General Mathematics

Written Examination 2

Question and Answer Book

2024 Insight Year 12 Trial Exam Paper

- **Reading time:** 15 minutes
- **Writing time:** 1 hour and 30 minutes
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and Answer Book of 27 pages
- Formula Sheet
- Working space is provided throughout the book.

Instructions

- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination

- You may keep the Formula Sheet.

Students are **not** permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Number of questions: 14
Number of questions to be answered: 14
Number of marks: 60

Instructions

- Answer **all** questions in the spaces provided.
- In all questions where a numerical answer is required, you should only round your answer when instructed to do so.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data analysis

Question 1 (7 marks)

Data obtained about the visitors staying at a camping ground over an Easter long weekend is shown in Table 1 below.

Table 1

Site number	Length of stay (nights)	Number of campers	Repeat booking	Distance travelled (km)
1	5	7	Y	0–<20
2	7	3	N	50+
3	4	2	Y	50+
4	3	4	Y	20–<50
5	3	1	Y	50+
6	7	4	N	20–<50
7	10	5	N	20–<50
8	7	3	Y	50+
9	14	6	N	20–<50
10	3	5	Y	50+
11	3	5	Y	50+
12	5	8	N	50+
13	7	3	Y	20–<50
14	7	5	N	20–<50
15	7	6	N	50+
16	14	4	Y	0–<20
17	3	1	N	20–<50
18	4	2	N	20–<50
19	8	2	N	50+
20	7	3	N	50+

The variables in this data set are:

- *site number:* the location of the camp sites in the camping ground
- *length of stay*: the number of nights each group stayed
- *number of campers*: the number of people who stayed at the site
- *repeat booking*: if the campers had previously booked at the camping ground (Y = yes, N = no)
- *distance travelled*: on average, how far the campers at each site had travelled to get to the site (0–<20 km, 20–<50 km, 50+ km).

a. Which variables are *categorical* variables?

1 mark

b. Determine the median number of nights for the *length of stay*.

1 mark

c. Determine the mean *number of campers*, rounded to the nearest whole number.

1 mark

d. What percentage of the sites had a *length of stay* less than five nights?

1 mark

e. Use the data from Table 1 to complete the two-way frequency table below.

1 mark

Length of stay (nights)	Repeat booking	
	Yes	**No**
1–3	4	
4–7	4	
8+	1	
Total	9	

The distribution of the age of the campers, in years, is shown in the five-number summary below.

Minimum	Q1	Median	Q3	Maximum
2	21	26	41	76

f. Show that the oldest camper is considered to be an outlier.

2 marks

Question 2 (4 marks)

Campers and locals swim at a river that is adjacent to the campsite.

A total of 400 swimmers visited the river one day. The amount of time they spent at the river was normally distributed.

Sixteen per cent of the swimmers stayed at the river for less than 55 minutes and 47.5% stayed for between 80 and 130 minutes.

a. Determine

i. how many swimmers stayed at the river between 80 and 130 minutes

1 mark

ii. how many swimmers stayed at the river for longer than 55 minutes.

1 mark

b. Using the $68–95–99.7\%$ rule, determine the mean and standard deviation of the amount of time spent at the river.

