2024
NHT

General Mathematics Examination 1

Multiple-Choice Question Book

VCE (NHT) Examination – Thursday 23 May 2024

- Reading time is **15 minutes**: 2.00pm to 2.15pm
- Writing time is **1 hour 30 minutes**: 2.15pm to 3.45pm

Approved materials

- One bound reference that may be annotated
- One approved CAS calculator or CAS software and one scientific calculator

Materials supplied

- Multiple-Choice Question Book of 28 pages
- Formula Sheet
- Multiple-Choice Answer Sheet

Instructions

- Follow the instructions on your Multiple-Choice Answer Sheet.
- You may keep this Question Book and the Formula Sheet.

Students are **not** permitted to bring mobile phones and/or any unauthorised electronic devices into the examination room.

Contents

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY
VICTORIA State Government

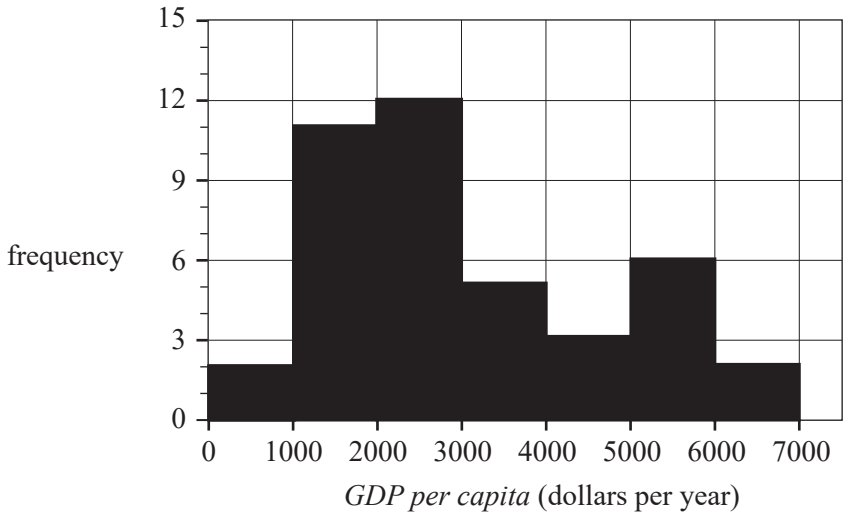
Instructions

- Answer **all** questions in pencil on the Multiple-Choice Answer Sheet.
- Choose the response that is **correct** for the question.
- A correct answer scores 1; an incorrect answer scores 0.
- Marks will **not** be deducted for incorrect answers.
- No marks will be given if more than one answer is completed for any question.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data analysis

Question 1

The histogram below shows the distribution of the *GDP per capita* (gross domestic product per capita), in dollars per year, for a sample of 41 African countries in 2021.



Data: <data.worldbank.org>

The median *GDP per capita*, in dollars per year, for this sample will be within the range

A. greater than or equal to 0 but less than 1000

B. greater than or equal to 1000 but less than 2000

C. greater than or equal to 2000 but less than 3000

D. greater than or equal to 3000 but less than 4000

E. greater than or equal to 4000 but less than 5000

Question 2

The variables *exercise type* (aerobic, boxing, circuit) and *recovery time* (short, medium, long) are

A. a nominal variable and a numerical variable respectively.

B. a nominal variable and an ordinal variable respectively.

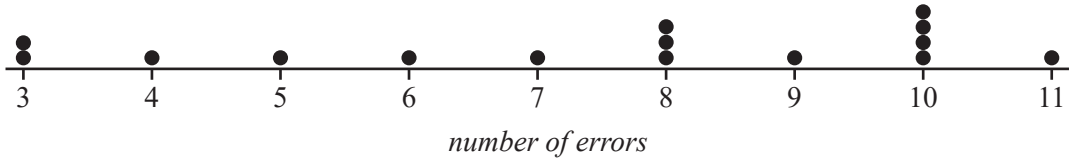
C. an ordinal variable and a nominal variable respectively.

D. both ordinal variables.

E. both nominal variables.

Question 3

The dot plot below displays the *number of errors* made in a test, for a sample of 15 students.



The mean and standard deviation for the number of errors are closest to

A. mean = 2.60 standard deviation = 7.47

B. mean = 2.70 standard deviation = 7.47

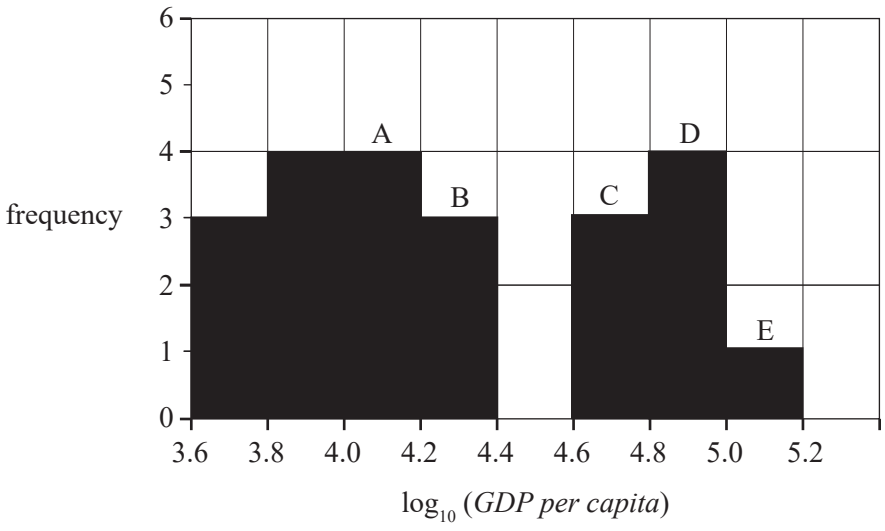
C. mean = 8.00 standard deviation = 3.00

D. mean = 7.47 standard deviation = 2.60

E. mean = 7.47 standard deviation = 2.70

Question 4

The histogram below shows the distribution of the *GDP per capita*, in dollars per year, for a sample of 22 countries in 2021 plotted on a logarithmic (base 10) scale.



