordinal data.

B is incorrect. Number of people is an example of numerical discrete data and gender is an example of categorical nominal data.

C is incorrect. Education level is an example of categorical ordinal data and annual salary is an example of numerical discrete data.

E is incorrect. An English grade is an example of categorical ordinal data, and an IQ test score is an example of numerical discrete data.

Question 2 A

Outliers are values that lie outside of the lower or upper fence of a data set. Therefore, a test score of 3 is an outlier if it lies outside the lower fence.

Finding the lower fence gives:

$$Q1 = 27$$

$$Q3 = 40$$

$$IQR = Q3 - Q1$$

$$= 40 - 27$$

$$= 13$$

$$\text{lower fence} = Q1 - 1.5 \times IQR$$

$$= 27 - 1.5 \times 13$$

$$= 7.5$$

Question 3 E

There are 16 countries with a *GDP* higher than $\$10^{10}$. Therefore, the percentage is $\frac{16}{20} \times 100 = 80\%$.

Question 4 E

Applying the formula to locate the median value gives:

$$\text{median} = \frac{n+1}{2}$$

$$= \frac{20+1}{2}$$

$$= 10.5$$

Therefore, the median is the average of the 10th and 11th values. Both the 10th and 11th values are between 11 and 12 on the logarithmic scale. Therefore, the median *GDP* is between 10^{11} and 10^{12} .

Question 5 D

The range will not change as the maximum and minimum values are not affected by the revised score.

Question 6 C

Reading from the scatterplot, the gradient is positive and the y-intercept is below 10. Therefore, the only possible equation is $y = 4.76x + 7.85$.

Question 7 C

By observing the scatterplot, it can be seen that the direction is positive. As all the points are relatively close to each other, the strength is moderate.

Question 8 B

Since the study investigates how the IQ test score affects the final test score, the IQ test score is the explanatory variable and the final exam score is the response variable.

Question 9 C

From the equation, the gradient of the data is 0.28. For every increase of 1 unit in the explanatory variable, it is expected that the response variable will increase by 0.28. Therefore, if the explanatory variable increases by 2, the response variable is expected to increase by $0.28 \times 2 = 0.56$.

Question 10 B

Substituting exam result = 80 and solving on a CAS calculator gives:

$\text{solve}(80=25+3.8 \cdot x, x)$	$x=14.4736842105$
--------------------------------------	-------------------

AREA OF STUDY 2

- Algebra, number and structure
- Discrete mathematics

Question 11 A

From observation, it can be determined that each term of the sequence is the previous term multiplied by 3.

$$2 \times 3 = 6$$

$$6 \times 3 = 18$$

$$18 \times 3 = 54$$

Therefore, $u_0 = 2$, $u_{n+1} = 3u_n$.

Question 12 E

Using the common ratio of 1.5, the first six terms of the sequence are 2, 3, 4.5, 6.75, 10.125 and 15.1875. Hence, the sum is 41.5625.

Question 13 D

$$\begin{aligned}\text{simple interest every year} &= 12\,000 \times \frac{5}{100} \\ &= 600\end{aligned}$$

$$\begin{aligned}\text{total interest over the first three years} &= 3 \times 600 \\ &= 1800\end{aligned}$$

After the first three years, John owes $12\,000 + 1800 = \$13\,800$.

Question 14 D

The useable value of the printer is $350 - 50 = \$300$.

Finding the number of pages that can be printed gives:

$$\begin{aligned}\text{number of pages} &= \frac{\text{useable value}}{\text{depreciation per page}} \\ &= \frac{300}{0.002} \\ &= 150\,000\end{aligned}$$

Question 15 C

The ratio in the scale is 1 cm : 50 km.

Hence:

$$\begin{aligned}\text{distance between town A and town B} &= 50 \times 2.5 \text{ cm} \\ &= 125 \text{ km}\end{aligned}$$

Question 16 B

The degrees of the vertices are:

$$\text{deg}(A) = 4$$

$$\text{deg}(B) = 3$$

$$\text{deg}(C) = 4 \text{ (as there is a loop at } C\text{)}$$

$$\text{deg}(D) = 3$$

Hence, the sum is $4 + 3 + 4 + 3 = 14$.

Question 17 C

Since the graph is planar, substituting five vertices and five faces into Euler's formula gives:

$$v + f - e = 2$$

$$5 + 5 - e = 2$$

$$e = 8$$

As there are five edges in the incomplete graph, there are $8 - 5 = 3$ edges missing from the graph.