2 marks

mean time = ☐ standard deviation = ☐

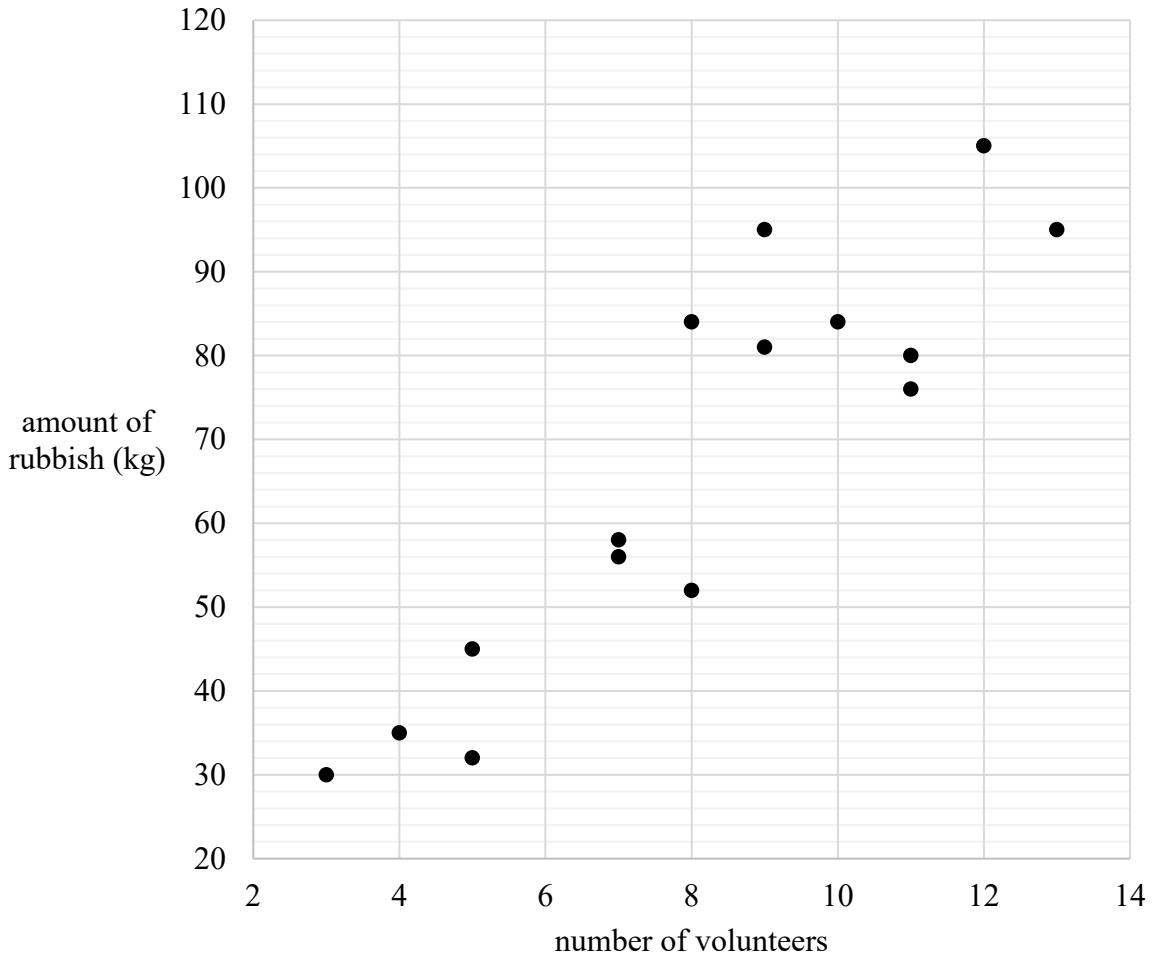
Question 3 (6 marks)

The campsite is situated next to a national park. Each Sunday the local council runs a clean-up day for the national park.

The number of volunteers for the clean-up each Sunday for 15 weeks and the amount of rubbish they collected, in kilograms, was recorded and a scatterplot of the data drawn.

Table 2

No. of volunteers	5	9	12	7	3	8	8	13	11	5	4	10	9	11	7
Amount of rubbish (kg)	45	95	105	58	30	52	84	95	76	32	35	84	81	80	56



When a least squares line is fitted to the scatterplot, the equation is found to be

$$Amount\ of\ rubbish\left(kg\right) = 6.55 + 7.46 \times number\ of\ volunteers$$

a. Draw the least squares line on the scatterplot graph above.

1 mark

b. Interpret the slope of the regression line in terms of the variables *number of volunteers* and *amount of rubbish*.

1 mark

c. Calculate the correlation coefficient, rounding your answer to three decimal places.

1 mark

d. Determine the percentage of the variation in *amount of rubbish* that cannot be explained by the variation in *number of volunteers*.

Round your answer to one decimal place.

1 mark

e. The council wants to predict the amount of rubbish collected by 30 volunteers.

Use the equation of this least squares line to predict the weight of rubbish collected, in kilograms, by 30 volunteers.

Round your answer to the nearest whole number.

1 mark

f. On a day when there are 10 volunteers, 89 kg of rubbish is collected.

