THE
HEFFERNAN
GROUP

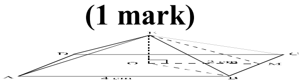
P.O. Box 1180
Surrey Hills North VIC 3127
ABN 20 607 374 020
Phone 9836 5021
Fax 9836 5025

FURTHER MATHEMATICS
TRIAL EXAM 2
2000
SOLUTIONS

Section A : Core - solutions

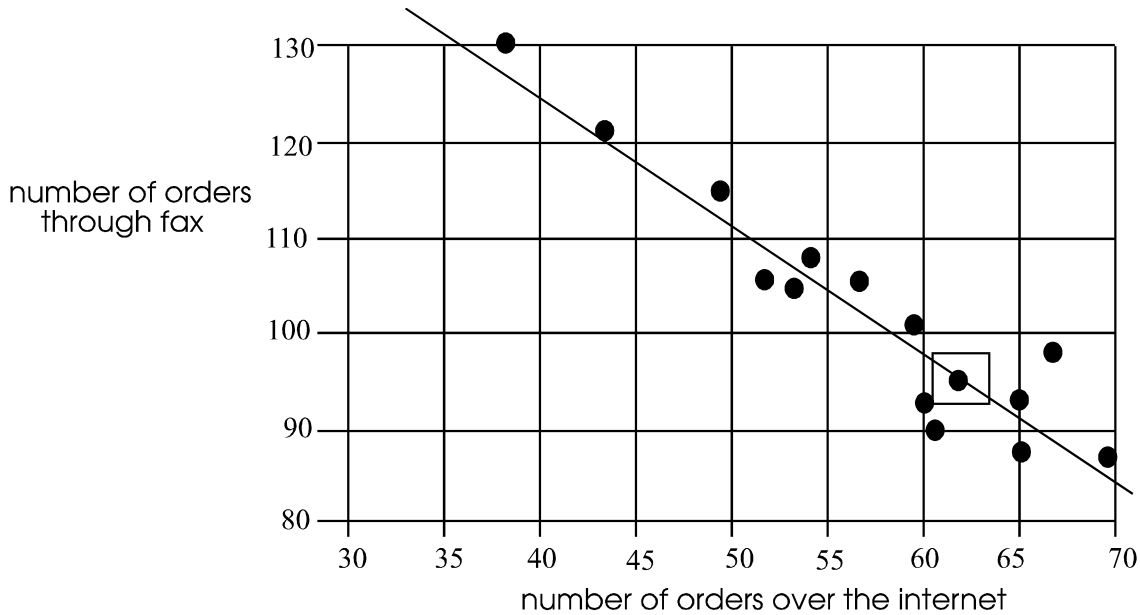
Question 1

a. The data can be described as <u>numerical</u> and <u>discrete</u>. **(2 marks)**
b. The stems, in order, going down the page are 3, 4, 5 and 6. **(2 marks)**
c. The distribution is negatively skewed, since the data "trails off" in the lower values. **(1 mark)**
d. Use your graphics calculator to obtain these values.

i. mean = t_1 = 7 (to 2 decimal places) **(1 mark)**
ii. standard deviation = $S\text{x} = 8.85$ (to 2 decimal places) **(1 mark)**
iii. the median = Med = 59 **(1 mark)**
iv. the interquartile range =  **(1 mark)**

Question 2

a. i. It is appropriate to use Pearson's correlation coefficient since there appears from the scatterplot to be a linear relationship between the two variables. **(1 mark)**
ii. Using a graphics calculator to calculate *r*, we have *r* = -0.95 (correct to 2 places) **(1 mark)**
iii. Pearson's correlation coefficient might suggest to us that there is strong association between the number of orders on the Internet and the number of orders through the fax but this is not to say that the number of orders over the Internet has caused a drop in the number of orders through the fax since there are many other factors that may be involved. **(1 mark)**
b. i. Use your graphics calculator to find these two values and complete the equation of the regression line. We have: orders by fax $= 178.57 - 1.34 \times$ orders over Internet **(1 mark)**
ii. Graph the regression line using your graphics calculator. **(1 mark)**



iii.
would be the point (62, 95) since it is sitting almost right on the regression line. This point is shown in the diagram above with a square drawn around it. **(1 mark)**

