VCE General Mathematics Units 1&2

AT1.4

Matrices Test

Name: _Solutions_ _____

Circle teacher's name: Ms Jabeen   Ms Le   Mr Rossignolo   Ms Yang

Total _____/50

Assessment Criteria

Students should be able to:

- Define and explain key concepts and apply a range of related mathematical routines and procedures.
- Apply mathematical processes in non-routine contexts including situations requiring problem-solving, modelling or investigative techniques or approaches and analyse and discuss these applications of mathematics.
- Use numerical, graphical and symbolic functionalities of technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches.

Instructions

A single bound reference and a CAS and scientific calculator permitted.

Answer all questions in the spaces provided.

Round values to 2 decimal places where not specified.

In questions where more than one mark is available, appropriate working must be shown.

Multiple choice questions are worth one mark each.

<u>**Section A**</u>   **Multiple Choice Questions**   **12 marks**

Circle the letter corresponding to the correct response.

1   A matrix has 4 columns and 3 rows. What is the order of this matrix?

   A. 4×3

   B. 4×4

   C. 3×3

   D. 3×4

   E. 3×12

2   Which term most accurately describes the following matrix?

   $$[1 \quad 9 \quad 4 \quad 16]$$

   A. Column matrix

   B. Row matrix

   C. Square matrix

   D. Scalar matrix

   E. Identity matrix

3   If $A = \begin{bmatrix} 4 & 3 \\ 2 & 7 \\ 7 & 8 \end{bmatrix} - \begin{bmatrix} 6 & 5 \\ 1 & 3 \\ 7 & 9 \end{bmatrix}$, what is a_{12} ?

   A. -2

   B. 1

   C. 2

   D. 3

   E. 5

The following information is relevant for Questions 4 to 6.

$$A = \begin{bmatrix} 1 & 9 \\ 6 & 5 \\ 2 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 3 \\ 7 & 5 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 9 & 4 \\ 5 & 3 & 2 \end{bmatrix}$$

4 Which of the following matrix multiplication cannot be defined?

  A. AB

  B. BB

  C. AC

  D. CA

  E. BA

5 The order of the matrix BC is

  A. 2×2

  B. 2×3

  C. 3×2

  D. 3×3

  E. 2×6

6 If we let D = AC, what would be d_{31} ?

  A. 46

  B. 69

  C. 12

  D. 22

  E. d_{31} does not exist

7   If $M = \begin{bmatrix} 1 & 3 \\ 7 & 5 \end{bmatrix}$, and $MB = \begin{bmatrix} 1 & 3 \\ 7 & 5 \end{bmatrix}$, what is matrix B?

A. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ D. $\begin{bmatrix} 1 & 3 \\ 7 & 5 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ E. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

8   On a Monday, Maria's store sold 18 pillows, 4 mattresses, and 6 quilts, costing $20, $115, and $45 respectively. Which of the following will correctly result in the total value of the sales of pillows and quilts on the day?

A. $\begin{bmatrix} 18 & 4 & 6 \end{bmatrix} \times \begin{bmatrix} 20 \\ 115 \\ 45 \end{bmatrix}$

B. $\begin{bmatrix} 0 & 0 & 6 \end{bmatrix} \times \begin{bmatrix} 20 \\ 115 \\ 45 \end{bmatrix}$

C. $\begin{bmatrix} 0 & 4 & 0 \end{bmatrix} \times \begin{bmatrix} 20 \\ 115 \\ 45 \end{bmatrix}$

D. $\begin{bmatrix} 18 & 0 & 6 \end{bmatrix} \times \begin{bmatrix} 20 \\ 115 \\ 45 \end{bmatrix}$

E. $\begin{bmatrix} 18 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 20 \\ 115 \\ 45 \end{bmatrix}$

A map of the roads connecting villages P, Q, R and S is shown below:
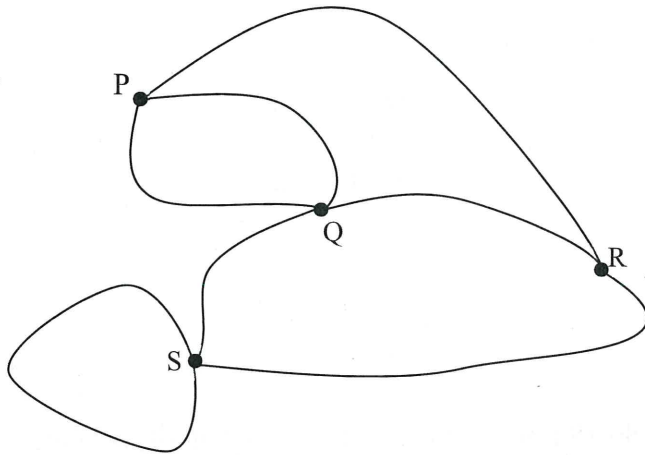
9 What is matrix M that corresponds to the road network?