Determine the residual value.

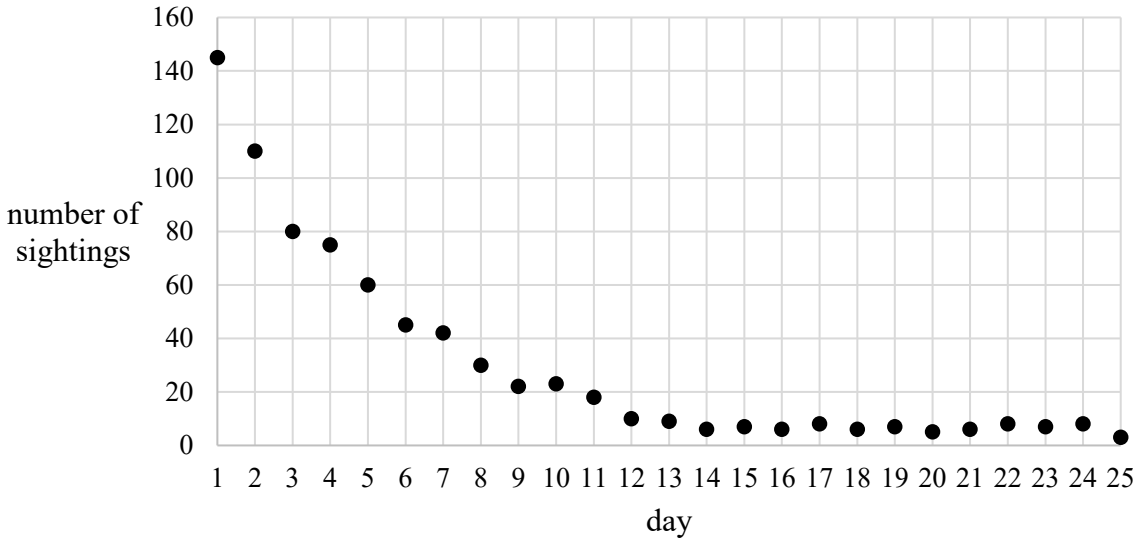
1 mark

Question 4 (3 marks)

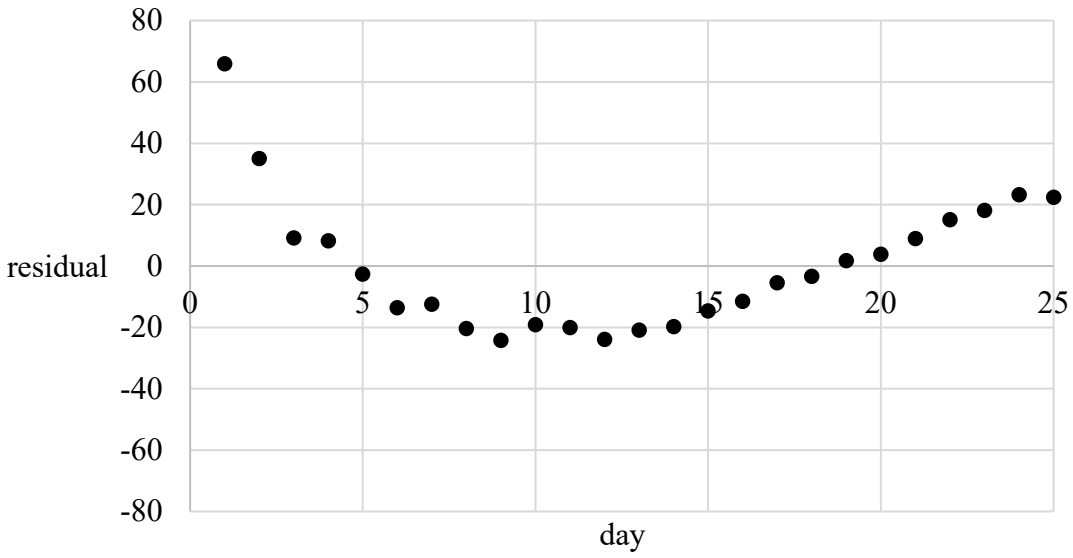
To raise awareness for an endangered bird that resides in the area, a local wildlife organisation ran a campaign asking campers to record sightings of the bird.