Total 15 marks

Section B - Modules - solutions

Module 1 - Arithmetic and applications

Question 1

a. i. 16 km **(1 mark)**

ii. 5 km **(1 mark)**

iii. $t_n = a + (n-1)d$ where $a = 1$ and $d = 5$ **(1 mark)**

So, $t_n = 1 + (n-1) \times 5$

$= 1 + 5n - 5$

So, $t_n = 5n - 4$ **(1 mark)**

Question 2

a. i. $r = \dfrac{15}{10} = 1.5$ **(1 mark)**

ii. $t_n = ar^{n-1}$

So, $t_7 = 10 \times 1.5^6 = 114$ (to the nearest km) **(1 mark)**

b. Now, $S_n = \dfrac{a(r^n - 1)}{r - 1}$

so, $S_{11} = \dfrac{10(1.5^{11} - 1)}{0.5}$

$= 1709.95$ km (to 2 decimal places) **(1 mark)**

Since this is the sum of the distances from the post office to the phones, we need to double this distance since the technician kept returning to Burkeville after upgrading each phone. So, the required distance is 3420 km (to the nearest km) **(1 mark)**

Question 3

To find the number of emergency phones on the first 1000 km of highway, we use the formula

$$t_n = ar^{n-1}$$

We have $1000 = 10 \times 1.5^{n-1}$ and we need to solve for *n*. **(1 mark)**

__Method 1__ - using logarithms

Now, $1000 = 10 \times 1.5^{n-1}$

$100 = 1.5^{n-1}$

$10^2 = 10^{0.1761(n-1)}$ OR $\log_{10} 100 = \log_{10} 1.5^{n-1}$

$2 = 0.1761(n-1)$ $\log_{10} 100 = (n-1)\log_{10} 1.5$

$n - 1 = 11.3577$ $\dfrac{\log_{10} 100}{\log_{10} 1.5} = n - 1$

$n = 12.3577$ $n = 12.3577$

Method 2 - using trial and error
From above, we have, $1.5^{n-1} = 100$
For $n = 10$, left side = 38
For $n = 11$, left side = 57
For $n = 12$, left side = 86
For $n = 13$, left side = 129
The value of *n* lies between 12 and 13.

Method 3 - using a graphics calculator
Graph the function $y = 1.5^{x-1}$
By either looking at the table of values or the graph (the table of values is probably quicker), find the value of *x* for which $y = 100$. We see that the value lies between 12 and 13.

(1 mark) for finding the value of *n* regardless of the method used.

Note that "technically" there are about 12.35 emergency phones in the first 1000 km of highway. Practically, this makes no sense - there are 12 "whole" emergency phones in the first 1000 km. The 13th emergency phone lies beyond 1000 km.
The number of speed limit signs is found by solving $t_n = 5n - 4$ where $t_n = 1000$

$$1000 = 5n - 4$$
$$n = \frac{1004}{5}$$
$$\text{so, } n = 200.8$$

There are 200 speed limit signs . **(1 mark)**

Again, note that the 201st speed limit sign lies outside the 1000 km range.
The ratio of the number of emergency phones compared to the number of speed limit signs is 12 : 200 which simplifies to give 3 : 50 **(1 mark)**

Question 4

Now, $t_2 = 2t_1 + 1 = 11$
so, $2t_1 = 10$
$t_1 = 5$ **(1 mark)**
By using a graphics calculator or by generating the terms manually, we see that $t_7 = 383$ and $t_8 = 767$. So, in the first 500 km there are 7 billboards. **(1 mark)**
So the cost is $7 \times \$455 = \$3185$ **(1 mark)**

Total 15 marks

Module 2 - Geometry and trigonometry

Question 1

a. i. Let the point where the toy lands be point F. ΔDEF is a right-angled triangle with $DE = 30$ m and DF = 5.724 m