	P	Q	R	S
P	0	1	1	0
A. Q	1	0	1	1
R	1	1	0	1
S	0	1	1	1

	P	Q	R	S
P	0	2	1	1
D. Q	2	0	1	1
R	1	1	0	1
S	1	1	1	1

	P	Q	R	S
P	0	2	1	0
B. Q	2	0	1	1
R	1	1	0	1
S	0	1	1	0

	P	Q	R	S
P	0	1	1	1
E. Q	1	0	1	1
R	1	1	0	1
S	1	1	1	0

	P	Q	R	S
P	0	2	1	0
C. Q	2	0	1	1
R	1	1	0	1
S	0	1	1	1

10 In matrix M, what does m_{14} mean, in context of the road network?

A. The number of roads connecting villages P and Q

B. The number of roads connecting villages P and R

C. The number of roads connecting villages S and Q

D. The number of roads connecting villages Q and R

E. The number of roads connecting villages S and P

11 Given that $Q = \begin{bmatrix} 6 & 4 \\ -1 & 3 \end{bmatrix}$, what is Q^2 ?

(A) $\begin{bmatrix} 32 & 36 \\ -9 & 5 \end{bmatrix}$

B. $\begin{bmatrix} 32 & -36 \\ 9 & 5 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 36 & 16 \\ -1 & 9 \end{bmatrix}$

E. $\begin{bmatrix} 36 & 16 \\ 1 & 9 \end{bmatrix}$

12 What are the conditions for matrix multiplication to be defined?

(A) The number of columns in the first matrix is equal to the number of rows in the second matrix.

B. The number of columns in the first matrix is equal to the number of columns in the second matrix.

C. The number of rows in the first matrix is equal to the number of rows in the second matrix.

D. The number of rows in the first matrix is equal to the number of rows in the second matrix.

E. The number of rows in the first matrix is equal to the very first element of the second matrix.

1. Use the information below to determine the resulting matrices:

$$U = \begin{bmatrix} 4 & 2 \\ 0 & 0 \\ 1 & 2 \end{bmatrix} \quad V = \begin{bmatrix} 1 & 1 & 0 \\ 3 & 1 & 9 \\ 2 & 3 & 8 \end{bmatrix} \quad W = \begin{bmatrix} 0 & 1 \\ 2 & 0 \\ 1 & 3 \end{bmatrix} \quad X = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix} \quad Y = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad Z = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

a) $Y + Z$

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad ①$$

1 mark

b) $-5X$

$$\begin{bmatrix} -15 & -5 & 5 \\ -5 & 0 & -5 \end{bmatrix} \quad ①$$

1 mark

c) $5U + 2W$

$$\begin{bmatrix} 20 & 12 \\ 4 & 0 \\ 7 & 16 \end{bmatrix} \quad ①$$

1 mark

d) $V^{-1}V$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad ①$$

1 mark

e) Calculate XY and state the order of the matrix.

$$XY = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \text{ORDER : } 2 \times 1 \quad ①$$

$①$

2 marks

2. If $\begin{bmatrix} 3 & 0 \\ 5 & a \end{bmatrix} + \begin{bmatrix} 2 & 23 \\ b & 1 \end{bmatrix} = \begin{bmatrix} c & 23 \\ 3 & -4 \end{bmatrix}$, find the values of a, b and c.

$$a + 1 = -4 \qquad\qquad c = 5 ①$$

$$a = -5 ①$$

$$5 + b = 3$$

$$b = -2 ①$$

3 marks

3.

If $P = \begin{bmatrix} 14.50 \\ 21.60 \\ 19.20 \end{bmatrix}$ and $Q = \begin{bmatrix} 2500 & 3400 & 1890 \\ 1765 & 4588 & 2456 \end{bmatrix}$. Is the product of P and Q defined?

