2016 Midyear exam solutions.

	1	2	3	4	5	6	7	8	9	10	11	12
Data	D	C	D	D	B	A	E	C	A	B	D	C
Financial	C	A	D	A	D	B	B	C				

Core – Data analysis

Question 1

There are 4 classes which have less than 10% of left handed students. These 4 classes have 4%, 8%, 8% and 9% of left handed students.

The answer is D.

Question 2

We are only interested in those people whose work car is a wagon. We have no interest in what they use at home so we focus on the third column of data $25+16+27=68$.
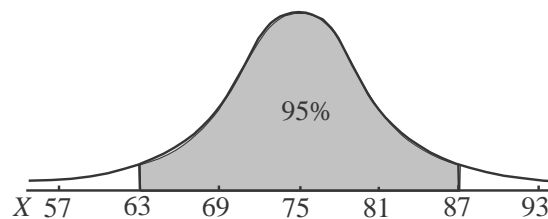
The answer is C.

Question 3

Draw a diagram.

95% of boys will weigh between 63kg and 87kg.

95% of $870=826.5$.

The closest answer is 827.

The answer is D.

Question 4
We focus on the right hand column which shows those living in a high-rise apartment.

The white block represents people aged between 16 – 55. The percentage is approximately $70\%-15\%=55\%$.

The answer is D.

Question 5

The variable *age* is categorical. This may seem strange because it involves numbers, but they are categories of numbers i.e. 0 – 15, 16 – 55 and over 55. The variable *type of housing* is also categorical.

The answer is B.

Question 6

It is true that on more than 75% of days in January the website had less hits than on any day in February. This is because the right hand end of the box for January coincides with approximately 1500 hits per day. The minimum value of the February distribution is approximately 1600.

The answer is A.

Question 7

The *x* values and the approximate corresponding residuals are shown in the table below.

x	0.6	0.8	1.2	1.7	2.0	2.4	2.8	3.4	3.8	4.4
Residual	+ 2	- 5	- 4	+ 5	+ 10	- 4	+ 4	0	- 2	+ 6

The only residual plot that represents this is option E.

The answer is E.

Question 8

Enter the data into your calculator and calculate *r*. It is $r = 0.86895...$

The closest answer is 0.87.

Note that in this question it doesn't matter whether you assign 2004 as the independent variable (*x*-values) or the dependent variable (*y*-values).

It WOULD MATTER though if you had been asked to find the equation of the least squares regression line.

The answer is C.

Question 9

The percentage of the variation in the number of students who tried out in 2014 that is explained by the variation in the number of students who tried out in 2004 is given by the coefficient of determination, r^2, expressed as a percentage.

From Question 8, $r = 0.86895$ so $r^2 = 0.75507...$

Expressed as a percentage, this becomes 75.507…%

The closest answer is 76%.

The answer is A.

Question 10

The smoothed value for the average monthly maximum temperature in September is given by
$$\frac{22+28+31}{3} = 27.$$

The answer is B.

Question 11

Start by finding the quarterly average.

$$\text{Quarterly average} = \frac{42500+63900+110500+138600}{4}$$

$$= \$88875$$

So the seasonal index for Quarter 3 is equal to $\frac{110\,500}{88\,875} = 1.2433...$

The closest answer is 1.24.
The answer is D.

Question 12

Start by finding the seasonal index for quarter 2.

The three other seasonal indices add to give 2.9. Since the four seasonal indices should add to give 4, the seasonal index for quarter 2 is 1.1.

To find the deseasonalised revenue for Quarter 2, use the equation given.

$$\text{deseasonalised revenue} = 628\,000 + 12\,300 \times 2$$

$$= 652\,600$$

$$\text{Since} \quad \text{seasonal index} = \frac{\text{actual figure}}{\text{deseasonalised figure}} \quad \text{(from formula sheet)}$$

We have, for quarter 2,

$$1.1 = \frac{\text{actual figure}}{652\,600}$$

$$\text{actual figure} = 1.1 \times 652\,600$$
$$= 717\,860$$

The actual revenue for quarter 2 was \$717 860.

The answer is C.

Recursion and Financial Mathematics

Question 2

$$I = \frac{PrT}{100} \text{ (simple interst formula from the formula sheet)}$$

$$1044 = \frac{12000 \times r \times 3}{100}$$

$$r = \frac{100 \times 1044}{12000 \times 3}$$

$$r = 2.9$$

The interest rate per annum is 2.9%.

The answer is A.

Question 4

The dryer has depreciated by \$9500 − \$4940 = \$4560

The number of loads it has dried equals $4560 \div 0.38 = 12000$.

The answer is A.

Question 5

In total, Sue will pay $\$140 \times 24 = \$3360$.

Note that there is no deposit paid.

So the interest she pays equals $\$3360 - \$3200 = \$160$.

Use the simple interest formula to find the flat rate.

$$I = \frac{PrT}{100}$$

$$160 = \frac{3200 \times r \times 2}{100}$$

$$r = 2.5$$

From the formula sheet, the effective rate of interest $\approx \dfrac{2n}{n+1} \times \text{flat rate} = \dfrac{2 \times 24}{25} \times 2.5 = 4.8\%$.