Using Pythagoras, we have $(EF)^2 = (DE)^2 - (DF)^2$

$$= 30^2 - (5.724)^2$$

$EF = 29.449$ m (correct to 3 decimal places) **(1 mark)**

ii. In ΔDEF, $\sin(\angle DEF) = \dfrac{5.724}{30}$

$= 11°\ 0'$ (to the nearest minute) **(1 mark)**

iii. Now, $\angle CDE = 20°$ (alternate angles in parallel lines)
Since $\angle DEF = 11°$ (from part **ii.**), $\angle EDF = 79°$ (angles in a triangle add to $180°$)
So, $\angle CDF = 79° + 20° = 99°$ Draw a horizontal line through point D. The ramp CD must make an angle of $99° - 90° = 9°$ with the horizontal. **(1 mark)**

b. i. $\angle CDE = 20°$ (alternate angles in parallel lines) **(1 mark)**

ii. Use the cosine rule. **(1 mark)**
In ΔCDE, $(CE)^2 = 30^2 + 40^2 - 2 \times 30 \times 40 \times \cos 20°$
So, $CE = 15.644\,\text{m}$ **(1 mark)**

iii. Now, $\angle ABC = 20°$ (alternate angles in parallel lines)
So, ΔABC is congruent to ΔCDE (side, angle, side)
So, $AC = CE$ and so AE $= 2 \times CE = 31.288$ m (correct to 3 decimal places) **(1 mark)**

iv. In ΔCED, $\dfrac{\sin(\angle CED)}{CD} = \dfrac{\sin(\angle CDE)}{CE}$ **(1 mark)**

So, $\dfrac{\sin(\angle CED)}{40} = \dfrac{\sin 20°}{15.644}$

So, $\angle CED = 180° - 60°59' = 119°1'$ **(1 mark)**
Note that the diagram indicates that $\angle CED$ is greater than $90°$ so it cannot equal $60°59'$.

v. Draw a vertical line from point A to the carpark level. Let the point where this line touches the carpark level be M. We want to find AM. **(1 mark)**

In the right angled triangle AEM, we have $AE = 31.288$ m (from part **b. iii.**)
Also, since $\angle DEF = 11°$ (part **a. ii.**) and $\angle CED = 119°1'$ (part **b. iv.**), then
$\angle AEM = 180° - 11° - 119°1' = 49°59'$ **(1 mark)**

So, in ΔAEM, $\sin(\angle AEM) = \dfrac{AM}{AE}$

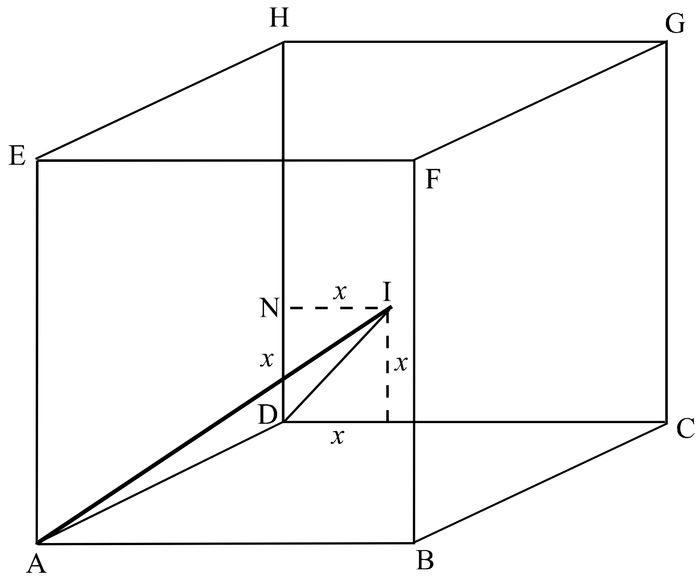
So, $\sin(49°59') = \dfrac{AM}{31.288}$

$AM = 23.96\,\text{m}$

So the vertical distance that the lift moves to get from carpark level to shop level is 23.96 m.
(1 mark)

Question 2

Let the distance required be x.

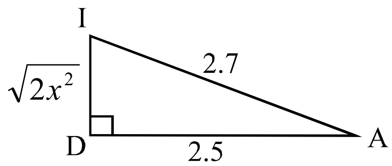


In ΔADI, we have $AD = 2.5,\ AI = 2.7$ and $\angle ADI = 90°$

Consider sidelength DI.