The total number of recorded sightings over 25 days is shown below.

Day	Number
1	145
2	110
3	80
4	75
5	60
6	45
7	42
8	30
9	22
10	23
11	18
12	10
13	9
14	6
15	7
16	6
17	8
18	6
19	7
20	5
21	6
22	8
23	7
24	8
25	3



The residual plot obtained when the least squares line was fitted to the data is shown below.



a. A residual plot can be used to test an assumption about the nature of the association between two numerical variables. What is this assumption?

1 mark

_____

b. Does the residual plot above support this assumption? Explain your answer.

1 mark

_____

_____

c. A $\log_{10}$ transformation is applied to the *number of sightings* and a least squares regression line is fitted to the transformed data.

The slope is calculated to be -0.0611 and the intercept is 1.9929.

Rounding the values to two significant figures, complete the equation of the least squares regression line.

1 mark

$$\log_{10}(\textit{number of sightings}) = \square + \square \times \textit{day}$$

Question 5 (4 marks)

The percentage of the council's staff that are working from home through each quarter in 2023 and 2024 is shown in Table 3.

Table 3

	Q1	Q2	Q3	Q4
2023	38	45	68	46
2024	32	39	63	35

a. The seasonal indices for Q1 and Q3 are shown in Table 4 below.

Table 4

	Q1	Q2	Q3	Q4
Seasonal index	0.76		1.44	

Find the seasonal indices for Q2 and Q4, rounding each answer to two decimal places.

2 marks

b. The percentage of staff working from home in Q3 of 2024 is 63%.

Calculate the deseasonalised percentage of staff working from home in Q3 of 2024. Round your answer to the nearest whole number.

1 mark

c. A least squares line is fitted to the deseasonalised data. The equation of the least squares line is:

$$\textit{Deseasonalised staff working from home (\%)} = 52.3 - 1.45 \times \textit{quarter number}$$

Calculate the actual percentage of staff working from home predicted for Q3 in 2025, rounding your answer to the nearest whole number.

1 mark

Recursion and financial modelling

Question 6 (4 marks)

Kathryn puts money into an investment account that earns compound interest at a rate of 3.9% per annum, compounding fortnightly.

The balance of her investment, in dollars, after n fortnights, B_n, can be modelled by the recurrence relation

$$B_0 = 1800 \qquad B_{n+1} = 1.0015B_n$$

a. How much was invested?

1 mark

b. Showing recursive calculations, determine the balance of the investment after two fortnights. Round your answer to the nearest cent.

1 mark

c. Show that the annual compound interest rate is 3.9% per annum.

1 mark

d. Calculate the balance after four years.

1 mark

Question 7 (4 marks)

Kathryn purchases items from a local second-hand store.

She buys a table that she believes is an antique for $\$2800$.

If the table is an antique, it is expected to increase in value at a flat rate of 8% per annum.

If the table is not an antique, it is expected to depreciate by 8% per annum on a reducing-balance basis.

a. If the table is an antique, after how many years will it have doubled in value?

1 mark

b. If the table is not an antique, after how many years will it have halved in value?

1 mark

Kathryn pays $\$1700$ for an automated cooking appliance called a Thermonator. It depreciates on a unit-cost basis.

On average, Kathryn uses the Thermonator four times a week. After five years the appliance is worth $\$712$.

c. By how much does the appliance depreciate per use?

1 mark

d. Kathryn borrowed money to cover some of the cost of the Thermonator. She was charged 4% per annum, compounding quarterly. Her repayments were $\$77.58$ per quarter. The loan was repaid in five years.

How much did Kathryn borrow, to the nearest dollar?

1 mark

Question 8 (4 marks)

Kathryn has money to invest.

She considers two options for the investment, both earning 4.8% per annum, compounding monthly:

- Option 1: a perpetuity, where she only withdraws the interest earned each month
- Option 2: where she makes withdrawals of $\$458.80$ each month for four years.

a. What is the name of the financial model described in Option 2?

1 mark

b. How much would Kathryn need to invest for the perpetuity to earn $\$2400$ interest over two years?