Data: <data.worldbank.org>

Australia is one of these 22 countries, and its *GDP per capita* value is $56 281.

Which one of the columns marked A, B, C, D or E would contain Australia's value?

A. A

B. B

C. C

D. D

E. E

Question 5

The heights of a group of Year 9 students were measured and the standard deviation was found to be 12.25 cm.

One student with a height of 174.6 cm had a standardised score of $z = 0.45$

The mean height of this group of students, in centimetres, was closest to

A. 161.9

B. 169.1

C. 180.1

D. 186.4

E. 187.3

Question 6

The weights of cans of fish on a production line are approximately normally distributed with a mean of 126.4 grams and a standard deviation of 2.4 grams.

$13\,600$ cans of fish will be produced today.

Using the 68–95–99.7% rule, the number of these cans that are expected to weigh between 121.6 and 128.8 grams is

A. 6460

B. 9248

C. $10\,812$

D. $11\,084$

E. $12\,920$

Question 7

Data was collected to investigate the association between two variables:

- *age* (in years)
- *uses public transport* (yes, no).

Which one of the following is appropriate to use in the statistical display of this data?

A. a histogram

B. a least squares line

C. parallel boxplots

D. a segmented bar chart

E. a scatterplot

Question 8

A class investigation considered 20 countries and any association between the *birth rate*, per 1000 people, and the *life expectancy*, in years.

Students were given the following table of summary statistics.

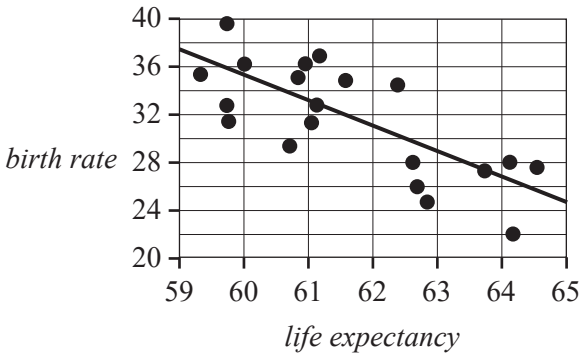
	birth rate	***life expectancy***
Mean	31.5	61.7
Standard deviation	4.70	1.64
Correlation coefficient (r)	–0.752	

Data: <data.worldbank.org> (life expectancy), CIA factbook (birth rate)

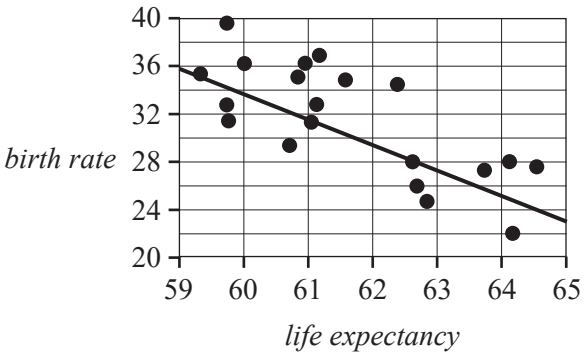
Scatterplots **A**, **B**, **C**, **D** and **E** show attempts by five students to fit the calculated least squares line to a scatterplot of the original data.

Which one of these attempts has been completed correctly?

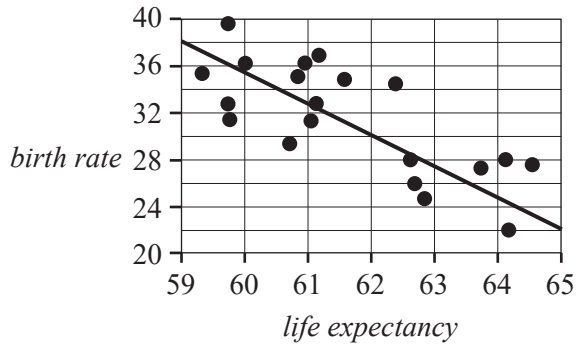
A.



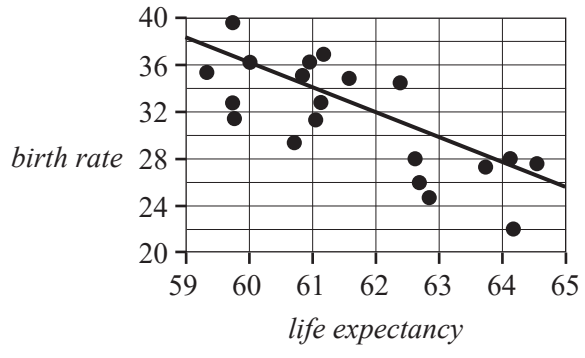
B.



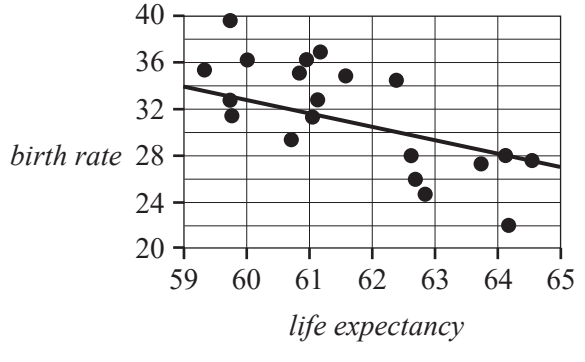
C.