The answer is D.

Question 6

Using the compound interest formula from the formula sheet, we have

$$A = PR^n \qquad \text{where} \quad R = 1 + \frac{r}{100}$$

$$R = 1 + \frac{r \div 52}{100}$$

So, $2805.02 = 2400 \times \left(1 + \dfrac{r \div 52}{100}\right)^{4 \times 52}$

Using your CAS, solve this equation for r.

$r = -10403.900...$ or $r = 3.9000...$

Clearly the first answer is nonsensical, so $r = 3.9\%$

The answer is B.

Question 7

Using *TVM*, we have

$$N : ?$$
$$I : 5.2$$
$$PV : -40\,000$$
$$PMT : 3500$$
$$FV : 0$$
$$P_pY : 4$$
$$C_pY : 4$$
$$N = 12.4525...$$

These payments will last for 12.4525... quarters or 3.11... years.

The closest answer is 3.1.

The answer is B.

Question 8

The final payment represents a part payment of a full week.

The proportion of the week is given by $\dfrac{627.54}{1300} = 0.48272\ldots$

So the number of weeks required for Vesna to completely pay out her home loan is 250.48272… weeks.

Now we can use TVM.

$$N : 250.48272$$
$$I : 7.2$$
$$PV : ?$$
$$PMT : -1300$$
$$FV : 0$$
$$P_p Y : 52$$
$$C_p Y : 52$$
$$PV = 274999.9985\ldots$$

So Vesna borrowed $275 000.

The answer is C.

<u>C<u>ore:</u></u>

Question 1 (3 marks)

a. From the dot plot we see that there are 4 farms which had a crop yield less than 2 tonnes per hectare.

$$\left(\frac{4}{32} \times \frac{100}{1}\right)\% = 12.5\%$$

So 12.5% of the farms had a crop yield less than 2 tonnes per hectare.

(1 mark)

b. We need to calculate $Q_3 + 1.5 \times IQR$ where Q_3 is the third quartile, and the *IQR* is the interquartile range.
Since there are 32 pieces of data, they divide neatly into 4 quartiles of 8 data values.

Counting in from the left, the eighth data value is 2 and so is the ninth so $Q_1 = 2$.

Counting in from the right, the eighth data value is 4 and so is the ninth. So $Q_3 = 4$.

So $IQR = Q_3 - Q_1 = 4 - 2 = 2$

$Q_3 + 1.5 \times IQR = 4 + 1.5 \times 2 = 7$ **(1 mark)**

(If you stop here you will lose a mark. We need to state that 8 is greater than this value and is hence an outlier.)

Since $8 > 7$ then the farm which has a crop yield of 8 tonnes per hectare is an outlier.

(1 mark)

Question 2 (5 marks)

a. From the time series plot, the highest crop yield was about 3.1 tonnes per hectare.
This occurred in 2012.

(1 mark)

b. **i.** $\text{crop yield} = -307.427 + 0.154 \times 2020$
                    $= 3.653$

In 2020 the crop yield is predicted to be 3.65 tonnes per hectare (correct to 2 decimal places).

(1 mark)

ii. To find the crop yield in 2020 requires extrapolating, in other words, going beyond the data that we have which can be unreliable.

(1 mark)

iii. In 2008,

$$\text{crop yield} = -307.427 + 0.154 \times 2008$$
$$= 1.805$$

$$\text{residual value} = \text{actual value} - \text{predicted value (formula sheet)}$$
$$= 2.3 - 1.805$$
$$= 0.495$$
$$= 0.50 \text{ (correct to 2 decimal places)}$$

(1 mark)

iv. The gradient of the regression line tells us by how much the crop yield will increase each year. So that increase is 0.15 tonnes per hectare (correct to 2 decimal places).

(1 mark)

Question 3 (3 marks)

a. To obtain the coefficients in this equation enter the data in the table and then get your calculator to find $\dfrac{1}{(crop\ yield)}$ for each of the data values of the variable *crop yield*. This creates a third column of data. Calculate the least squares regression line equation using *area* as the independent variable (*x*-variable) and $\dfrac{1}{(crop\ yield)}$ as the dependent variable (*y*-variable).

$$\frac{1}{(crop\ yield)} = 2.17642... - 0.10375... \times area$$

So $\dfrac{1}{(crop\ yield)} = 2.18 - 0.10 \times area$, where the coefficients have been expressed correct to two decimal places. **(1 mark)**

b. The explanatory variable is $\dfrac{1}{(crop\ yield)}$. **(1 mark)**

c. $\dfrac{1}{(crop\ yield)} = 2.18 - 0.10 \times 15 = 0.68$

$$crop\ yield = \frac{1}{0.68} = 1.47058... = 1.47 \text{ tonnes per hectare (correct to 2 decimal places)}$$

(1 mark)

Question 4 (4 marks)

a. It is not appropriate to find the equation of the least squares regression line because the relationship between the variables *crop yield* and *area* is clearly not linear.