1 mark

Kathryn decides to invest $\$20\,000$ via Option 2. After two years, the interest rate with Option 2 increases to 4.9% per annum. Her investment remains as a four-year investment.

c. What is the total interest earned over the four-year period with Option 2?

Round your answer to the nearest dollar.

2 marks

Matrices

Question 9 (4 marks)

There are four types of tickets for entry into an aquarium: child (C), adult (A), pensioner (P) and family (F). The cost of each ticket is shown in matrix T.

$$T = \begin{bmatrix} 12 \\ 18 \\ 15 \\ 55 \end{bmatrix} \begin{matrix} C \\ A \\ P \\ F \end{matrix}$$

a. Which element shows the cost for one adult?

1 mark

_____

b. Complete the following calculation to show that the cost of a ticket for an adult and a child is $\$30$.

1 mark

$$[\_\_\_ \quad \_\_\_ \quad \_\_\_ \quad \_\_\_] \times \begin{bmatrix} 12 \\ 18 \\ 15 \\ 55 \end{bmatrix} = [30]$$

c. On a particular Sunday, 140 tickets were sold to children, 72 to adults, 12 to pensioners and 55 to families. This is represented in matrix N.

The total sales, in dollars, is calculated by the matrix product $N \times T = \$6181$.

Write matrix N in the space below.

1 mark

$N =$

d. A 25% decrease in ticket sales occurs on Mondays for all types of tickets.

Matrix T is multiplied by a scalar value, k, to determine the expected number of sales for the Monday following the Sunday in **part c**.

Write the value of the scalar k.

1 mark

$k =$ ☐

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Question 10 (3 marks)

Ana conducts a walking tour of six exhibits: sharks (S), seals (L), turtles (T), exotic fish (E), stingrays (G) and penguins (P), before beginning the tour again in the same order.

a. Complete the matrix below to show Ana's movement between the six locations. Two rows have been completed.

1 mark

this exhibit

	E	G	L	P	S	T	
	0	0	0	0	0	1	E
	—	—	—	—	—	—	G
	—	—	—	—	—	—	L
	—	—	—	—	—	—	P
	—	—	—	—	—	—	S
	0	0	1	0	0	0	T

next exhibit

Staff at the aquarium use two-way radio to communicate with colleagues working the same shift.

The radios experience interference in some parts of the aquarium, so not all staff can communicate with each other.

The communication between Ana (A), Bernie (B), Celia (C), Diego (D) and Erin (E) is shown in matrix F.

receiver

$F =$ *sender*	A	B	C	D	E
A	0	1	0	1	1
B	0	0	0	0	1
C	0	0	0	1	1
D	1	0	1	0	0
E	1	0	1	0	0

In this matrix:

- The '1' in row A, column B indicates that Ana can send a message directly to Bernie.
- The '0' in row B, column C indicates that Bernie cannot send a message directly to Celia.

b. Which staff can Ana both directly receive a message from and send a message to?

1 mark

c. Using matrix F to calculate the two-step communication between colleagues, which of her colleagues can send a message to Ana only using two-step communication?

1 mark

Question 11 (5 marks)

When purchasing an annual membership to the aquarium (A), customers are also entitled to free entry into the local zoo (Z) and museum (M) once a month.

Matrix C shows the proportion of customers visiting each facility and their movement between them each month.

this month

$$C = \begin{matrix} \\ A \\ Z \\ M \end{matrix} \begin{matrix} \begin{matrix} A & Z & M \end{matrix} \\ \begin{bmatrix} 0.80 & 0.25 & 0.05 \\ 0.15 & 0.60 & 0.05 \\ 0.05 & 0.15 & 0.90 \end{bmatrix} \end{matrix} \begin{matrix} \\ \\ \textit{next month} \\ \\ \end{matrix}$$

a. What percentage of visitors at the museum this month are not expected to return next month?

1 mark

b. Interpret the element in c_{12}.

1 mark

c. If there are 100 people at the aquarium, how many are expected to go to the zoo next month?