You must explain your response.

$$P \times Q \qquad\qquad\qquad Q \times P$$

$$(3 \times 1) \times (2 \times 3) \qquad\qquad (2 \times 3) \times (3 \times 1)$$

Not equal $\qquad\qquad\qquad$ Equal

∴ Not defined $\qquad\qquad\qquad$ ∴ Defined

① $\qquad\qquad\qquad\qquad\qquad$ ①

2 marks

4. Show how the matrix $[1 \quad 1 \quad 1]$ can be used to sum the columns of $\begin{bmatrix} 9 & 0 & 1 \\ 2 & 8 & 5 \\ -2 & 4 & -5 \end{bmatrix}$

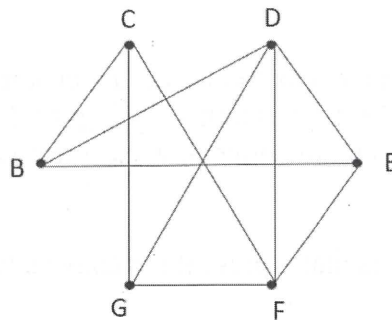$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 9 & 0 & 1 \\ 2 & 8 & 5 \\ -2 & 4 & -5 \end{bmatrix} \qquad ①$$

$$= \begin{bmatrix} 9 & 12 & 1 \end{bmatrix} \qquad ①$$

2 marks

5. Craig has drawn a network diagram showing the connections between himself and his group of 6 friends: Bonnie, Caroline, Damon, Elena, Finn and Georgina.

(a) Construct an adjacency matrix X to represent this information.   ⊝ per error.

$X =$

	B	C	D	E	F	G
B	O	1	1	1	O	O
C	1	O	O	O	1	1
D	1	O	O	1	1	1
E	1	O	1	O	1	O
F	O	1	1	1	O	1
G	O	1	1	O	1	O

3 marks

(b) The square matrix indicates the number of two-step connections between each pair of points. In this case, X^2 would indicate the number of mutual friends each pair of students has.

Calculate X^2 and use it to find the number of mutual friends between Caroline and Damon.

$X^2 =$

	B	C	D	E	F	G
B	3	O	1	1	3	2
C	O	3	3	2	1	1
D	1	3	4	2	2	1
E	1	2	2	3	1	2
F	3	1	2	1	4	2
G	2	1	1	2	2	3

① 2 marks

∴ Caroline and Damon have 3 friends in common ①

Section C   Extended Response   20 marks

Include working throughout.

Question 1 (5 marks)

Greer is working on her indoor plant collection. On a recent trip to Bunnings, she bought two ferns and three succulents for $58. Last month she bought 5 ferns and one succulent for $60.50. If the price of a fern is f and the price of a succulent is s, and the price of both ferns and succulents remain constant:

(a) Write two linear equations that represent the above situation.

$$2f + 3s = 58 \quad \ldots \text{(1)} \qquad ①$$

$$5f + s = 60 \cdot 5 \quad \ldots \text{(2)} \qquad ①$$

2 marks

(b) Set up a matrix equation to represent this information.

$$\begin{bmatrix} 2 & 3 \\ 5 & 1 \end{bmatrix} \begin{bmatrix} f \\ s \end{bmatrix} = \begin{bmatrix} 58 \\ 60 \cdot 5 \end{bmatrix} \qquad ①$$

1 mark

(c) Hence, determine the cost of each fern and succulent to the nearest cent.

$$\begin{bmatrix} f \\ s \end{bmatrix} = \begin{bmatrix} 9 \cdot 5 \\ 13 \end{bmatrix} \qquad \therefore \text{ferns cost } \$9.50 \text{ each}$$

$$\text{succulents cos } \$13 \text{ each} \qquad ①$$

①

2 marks

Question 2 (8 marks)

The diagram below involves three physical activities that are part of a weekly fitness program. At the beginning of the program participants can choose to start on the treadmills (*T*), weights (*W*) or bikes (*B*) to undergo their fitness program.
The transition diagram below shows the way in which the participants are expected to change their choice of activity from week to week.
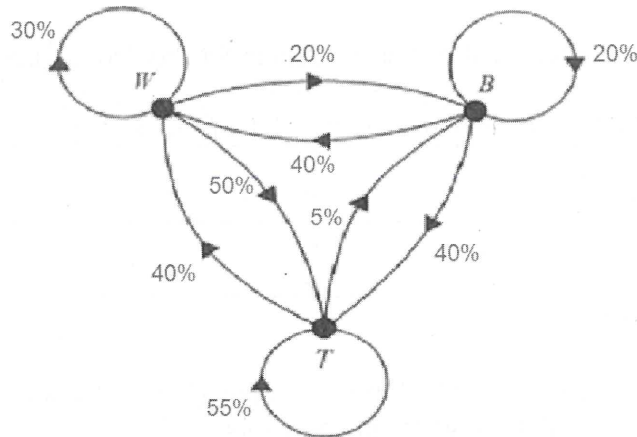
(a) What information does the 5% on the diagram provide?