Question 18 E

An Eulerian circuit starts and ends at the same vertex and travels along each edge once. For an Eulerian circuit to exist, the degree of all vertices must be even.

The degree of each town is:

$$\deg(A) = 2$$

$$\deg(B) = 2$$

$$\deg(C) = 4$$

$$\deg(D) = 3$$

$$\deg(E) = 2$$

$$\deg(F) = 3$$

Both D and F have an odd degree; therefore, for an Eulerian trial to exist, the new road should be built between towns D and F .

Question 19 A

The two Hamiltonian cycles in the graph are $E-H-I-G-F-E$ and $E-G-I-H-F-E$.

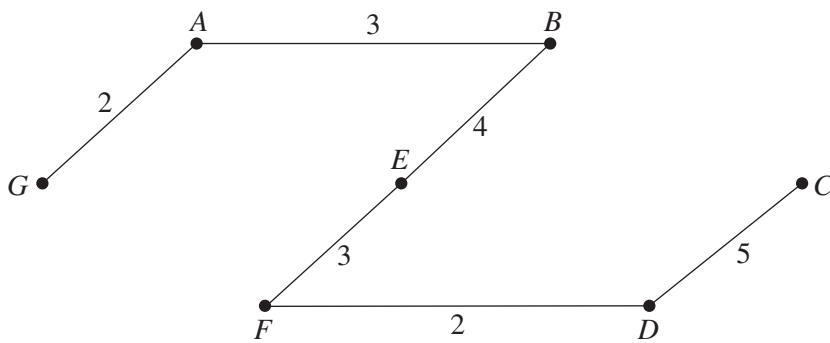
$$\begin{aligned} \text{distance of } E-H-I-G-F-E &= 6 + 10 + 3 + 4 + 3 \\ &= 26 \end{aligned}$$

$$\begin{aligned} \text{distance of } E-G-I-H-F-E &= 6 + 3 + 10 + 3 + 3 \\ &= 25 \end{aligned}$$

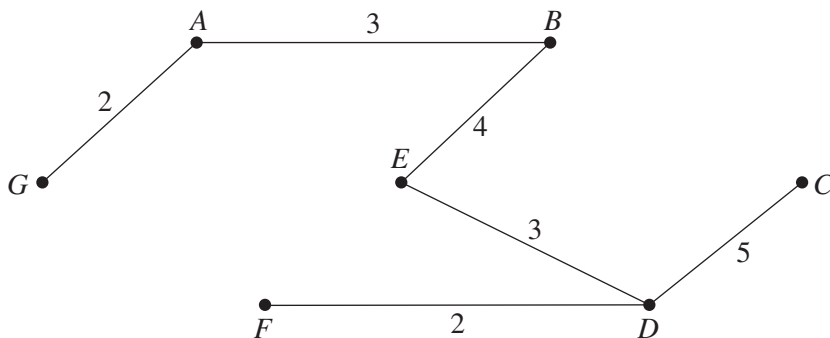
Therefore, the shortest distance is 25.

Question 20 B

There are two possible minimum spanning trees, as shown below.



OR



Hence, the length is $2 + 3 + 4 + 3 + 2 + 5 = 19$.

AREA OF STUDY 3

- Functions, relations and graphs
-

Question 21 C

Substituting $x = 1$, $y = 2$ and $z = 3$ into the expression gives:

$$(2 \times 1) - (2 \times 3) + (2 \times 2) = 0$$

Question 22 E

$$\begin{cases} \frac{120}{x} - \frac{120}{y} = -10 \\ 16x + 6y = 120 \end{cases}$$

OR

Using a CAS calculator gives:

<code>solve(2·m+7=5·m-8,m)</code>	<code>m=5.</code>
-----------------------------------	-------------------

Question 23 D

It is known that when $d = 0$, $C = 250$, and when $d = 50$, $C = 305$.

$(0, 250)$ gives the y -intercept.

Finding the gradient using $(0, 250)$ and $(50, 305)$ gives:

$$\begin{aligned} \text{gradient} &= \frac{C_2 - C_1}{d_2 - d_1} \\ &= \frac{305 - 250}{50 - 0} \\ &= 1.1 \end{aligned}$$

Therefore, the equation of the relationship is $C = 250 + 1.1d$.

Question 24 A

The linear equation has a negative gradient and a positive, non-zero y -intercept. Therefore, graph **A** best represents the equation.