1 mark

The number of visitors to each location in July is shown in matrix S_0.

$$S_0 = \begin{bmatrix} 12000 \\ 8000 \\ 7000 \end{bmatrix} \begin{matrix} A \\ Z \\ M \end{matrix}$$

d. How many visitors are expected to visit the museum in September?

1 mark

Management at all three locations decide that they want an equal number of visitors at each location each month. They decide that 9000 visitors at each location each month will help them plan staffing requirements more accurately.

For this to occur, the aquarium will need less visitors and the zoo and museum will both need more visitors.

e. How many additional visitors do the zoo and museum need each month to ensure that there is an equal number of visitors at each location?

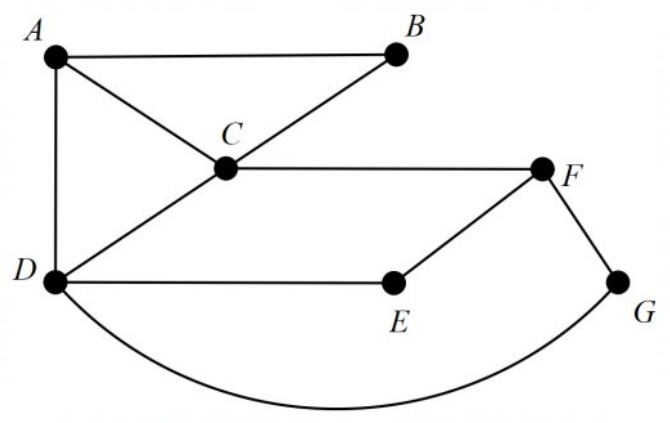
1 mark

Networks and decision mathematics

Question 12 (3 marks)

A mobile coffee van services seven buildings, A to G, in an industrial park.

The graph below shows the roads connecting each building.



a. How many faces does the network have?

1 mark

b. If the van is at D, which buildings can it visit next without passing any other buildings?

1 mark

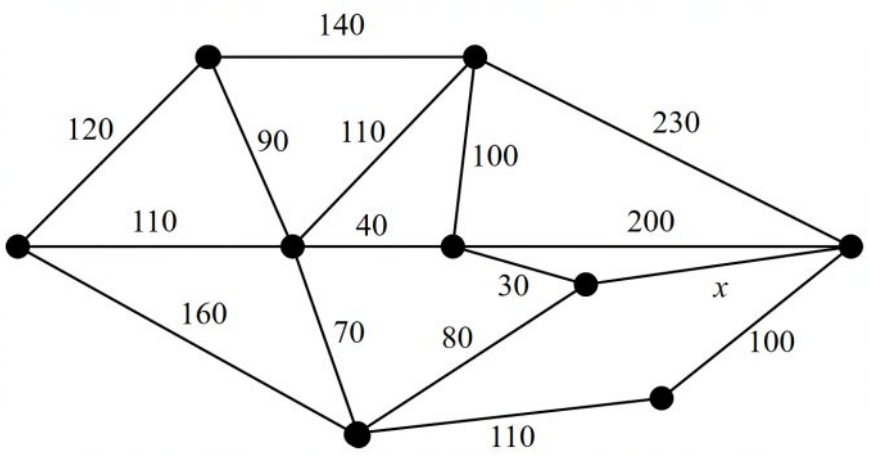
c. The coffee van starts at building A and travels all the roads exactly once, except one. In order to end at building G, which road must be travelled twice?

1 mark

Question 13 (4 marks)

Maintenance is being conducted on a set of stormwater drains in the industrial park. The network below shows the length of each drain, in metres.

In order to minimise costs, the contractor begins the maintenance on the drains that form a minimum spanning tree.