D.



E.

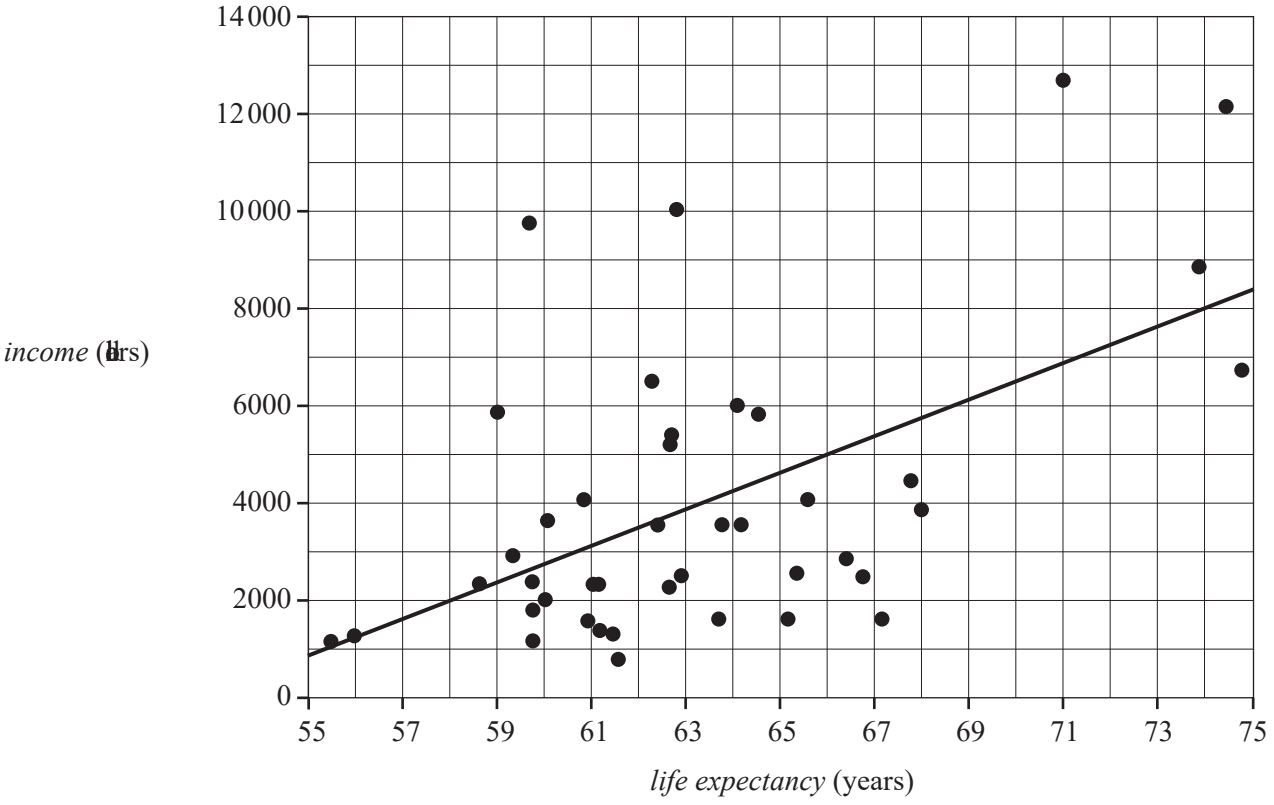


Use the following information to answer Questions 9 and 10.

The scatterplot below shows the average annual *income*, in dollars, plotted against *life expectancy*, in years, for 42 countries in 2020.

A least squares line has been fitted to the scatterplot.

The coefficient of determination is 0.306.



Data: United Nations Population Division, <data.worldbank.org>

Question 9

The equation of the least squares line is closest to

A. $income = -19\,000 + 345 \times life\ expectancy$

B. $income = -19\,250 + 355 \times life\ expectancy$

C. $income = -19\,500 + 365 \times life\ expectancy$

D. $income = -19\,750 + 375 \times life\ expectancy$

E. $income = -20\,000 + 385 \times life\ expectancy$

Question 10

Which one of the following statements is true?

A. The value of the correlation coefficient is 0.306

B. There are more data points above the least squares line than below.

C. 30.6% of the variation in annual *income* is not explained by the variation in *life expectancy*.

D. The country with the longest *life expectancy* has a positive residual associated with it.

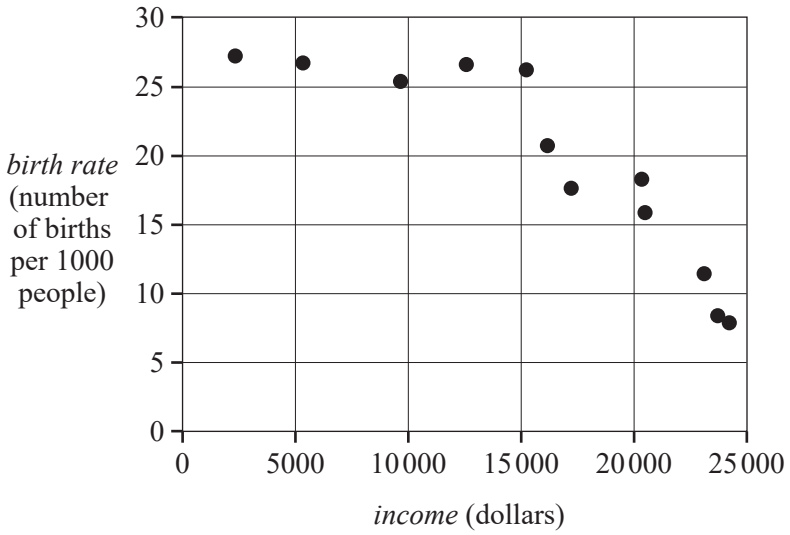
E. Using the least squares line to predict the annual *income* of a country whose citizens have a *life expectancy* of 54 years is an example of extrapolation.