Now in ΔDIN, we have $(DI)^2 = x^2 + x^2$ (Pythagoras)

So, $DI = \sqrt{2x^2}$ **(1 mark)**



So, in ΔADI, we have $(\sqrt{2x^2})^2 = 2.7^2 - 2.5^2$ (Pythagoras) **(1 mark)**

So, $2x^2 = 1.04$

So, $x = 0.72$ (to 2 decimal places)

So the required distance is 0.72 m **(1 mark)**

Total 15 marks

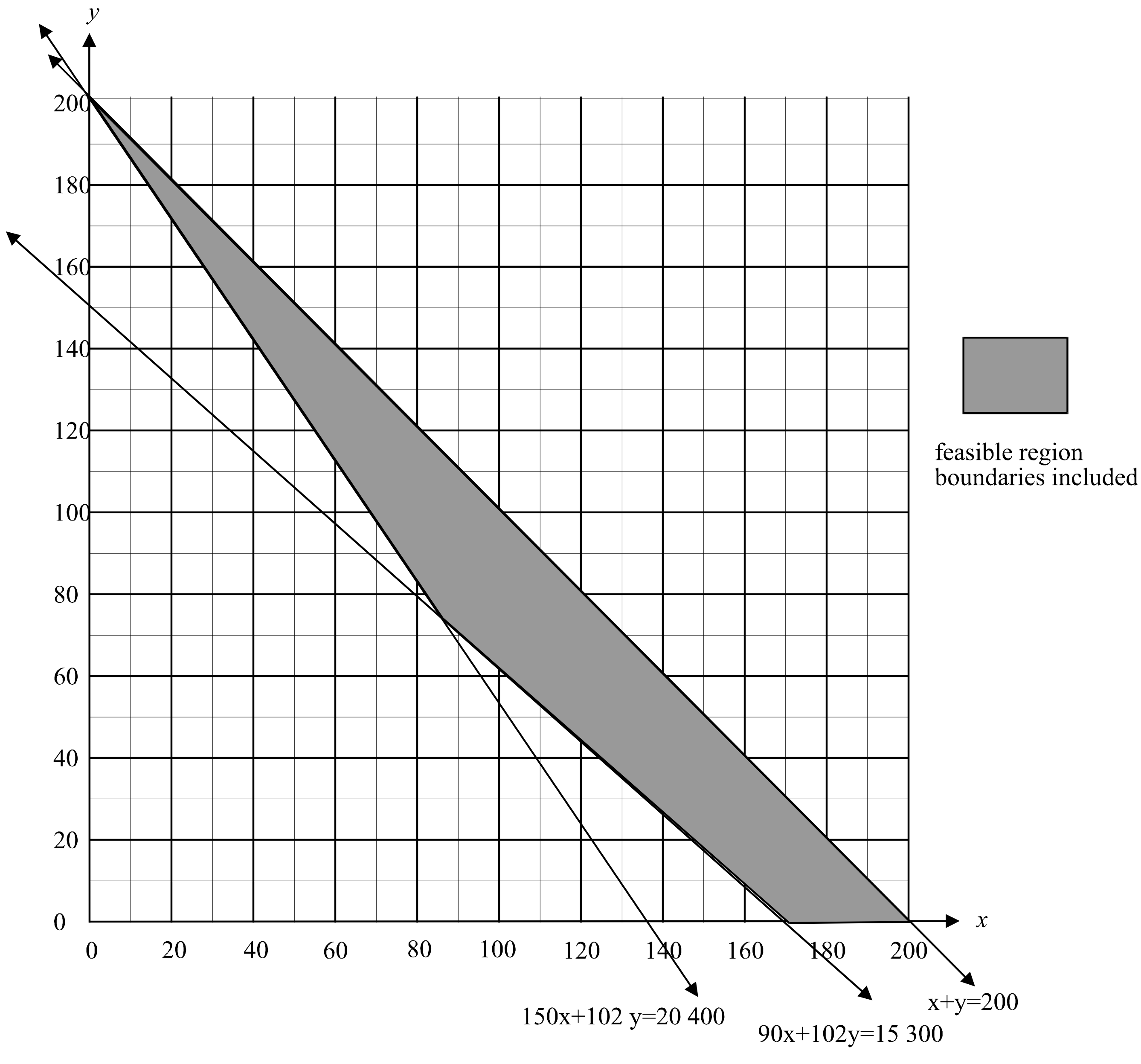
Module 3 - Graphs and relations

Question 1

a. i. $x + y \leq 200 \qquad x \geq 0, \quad y \geq 0$ **(1 mark)**

ii. $150x + 102y \geq 20\,400 \qquad x \geq 0, \quad y \geq 0$ **(1 mark)**

b.