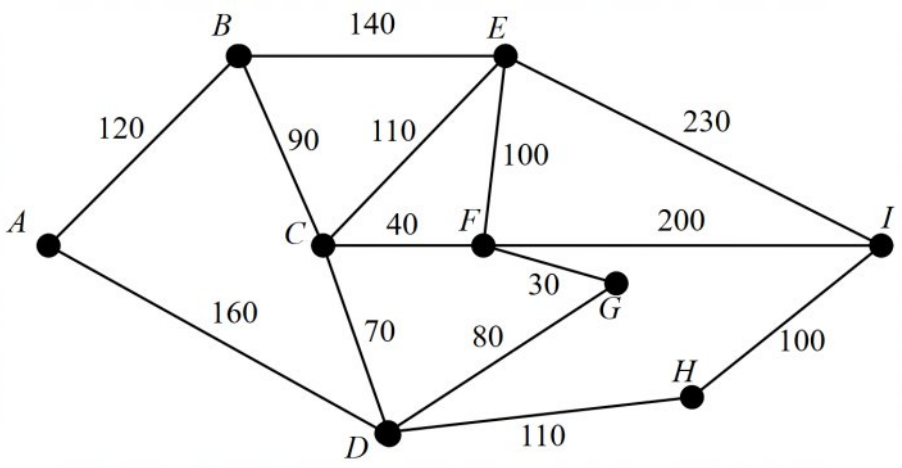


The length of the drains that are maintained is 600 metres.

a. One of the drains in the minimum spanning tree is x metres long. Determine the length of x, in metres.

1 mark

The maintenance team revises the network, removing some drains. The graph below shows the updated network.



b. Determine the shortest distance, in metres, from A to I.

1 mark

c. The team follows a Hamiltonian path around the network, starting at A and ending at H. The length is 870 metres.

Write down the path taken.

1 mark

The team assigned to this maintenance project have four tasks to complete. The four workers in the team are Bonnie, Sonny, Frankie and Joni. The time taken in hours for each of them to complete the four tasks is shown below.

	Task 1	Task 2	Task 3	Task 4
Bonnie	12	14	12	10
Sonny	13	12	15	10
Frankie	10	10	11	9
Joni	11	14	15	11

d. In order to minimise the time taken to complete the tasks, each worker is assigned just one task.

Complete the table below to show the allocation required to minimise the total time.

1 mark

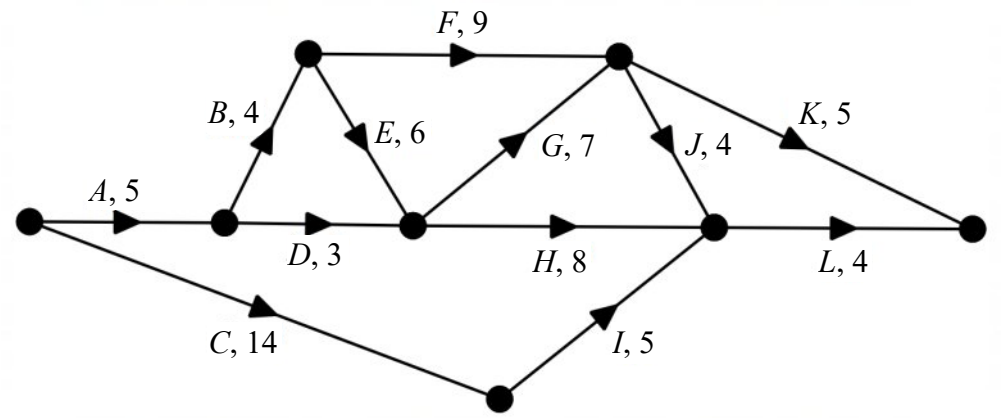
Worker	Task
Bonnie	
Sonny	
Frankie	
Joni	

Question 14 (5 marks)

A new warehouse is to be built in the industrial park.

The project involves 12 activities, A to L. The directed network below shows these activities and their completion times, in days.

The minimum completion time for the project is 30 days.



a. Write down the activities that have exactly two immediate predecessors.

1 mark

_____

b. Write down the activities on the critical path.

1 mark

_____

c. Which activities have an earliest start time of 22 days?

1 mark

_____

The project manager wants to reduce the length of the project.

The maximum reduction in time of each activity and the cost of the reduction is shown.

Activity	A	C	D	G	H	J	L
Maximum reduction (days)	2	1	1	2	3	3	2
Cost of reduction per day ($)	1000	500	2000	2000	1500	1000	2000

d. If activities A and D have their completion times reduced by the maximum reduction shown, what is the new minimum completion time, in days?

1 mark

e. Applying all the suitable reductions, what is the minimum cost of the reductions required to complete the project in the shortest time possible?

1 mark

End of Question and Answer Book