Use the following information to answer Questions 11 and 12.

The table below shows the *birth rate*, in number of births per 1000 people, and the average annual *income*, in dollars per person, for a sample of 12 countries.

A scatterplot displaying the data is also shown.

birth rate	*income*
27.18	2460
26.78	5211
25.21	9846
26.44	12 706
26.14	15 176
20.60	16 304
17.55	17 402
18.24	20 463
15.69	20 615
11.35	23 285
8.15	23 926
7.72	24 364



Data: <data.worldbank.org>, <www.cia.gov/the-world-factbook/>

Question 11

A squared transformation applied to the variable *birth rate* can be used to linearise the scatterplot.

The equation of the least squares line is

$$(birth\ rate)^2 = 953 - 0.0333 \times income$$

Using this equation, the predicted *birth rate*, in number of births per 1000 people, for a country with an average annual income of $\$18\,500$ is closest to

A. 17.7

B. 18.0

C. 18.4

D. 337

E. 113 535

Question 12

Coefficients of determination were calculated for

- *birth rate* vs *income* (coefficient 1)
- $(birth\ rate)^2$ vs *income* (coefficient 2)
- *birth rate* vs $(income)^2$ (coefficient 3)

These coefficients were ranked in order from largest to smallest.

The order would be

A. coefficient 1, coefficient 2, coefficient 3

B. coefficient 2, coefficient 3, coefficient 1

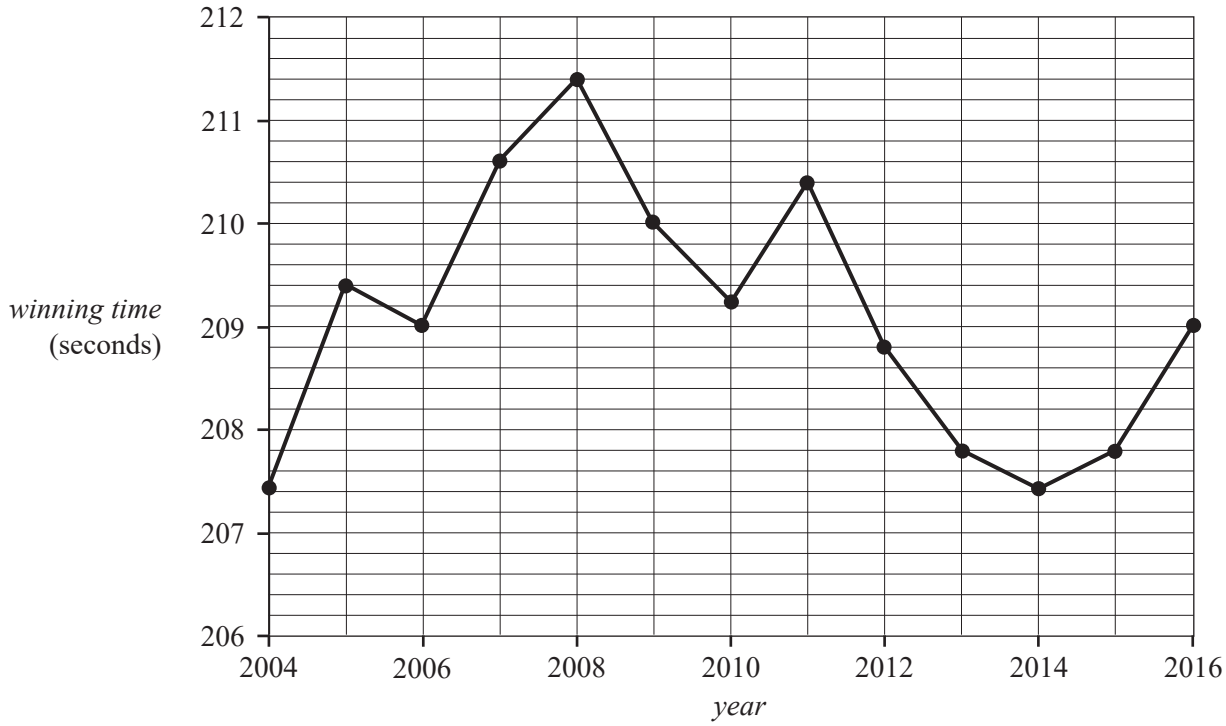
C. coefficient 2, coefficient 1, coefficient 3

D. coefficient 3, coefficient 1, coefficient 2

E. coefficient 3, coefficient 2, coefficient 1

Question 13

The following graph shows the *winning time*, in seconds, for each *year* from 2004 to 2016 for a men's 1500 m track event.



Data based on: <https://www.worldathletics.org>

The time series is smoothed using nine-median smoothing.

The smoothed value for the *winning time* in 2009, in seconds, is closest to

A. 209.0

B. 209.2

C. 209.4

D. 210.0

E. 210.4

Question 14

The number of fruit drinks sold by a fast-food shop each day for 14 consecutive days was recorded.

These results are shown in the table below.

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number	65	84	123	154	129	187	179	71	89	131	147	141	190	185

The six-mean smoothed value with centring for day 8 is

A. 132

B. 132.5

C. 133

D. 133.5

E. 134

Question 15

The sales revenue, in dollars, from the sale of chocolate eggs is seasonal.