(1 mark) for each line and **(1 mark)** for the correct feasible region.

c. i. The number of members of the bowling club is given by $x + y$.
The point on the feasible region for which $x + y$ is a minimum occurs at a corner point. To find the corner point which occurs at the intersection of the fees and fundraising lines we need to solve simultaneously the equations:

$$90x + 102y = 15300 \qquad \text{(A)}$$
$$150x + 102y = 20\ 400 \qquad \text{(B)}$$

(B) - (A) gives $\quad 60x = 5100$

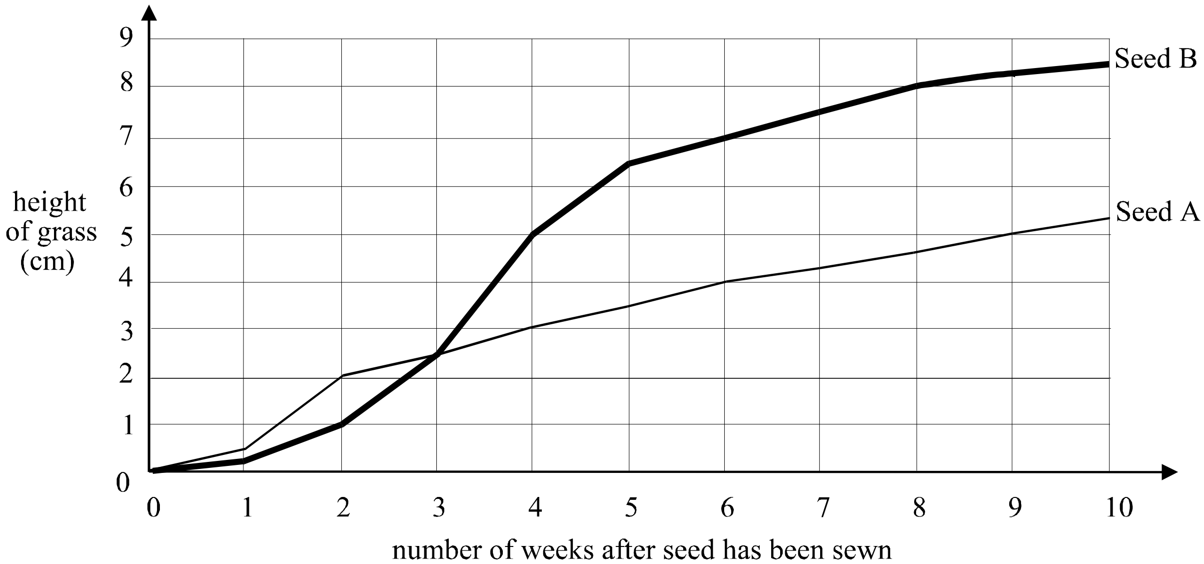
$$x = 85$$

Substituting this into equation (A), we have

$$7650 + 102y = 15\ 300$$
$$y = 75 \qquad \textbf{(1 mark)}$$

The corner points of the feasible region are (85, 75), (0, 200), (200, 0), (170,0)
For (85, 75), $x + y = 160$
For (0, 200), $x + y = 200$
For (200, 0), $x + y = 200$
For (170, 0), $x + y = 170$ **(1 mark)**
The minimum occurs at the point (85, 75), and so the minimum number of members required equals $x + y = 85 + 75 = 160$ **(1 mark)**

ii. Of these 160 members, 85 are women. **(1 mark)**
Question 2
a.



(1 mark)

b. i. 3 weeks after planting the height of the grass on each of the greens is the same, that is, about 2.5 cm high. **(1 mark)**
ii. During the 4th week the height of the grass sewn with seed B first exceeded the height of the grass sewn with seed A. **(1 mark)**

iii. The height of the grass sewn using seed A 12 weeks after sewing would be about 6 cm.
(1 mark)
iv. During the 4th week the grass sewn using seed B grew the most, that is, the gradient of the graph during that period is the steepest. **(1 mark)**

Total 15 marks

Module 4 : Business related mathematics
Question 1

a. i. simple interest $= \dfrac{\Pr T}{100}$ Now, $90 = \dfrac{1500 \times 3 \times T}{100}$

So, $T = 2$ Andrew needs to leave his money in for 2 years. **(2 marks)**

ii. $A = PR^n$ So, $A = 1500 \times (1.03)^2 = \$1591.35$ **(1 mark)**

b. i. $12 \times \$100 = \$1200$ He owes \$1000.
The amount of interest is \$200. **(1 mark)**

ii. The flat rate of interest $\dfrac{200}{1000} \times 100\% = 20\%$ **(1 mark)**

c. $A = P(1 + \dfrac{r}{100})^n$ where r represents the rate per period, that is, the rate per quarter which is 1%