5% are predicted /expected to change from treadmills to bikes in the following week. ①

1 mark

(b) Complete the transition matrix *X*, below.

$$
\begin{array}{c}
\quad\quad\quad \textit{this week} \\
\quad\quad T \quad\quad W \quad\quad B \\
X = \begin{bmatrix} 0.55 & 0.50 & 0.40 \\ 0.40 & 0.30 & 0.40 \\ 0.05 & 0.20 & 0.20 \end{bmatrix} \begin{array}{l} T \\ W \ \textit{next week} \\ B \end{array}
\end{array}
$$

①

Initially 100 people chose treadmills, 80 people chose weights and 5 people chose bikes in Week 1 of the program.

1 mark

(c) Write down a column matrix to describe the situation. Call the matrix S_0.

$$
S_0 = \begin{array}{c} T \\ W \\ B \end{array} \begin{bmatrix} 100 \\ 80 \\ 5 \end{bmatrix}
$$

①

1 mark

(d)(i) Using $S_{n+1} = X S_n$ find S_1

$$S_1 = X S_0 = \begin{bmatrix} 97 \\ 66 \\ 22 \end{bmatrix} \begin{matrix} T \\ W \\ B \end{matrix}$$ ①

1 mark

(ii) How many members will **not** change their activity from week 1 to week 2?

$$(0.55 \times 100) + (0.3 \times 80) + (0.2 \times 5)$$

$$= 55 + 24 + 1$$

$$= 80$$ ①

1 mark

(iii) 66 members are expected to choose weights (W) in Week 2.
Complete the calculation below to show this:

$$100 \times \boxed{0.4} + 80 \times \boxed{0.3} + 5 \times \boxed{0.4} = 66$$ ①

1 mark

(e) (i) In week 4, state the expected number of participants in treadmill (**T**) and bike (**B**).

Need to calculate S_3; $S_3 = X^3 S_0 = \begin{bmatrix} 95.0125 \\ 67.26 \\ 22.7275 \end{bmatrix}$ ~95 (Treadmill)

~23 (Bike) ①

1 mark

(ii) In the long term how many participants do you expect to choose weights?

Long term (could calculate S_{20}) : $\begin{bmatrix} 94.97 \\ 67.27 \\ 22.75 \end{bmatrix}$ ∴ 67–68 (or just 67) choose weights in the long term. ①

1 mark

Question 3 (7 marks)

The following shows the number of target areas are hit in archery by 4 people. Hitting an area of the archery target gives a certain number of points. Hitting the outer ring gives 2 points, the middle ring 5 points, the inner ring 7 points and a bullseye 10 points.

	Outer Ring	Middle Ring	Inner Ring	Bullseye
Robb	1	2	7	10
Jon	1	3	6	9
Arya	2	2	4	12
Bran	4	6	5	4

Let the matrix above be denoted A.

(a) What matrix B is needed such that AB results in the total scores of the players?

$$B = \begin{bmatrix} 2 \\ 5 \\ 7 \\ 10 \end{bmatrix}$$ ①

1 mark

(b) Using the result in (a) who scored the most? How much did they beat the second player by?

$$AB = \begin{bmatrix} 161 \\ 149 \\ 162 \\ 113 \end{bmatrix} \begin{matrix} Robb \\ Jon \\ Arya \\ Bran \end{matrix}$$ ①
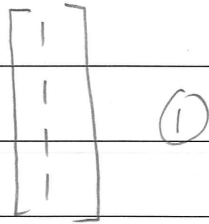
Arya score the most (beat Robb by 1) ①

2 marks

(c) If you wanted to know how many points each player gained from hitting the inner target what matrix would you multiply A with?

$$\begin{bmatrix} 0 \\ 0 \\ 7 \\ 0 \end{bmatrix}$$ ①

1 mark

(d) Suppose you wanted to know how times each player hit the target. What matrix would you multiply A with?

$$\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$ ①

1 mark

(e) Massimo wants to record how many times each target area is hit by all players. Explain in detail how this can be achieved using matrix operations. Specify which matrix he could use with A to form a matrix operation to show the number of times each target area has been hit.

Massimo can use matrix $\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$ (can call it C or other) ①

Next: calculate C × A (in that order)

$$CA = \begin{bmatrix} 8 & 13 & 22 & 35 \end{bmatrix}$$ ①.
        outer Mid Inner Bullseye
        (include labels).

2 marks

End of Test