To correct the sales revenue in May for seasonality, the actual sales revenue, to the nearest percent, is decreased by 17%.

The seasonal index for that month is closest to

A. 0.77

B. 0.83

C. 1.17

D. 1.20

E. 1.25

Question 16

Seasonal indices for visitor numbers to a theme park in a particular year are given in the table below. The seasonal index for summer is not given.

Season	Spring	Summer	Autumn	Winter
Seasonal index	0.85		0.96	0.45

In this particular year, 33 120 visitors attended during summer.

The total annual attendance for this particular year is closest to

A. 73 960

B. 74 520

C. 75 820

D. 76 140

E. 77 380

Recursion and financial modelling

Question 17

Mel bought a new car for $\$60\,000$. She will depreciate the value of the car using the reducing balance method. A recurrence relation that models the year-to-year value of her car, M_n, is

$$M_0 = 60\,000, \qquad M_{n+1} = 0.85\,M_n$$

An equivalent rule to determine the value of the car after n years is

A. $M_n = 60\,000 - 0.85n$

B. $M_n = 60\,000 + 0.85n$

C. $M_n = 60\,000 + 0.85^n$

D. $M_n = 60\,000 \times 0.85^{n-1}$

E. $M_n = 60\,000 \times 0.85^n$

Question 18

A sequence of numbers is generated by a recurrence relation of the form

$$T_0 = 5, \qquad T_{n+1} = k - T_n$$

All terms of the sequence have the same value.

The constant k is equal to

A. -10

B. -5

C. 0

D. 5

E. 10

Use the following information to answer Questions 19 and 20.

Edo invests $\$10\,000$ in an account earning 3% interest per annum compounding monthly.

Question 19

The value, V_n, of Edo's investment after n months is given by

A. $V_n = 10\,000 \times 1.0025^n$

B. $V_n = 10\,000 \times 1.003^n$

C. $V_n = 10\,000 \times 1.03^n$

D. $V_n = 10\,000 \times 1.003^{12n}$

E. $V_n = 10\,000 \times 1.03^{12n}$

Question 20

The effective interest rate for Edo's investment is closest to

A. 2.96%

B. 2.98%

C. 3.00%

D. 3.02%

E. 3.04%

Question 21

Yolanda purchased a motorcycle for $\$30\,000$. She explores two options for predicting the value of the motorcycle after four years.

Option 1: For the first two years, the value of the motorcycle is depreciated by 10% per annum using flat rate depreciation. For the next two years, the value of the motorcycle is depreciated by 10% per annum using reducing balance depreciation.

Option 2: The value of the motorcycle is depreciated using reducing balance depreciation with a constant depreciation rate per annum for four years.

For both options to predict the same value after four years, the rate per annum used for Option 2 is closest to

A. 9.4%

B. 9.7%

C. 10.0%

D. 10.3%

E. 10.6%

Question 22

The recurrence relation below models the value, P_n, in a financial context after n time periods.

$$P_0 = a, \qquad P_{n+1} = RP_n - d$$

All constants, a, R and d, are greater than 1.

Four options of what the value of P_n could represent are listed below.

- a reducing balance loan
- an annuity
- an asset depreciated using the unit cost method
- a perpetuity

How many of these four options could be represented by the recurrence relation?

A. 0

B. 1

C. 2

D. 3

E. 4

Question 23

Todd invested $\$450\,000$ in an annuity at the start of 2024.

The interest rate for this annuity is 3.75% per annum compounding monthly.

He will receive regular monthly payments for the 15-year life of the annuity.

In which year will the balance of the annuity first fall below $\$350\,000$?

A. 2027

B. 2028

C. 2029

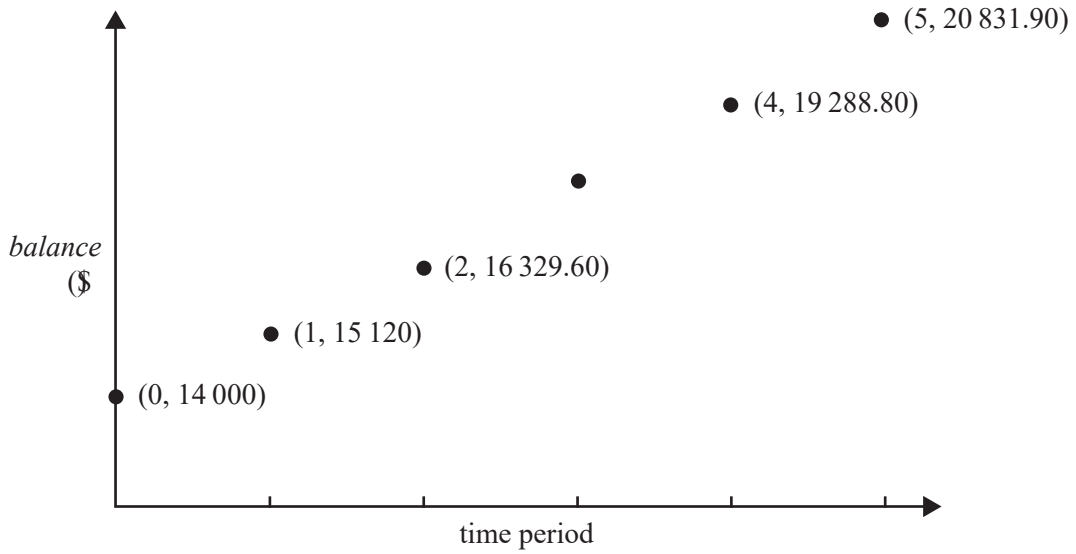
D. 2030

E. 2031

Question 24

Jarryd invested $\$14\,000$ into an account earning compound interest at a fixed rate per time period.