Question 25 B

By letting x be the price of one sausage and y be the price of one soft drink, the following simultaneous equations can be formed.

$$\begin{cases} 60x + 55y = 200 \\ 45x + 45y = 200 - 42.5 \end{cases}$$

Using a CAS calculator to solve the simultaneous equations gives:

$\text{solve}\left(\begin{cases} 60 \cdot x + 55 \cdot y = 200 \\ 45 \cdot x + 45 \cdot y = 200 - 42.5 \end{cases}, \{x, y\}\right)$ $x = 1.5 \text{ and } y = 2.$
--

Question 26 D

Given that y is directly proportional to x , $y = kx$.

Substituting $x = 1.5$ and $y = 3.75$ into the equation gives:

$$\begin{aligned} y &= kx \\ 3.75 &= k \times 1.5 \\ k &= 2.5 \end{aligned}$$

Question 27 E

The graph of an inverse variation is a hyperbola. Therefore, graph **E** is correct.

Question 28 B

As the graph can be linearised by applying an x^2 transformation, the relationship between x and y can be written as $y = kx^2 + c$.

Finding the gradient using the points (3, 26) and (5, 74) gives:

$$\begin{aligned} k &= \frac{y_2 - y_1}{x_2^2 - x_1^2} \\ &= \frac{74 - 26}{5^2 - 3^2} \\ &= 3 \end{aligned}$$

Therefore, $y = 3x^2 + c$

Substituting (3, 26) into the equation gives:

$$\begin{aligned} 26 &= 3 \times 3^2 + c \\ c &= -1 \end{aligned}$$

Therefore, the equation of the linearised graph is $y = 3x^2 - 1$.

Question 29 C

Using the logarithmic scale, the strengths of the two earthquakes are $10^{8.2}$ and $10^{5.9}$.

$$\frac{10^{8.2}}{10^{5.9}} = 199.53$$

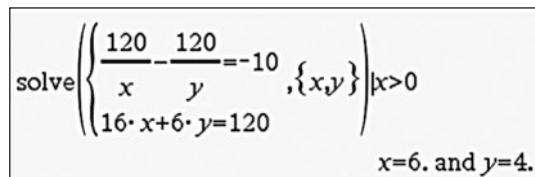
Therefore, an earthquake with a magnitude of 8.2 is 199.53 times stronger than an earthquake with a magnitude of 5.9.

Question 30 C

By assuming that pipe A pours x litres of water per hour and pipe B pours y litres of water per hour, the following simultaneous equations can be formed.

$$\begin{cases} \frac{120}{x} - \frac{120}{y} = -10 \\ 16x + 6y = 120 \end{cases}$$

Using a CAS calculator to solve the simultaneous equations gives:



solve $\left(\left(\frac{120}{x} - \frac{120}{y} = -10, \{x,y\} \right) \mid x > 0 \right. \\ \left. 16 \cdot x + 6 \cdot y = 120 \right)$
 $x=6.$ and $y=4.$

AREA OF STUDY 4

- Discrete mathematics
- Space and measurement

Question 31 C

Given that $C(A + B)$ is defined, matrices A and B must be of the same order. Hence, matrix B is a 2×3 matrix and the result of $A + B$ is also a 2×3 matrix.

Since matrix C times a 2×3 matrix is defined, matrix C must have 2 columns. Given that matrix C has the same number of rows as the number of columns in matrix B , matrix C is a 3×2 matrix.

Question 32 A

$$\begin{aligned} A &= \begin{bmatrix} 1 \times 1 - 1 & 1 \times 2 - 1 & 1 \times 3 - 1 \\ 2 \times 1 - 1 & 2 \times 2 - 1 & 2 \times 3 - 1 \\ 3 \times 1 - 1 & 3 \times 2 - 1 & 3 \times 3 - 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 1 & 2 \\ 1 & 3 & 5 \\ 2 & 5 & 8 \end{bmatrix} \end{aligned}$$

Question 33 C

C is correct. Each edge in the network diagram should be represented by a '1' in the relevant elements of the matrix. Therefore:

- For the edge between vertices A and B , row A , column B and row B , column A should contain a '1'.
- For the edge between vertices B and C , row B , column C and row C , column B should contain a '1'.
- For the edge between vertices B and E , row B , column E and row E , column B should contain a '1'.
- For the edge between vertices C and D , row C , column D and row D , column C should contain a '1'.
- For the edge between vertices D and E , row D , column E and row E , column D should contain a '1'.
- For the loop at vertex E , row E , column E should contain a '1'.

A is incorrect. In this matrix, row D , column B contains a '1', but there is no direct link between vertices B and D .