(1 mark)
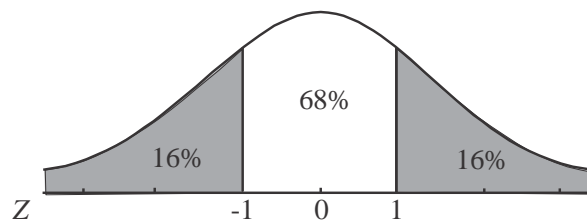
b. **i.** $z = \dfrac{x - \bar{x}}{s_x}$ (from formula sheet)

$z = \dfrac{0.5 - 2.07}{0.81}$

$= -1.93827...$

$z = -1.94$ (correct to 2 decimal places)

(1 mark)

ii. Because we have a normal distribution (or at least one is assumed), we can use the $68 - 95 - 99.7\%$ rule.

68% of data lies between one standard deviation either side of the mean.

Because of the symmetry of the normal curve, 16% of data is one standard deviation or more below the mean (and of course 16% of data is one standard deviation or more above the mean).

16% of $41 = 6.56 = 7$ farms (to the nearest whole number) **(1 mark)**

iii. For the variable *area*, the mean is 36.24 and the standard deviation is 17.46. So one standard deviation below the mean is $36.24 - 17.46 = 18.78$. Looking at the scatterplot we see that 7 farms actually have an area (over which the new type of crop is growing) that is less than 18.78 hectares.

(1 mark)

Financial:

Question 1 (4 marks)

a. $\dfrac{15}{100} \times 1400 = 210$

(1 mark)

b. $V_3 = 0.85^3 \times 1400\ \$859.78$

(1   mark)

c. i. $V_3 = 1400 - 3 \times 30 \times 10 = \$500$

(1 mark)

   ii. $solve(250 = 1400 - 30 \times 10 \times x), x = 3.8\ yrs$

(1 mark)

Question 2 (2 marks)

a.
$$A = PR^n \qquad\qquad R = 1 + \frac{r}{100} \quad \text{(formula sheet)}$$
$$= 65000 \times 1.003^6 \qquad = 1 + \frac{3.6 \div 12}{100}$$
$$= 66178.81 \qquad\qquad = 1.003$$

The amount of interest earned will be $\$66178.81 - \$65\,000 = \$1178.81$.

(1 mark)

b.
$$A = PR^n \qquad\qquad\qquad\qquad R = 1 + \frac{r}{100} \quad \text{(formula sheet)}$$
$$66711.29 = 65000 \times \left(1 + \frac{r \div 52}{100}\right)^{26} \qquad = 1 + \frac{r \div 52}{100}$$

Using your CAS, solve this equation for r.

$r = -10405.199...$ or $r = 5.199985...$

The first answer is clearly not feasible.

The annual interest rate is 5.2% (correct to one decimal place).

(1 mark)

Question 3 (5 marks)

a.   For a perpetuity, $P = \dfrac{100Q}{r}$

$$30\,000 = \frac{100 \times Q}{4}$$

$$Q = 1\,200$$

The annual amount of interest paid to the PFA would be $1200.

(1 mark)

b.   The amount remaining invested in the perpetuity would be $30 000.

(1 mark)

c.   This is an investment annuity so use TVM.

$N : ?$

$I\% : 3.8$

$PV : -10\,000$

$PMT : -2000$

$FV : 30\,000$

$P_PY : 4$

$C_PY : 4$

$N = 9.1814...$

It will take 9 quarters (to the nearest quarter).

(1 mark)

d.   Start by finding the value of the original investment at the end of one year using TVM.

$N : 4$

$I\% : 3.8$

$PV : -10\,000$

$PMT : -2000$

$FV : ?$

$P_PY : 4$

$C_PY : 4$

$FV = 18\,500.17309...$

(1 mark)

Now, we need to find the new payment required, again using TVM.

$N : 12$ (i.e. 3 years to go)

$I\% : 3.8$

$PV : -18500.17309...$

$PMT : ?$

$FV : 30000$

$P_PY : 4$

$C_PY : 4$

$PMT = -733.5206...$

The PFA will need to make quarterly payments of $733.52.

(1 mark)

Question 4 (4 marks)

$$V_0 = 320\,000, V_{n+1} = 1.004V_n - 5000$$

(1 mark)

a.   Using *TVM*.

$N : ?$

$I : 4.8$

$PV : 320\,000$

$PMT : -5000$

$FV : 0$

$P_PY : 12$

$C_PY : 12$

$N = 74.0763...$

It will take 74 months (to the nearest month).

(1 mark)

b.   Using the equation from part a.,

$$\text{balance next month} = 105\,446.72\left(1 + \frac{4.8 \div 12}{100}\right) - 5000$$

$$= 100\,868.50688...$$

(1 mark)

The difference between this month's balance and next month's balance is

$\$105\,446.72 - \$100\,868.51 = \$4\,578.21$

So $\left(\dfrac{4578.21}{5000} \times \dfrac{100}{1} \right)\%$

$= 91.5642$

So 92% (to the nearest whole percent) of the next payment will be directed to reducing the balance of the loan. (The remainder of the payment pays the interest.)

(1 mark)