The graph below shows the balance of the account for four of the first five time periods after the initial investment. The information for time period 3 is not shown.



Immediately after the interest was calculated for time period 3, Jarryd added an extra one-off amount into the account.

This amount was closest to

A. $\$224.03$

B. $\$225.97$

C. $\$228.62$

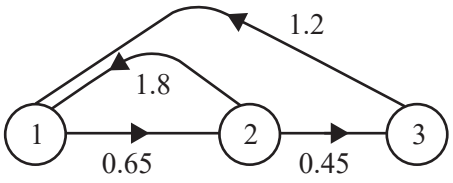
D. $\$229.38$

E. $\$231.46$

Matrices

Use the following information to answer Questions 25 and 26.

The following life cycle transition diagram shows changes in a female population of mammals with three age groups (1, 2 and 3).



Question 25

On average, what percentage of the female population from group 2 will survive to group 3?

A. 12%

B. 18%

C. 45%

D. 50%

E. 65%

Question 26

The associated Leslie matrix, L, for the above transition diagram is

A. $L = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1.8 & 1.2 \\ 0 & 0.65 & 0.45 \end{bmatrix}$

B. $L = \begin{bmatrix} 1 & 1.8 & 1.2 \\ 0 & 0.65 & 0 \\ 0 & 0 & 0.45 \end{bmatrix}$

C. $L = \begin{bmatrix} 0 & 1.8 & 1.2 \\ 0.65 & 0 & 0 \\ 0.45 & 0 & 0 \end{bmatrix}$

D. $L = \begin{bmatrix} 1.8 & 1.2 & 0 \\ 0 & 0.65 & 0.45 \\ 0 & 0 & 0 \end{bmatrix}$

E. $L = \begin{bmatrix} 0 & 1.8 & 1.2 \\ 0.65 & 0 & 0 \\ 0 & 0.45 & 0 \end{bmatrix}$

Question 27

Matrix V is an $n \times n$ matrix with a determinant equal to 1.

The product of $V \times V^{-1}$ will result in

A. an identity matrix.

B. a Leslie matrix.

C. a column matrix.

D. a zero matrix.

E. a row matrix.

Question 28

Matrix D is a 2×2 matrix where each element is given by d_{ij}

Which rule will result in a binary matrix?

A. $d_{ij} = i + j$

B. $d_{ij} = i - j$

C. $d_{ij} = i \times j$

D. $d_{ij} = i \div j$

E. $d_{ij} = (i - j)^2$

Question 29

Matrix J is a row matrix of order $1 \times n$.

Matrix K is a column matrix of order $n \times 1$.

Matrix J^T is the transpose of Matrix J.

Matrix K^T is the transpose of Matrix K.

Consider the following matrix products where n is a whole number greater than or equal to 2:

- J^2
- JK
- KJ
- $J^T K^T$
- $K^T J^T$

How many of the above matrix products are defined?

A. 1

B. 2

C. 3

D. 4

E. 5

Question 30

Matrix R is a column matrix.

$$R = \begin{bmatrix} T \\ A \\ L \\ L \\ Y \end{bmatrix}$$

A permutation matrix, P, is multiplied by matrix R to form the product matrix $Q = PR$.

If Q is equal to R, how many different permutation matrices could have been used?

A. 1

B. 2

C. 3

D. 4

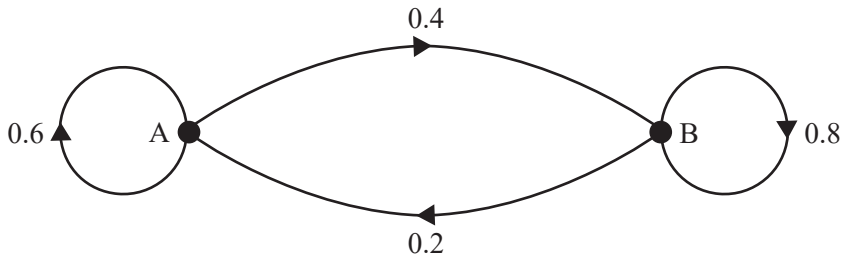
E. 5

Question 31

A group of meerkats lives in an enclosure at a zoo.

The meerkats sleep during the night in one of two chambers, chamber A or chamber B.

The transition diagram below shows the proportion of meerkats that stay in the same sleeping location or change sleeping location from night to night.



Every night there are a meerkats in chamber A.

Every night there are b meerkats in chamber B.

Of the meerkats sleeping in chamber A on Friday night, eight had slept in chamber B on the previous night.

How many meerkats live in the enclosure?

A. 20

B. 30

C. 40

D. 50

E. 60

Question 32

An online shop offers monthly subscriptions for protein powder.

The shop offers protein powder in three flavours: vanilla (V), chocolate (C) and malt (M).

Let P_n be the state matrix that shows the expected number of subscribers for each flavour n months after sales of the protein powder began.