B is incorrect. In this matrix, row E , column E contains a '2', but there is only one way to go from vertex E to vertex E .

D is incorrect. In this matrix, row D , column D contains a '1', but there is no loop at vertex D .

E is incorrect. In this matrix, row A , column A ; row B , column B ; row C , column C ; and row D , column D contain a '1', and row E , column E contains a '2'. Only row E , column E should contain a '1', as there is only one loop at vertex E in the diagram.

Question 34 C

To find the number of communications that can occur via one intermediary, the communication matrix needs to be squared.

$$C^2 = C \begin{matrix} & A & B & C & D & E \\ \begin{matrix} A \\ B \\ C \\ D \\ E \end{matrix} & \begin{bmatrix} 1 & 1 & 1 & 0 & 2 \\ 1 & 1 & 2 & 1 & 2 \\ 2 & 1 & 3 & 2 & 0 \\ 2 & 2 & 1 & 0 & 1 \\ 2 & 1 & 2 & 2 & 1 \end{bmatrix} \end{matrix}$$

From row A , column E , there are two ways that A can send a message to E via one intermediary.

Question 35 D

D is correct. A matrix is singular when its determinant is 0.

Using a CAS calculator to find the determinant of the matrix gives:

$$\det \begin{pmatrix} 0 & -1 & 2 \\ 1 & 0 & 1 \\ 4 & 1 & 2 \end{pmatrix} = 0.$$

A, B, C and **E** are incorrect. These matrices do not have determinants of 0.

$$\det \begin{pmatrix} 5 & 2 \\ 7 & 3 \end{pmatrix} = 1.$$

$$\det \begin{pmatrix} 2 & -1 & 3 \\ 1 & 0 & 2 \\ -6 & 3 & 1 \end{pmatrix} = 10.$$

$$\det \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix} = 1.$$

$$\det \begin{pmatrix} 5 & 3 & 1 \\ 2 & 6 & 8 \\ 1 & 0 & 0 \end{pmatrix} = 18.$$

Question 36 C

$\cos(\theta)$ can be found using the hypotenuse and adjacent side.

Using Pythagoras' theorem to find the adjacent side gives:

$$a^2 + b^2 = c^2$$

$$a^2 + 7^2 = 10^2$$

$$a = \sqrt{51}$$

Therefore:

$$\begin{aligned} \cos(\theta) &= \frac{\text{adjacent}}{\text{hypotenuse}} \\ &= \frac{\sqrt{51}}{10} \end{aligned}$$

Question 37 D

The shortest side in triangle ABC is BC . As triangles ABC and JKL are similar, the ratio of their shortest sides is:

$$5:15$$

$$1:3$$

Therefore, multiplying the longest side in triangle ABC by 3 to find the length of the longest side in triangle JKL gives:

$$46 \times 3 = 138$$

Question 38 A

Using the perimeter of a rectangular prism to find the height gives:

$$P = 4l + 4w + 4h$$

$$72 = (4 \times 9) + (4 \times 6) + 4h$$

$$h = 3$$

Using a CAS calculator gives:

<code>solve(72=4*9+4*6+4*l,l)</code>	<code>l=3</code>
--------------------------------------	------------------

Question 39 B

Assuming that the cube has a side length of x , its original volume is $V = x^3$.

When all the sides are halved, the cube's new side length is $\frac{1}{2}x$.

Substituting the new side length into the original volume gives:

$$\begin{aligned} V &= \left(\frac{1}{2}x\right)^3 \\ &= \frac{1}{8}x^3 \end{aligned}$$

Therefore, the new volume is $\frac{1}{8}$ of the original volume; therefore, the percentage is $\frac{1}{8} \times 100 = 12.5\%$.

Question 40 C

As the ratio of the cone's base to the cylinder's base is 5 : 3:

$$\text{base}_{\text{cone}} = \frac{5}{3} \times \text{base}_{\text{cylinder}}$$

Finding the height of the cone gives:

$$V_{\text{cone}} = V_{\text{cylinder}}$$

$$\frac{1}{3} \text{base}_{\text{cone}} \times \text{height}_{\text{cone}} = \text{base}_{\text{cylinder}} \times \text{height}_{\text{cylinder}}$$

$$\frac{1}{3} \times \frac{5}{3} \times \text{base}_{\text{cylinder}} \times \text{height}_{\text{cone}} = \text{base}_{\text{cylinder}} \times 8$$

$$\text{height}_{\text{cone}} = 14.4$$