The expected number of subscribers for each flavour can be determined by the matrix recurrence rule

$$P_{n+1} = TP_n + K$$

where

$$T = \begin{array}{c} \\ V \\ C \\ M \end{array} \begin{array}{c} \begin{array}{ccc} \multicolumn{3}{c}{this\ month} \\ V & C & M \end{array} \\ \begin{bmatrix} 0.2 & 0.2 & 0.1 \\ 0.4 & 0.2 & 0.1 \\ 0.4 & 0.6 & 0.8 \end{bmatrix} \end{array} \begin{array}{l} V \\ C \\ M \end{array} \quad next\ month \qquad \text{and} \qquad K = \begin{bmatrix} 93 \\ 59 \\ 9 \end{bmatrix} \begin{array}{l} V \\ C \\ M \end{array}$$

The state matrix, P_2, below shows the expected number of subscribers for each flavour two months after sales began.

$$P_2 = \begin{bmatrix} 147 \\ 137 \\ 199 \end{bmatrix}$$

The increase in the expected number of subscribers for vanilla (V) between the initial sales, P_0, and the first month after sales began, P_1, is equal to

A. 27

B. 54

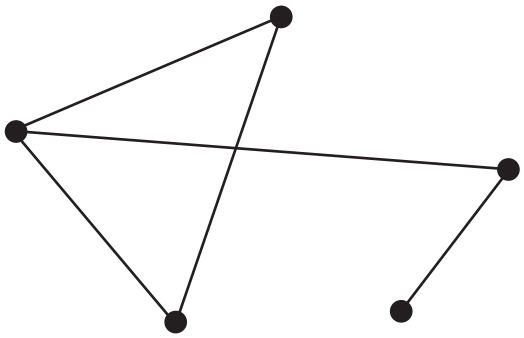
C. 60

D. 87

E. 93

Networks and decision mathematics

Question 33



In the graph shown above, the number of vertices of even degree is:

A. 0
B. 1
C. 2
D. 3
E. 4

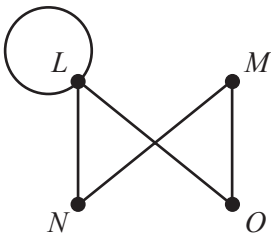
Question 34

The adjacency matrix below represents the road connections between four towns, labelled L, M, N and O.

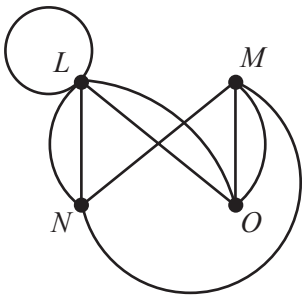
$$\begin{array}{c} \\ L \\ M \\ N \\ O \end{array} \begin{array}{c} \begin{array}{cccc} L & M & N & O \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 2 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 \end{bmatrix} \end{array}$$

A graph that represents all the road connections in the adjacency matrix is:

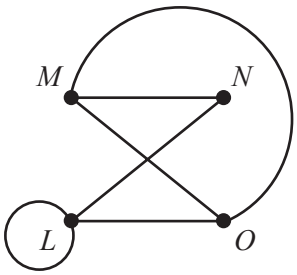
A.



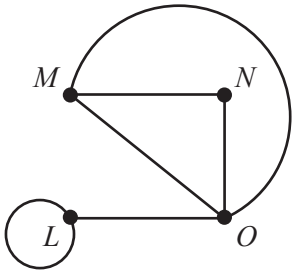
B.



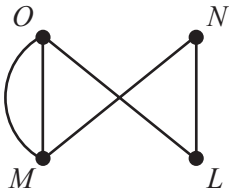
C.



D.



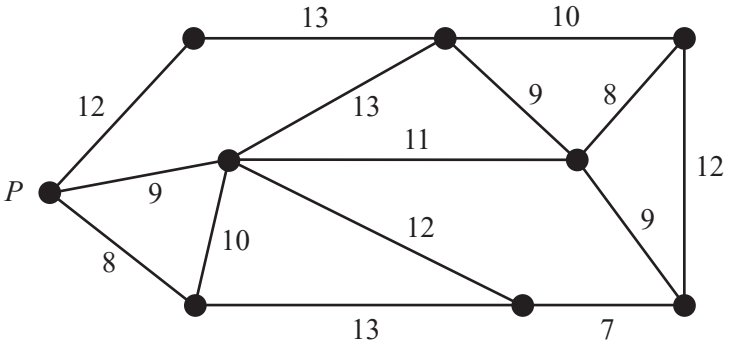
E.



Do not write in this area.

Question 35

The vertices of the graph below represent cabins in a holiday park, and the water pump (P) that will supply them. The numbers on the edges show the length, in metres, of water pipe required to connect the cabins and the pump.



The water pipes will cost $\$52$ per metre. What is the minimum cost to link all the cabins to the water pump (P)?

A. $\$3744$

B. $\$3796$

C. $\$3848$

D. $\$3900$

E. $\$3952$

Question 36

Four students, Peggy, Quincy, Radley and Sarah, are grouped together to complete a project. The project is in four parts, labelled W, X, Y and Z. Each student must complete one part of the project.

The table below shows each student's estimate of the score they will receive if they complete each section.

	Peggy	Quincy	Radley	Sarah
W	12	19	18	16
X	16	15	15	16
Y	10	16	17	15
Z	19	20	18	18

Based on the estimates, which allocation of project parts will maximise the students' group score on the project?

A.

W	Quincy
X	Sarah
Y	Radley
Z	Peggy

B.

W	Radley
X	Peggy
Y	Quincy
Z	Sarah

C.

W	Sarah
X	Quincy
Y	Peggy
Z	Radley

D.

W	Radley
X	Peggy
Y	Sarah
Z	Quincy

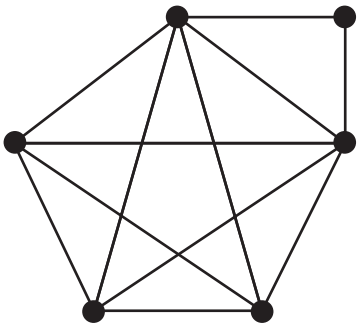
E.

W	Sarah
X	Peggy
Y	Radley
Z	Quincy

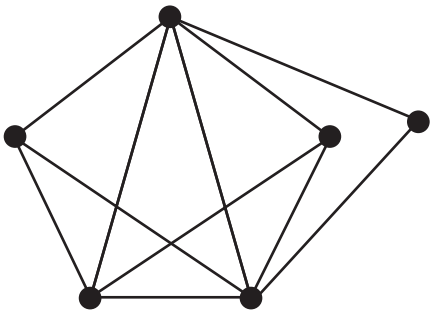
Question 37

Euler's formula can be applied to which of the following graphs?

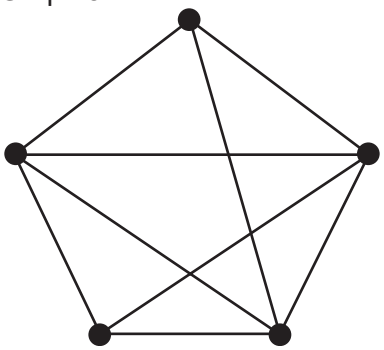
Graph 1



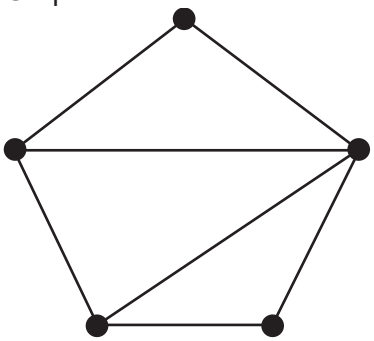
Graph 2



Graph 3



Graph 4



A. Graph 4 only

B. Graphs 1 and 2 only

C. Graphs 1, 2, 3 and 4

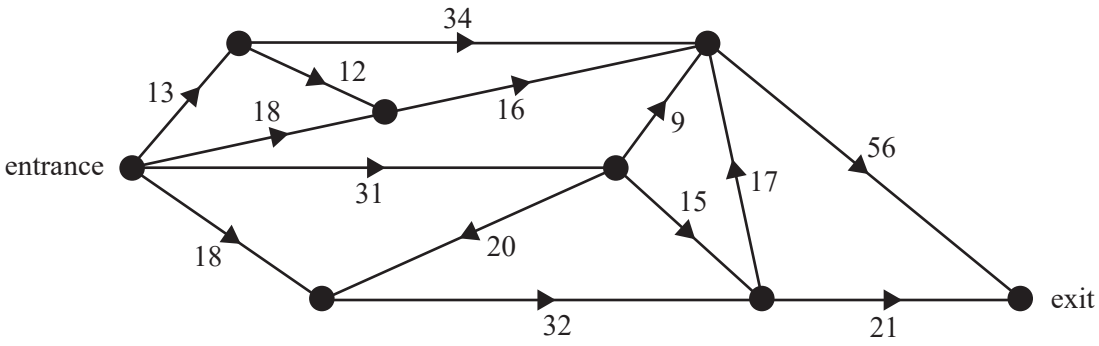
D. Graphs 3 and 4 only

E. Graphs 2, 3 and 4 only

Use the following information to answer Questions 38 and 39.

The following directed graph represents the one-way paths between attractions at an historical site. The entrance, exit and attractions are represented by vertices.

The numbers on the edges represent the maximum number of visitors allowed along each path per hour.



Question 38

What is the maximum number of visitors able to walk from the entrance to the exit each hour?

A. 75

B. 76

C. 77

D. 88

E. 96

Question 39

A group of students set out from the entrance and walk to the exit. The students all walk together and travel along the same route. They are the only people visiting the site that hour. What is the maximum number of students that could be in the group?

A. 9

B. 15

C. 16

D. 20

E. 31

Question 40

A project has 10 activities, labelled A to J. The table below shows the immediate predecessor(s) for each activity. Each activity has a duration of at least one day.

Activity	Immediate predecessor(s)
A	–
B	–
C	A
D	B
E	B
F	D
G	D, E
H	C, F
I	E
J	G, H

Which **one** of the following statements about this project is **not** true?

A. The earliest starting time of activity H could be two days.

B. In the network for this project, there would be a dummy activity from the end of activity D to the start of activity G.

C. One of the paths through the network of this project is $BDGJ$.

D. The latest starting time of activity E could be three days.

E. The network for this project would require two dummy activities.

End of examination questions

This page is blank.

N
H
T

Do not write in this area.

This page is blank.

2024

NHT

General Mathematics Examination 1

Formula Sheet

You may keep this Formula Sheet.

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

VICTORIA State Government

General Mathematics formulas

Data analysis

standardised score	$z = \frac{x - \bar{x}}{s_x}$
lower and upper fence in a boxplot	lower $\text{Q1} - 1.5 \times \text{IQR}$ upper $\text{Q3} + 1.5 \times \text{IQR}$
least squares line of best fit	$y = a + bx,$ where $b = r\frac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$
residual value	residual value = actual value – predicted value
seasonal index	$\text{seasonal index} = \frac{\text{actual figure}}{\text{deseasonalised figure}}$

Recursion and financial modelling

first-order linear recurrence relation	$u_0 = a, \quad u_{n+1} = Ru_n + d$
effective rate of interest for a compound interest loan or investment	$r_{effective} = \left[\left(1 + \frac{r}{100n}\right)^n - 1\right] \times 100\%$

Matrices

determinant of a 2×2 matrix	$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \qquad \det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
inverse of a 2×2 matrix	$A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}, \qquad \text{where} \qquad \det A \neq 0$
recurrence relation	$S_0 = \text{initial state}, \qquad S_{n+1} = T S_n + B$
Leslie matrix recurrence relation	$S_0 = \text{initial state}, \qquad S_{n+1} = L S_n$

Networks and decision mathematics

Euler's formula	$v + f = e + 2$

End of Formula Sheet