Method 1 - Using logs

Now, $1590 = 1500(1.01)^n$ **(1 mark)**

$$1.06 = 1.01^n$$
$$\log_{10} 1.06 = \log_{10}(1.01)^n$$
$$\log_{10} 1.06 = n \log_{10} 1.01$$

So, $n = \dfrac{\log_{10} 1.06}{\log_{10} 1.01}$ **(1 mark)**

$n = 5.8560$ (to 4 places)

So, it takes 5.8560 quarters. We express our answer to the nearest quarter as requested in the question. It takes 6 quarters. **(1 mark)**

Method 2 - Using trial and error **(1 mark)**

From above, we have $1.06 = 1.01^n$

Let $n = 2$, $1.01^2 = 1.0201$ (to 4 places)

Let $n = 3$, $1.01^3 = 1.0303$ (to 4 places)

Let $n = 4$, $1.01^4 = 1.0406$ (to 4 places)

Let $n = 5$, $1.01^5 = 1.0510$ (to 4 places)

Let $n = 6$, $1.01^6 = 1.0615$ (to 4 places) **(1 mark)**

So, Andrew's money amounts to \$1590 after 6 quarters (to the nearest quarter) **(1 mark)**

Question 2

Now, \$32 000 ÷\$0.30 =106 667 (to the nearest whole number) So, after 106 667 km, the book value of the car would be zero. **(2 marks)**

Question 3

i. $R = 1 + \dfrac{r}{100}$ The loan is over 5 years and there are monthly repayments

$= 1 + \dfrac{1}{100} = 1.01$ made. So, $n = 60$ **(2 marks)**

ii. Using the annuities formula we have

$0 = 80\ 000 \times (1.01)^{60} - \dfrac{Q((1.01)^{60} - 1)}{1.01 - 1}$ So, $Q = \$1779.56$ **(2 marks)**

iii. $\$1779.56 \times 60 = \$106\ 773.60 - \$80\ 000$

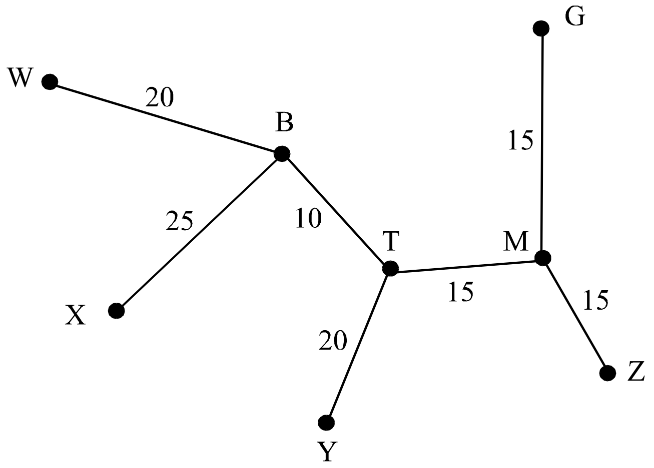
$= \$26\ 773.60$ in interest **(1 mark)**

Total 15 marks

Module 5 : Networks and decision mathematics

a. YTMG = 20 + 15 + 15 = 50 m **(1 mark)**

b. Use Prim's algorithm. Choose the path with the least distance, that is, choose BT. Choose the path leading from B or T with the least distance, that is, TM. Choose the path leading from either B, T or M with the least distance. Now, GM and MZ are both 15 m. Choose either and repeat the previous step. If GM was chosen first then MZ is chosen now or vice-versa. Choose the path leading from B, T, M, G or Z with the least distance (but of course not leading to any of these points but rather to a new "unreached" point. Choose say TY (BW would be fine also since they are each 20m. Repeat the process and choose this time BW. Repeat the step and choose BX. All vertices are now connected. **(2 marks)**



c. 120 m **(1 mark)**

d.

$$\begin{array}{c} \\ B \\ G \\ M \\ T \\ W \\ X \\ Y \\ Z \end{array}\begin{array}{c} \begin{array}{cccccccc} B & G & M & T & W & X & Y & Z \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 2 \\ 0 & 1 & 1 & 1 & 1 & 0 & 2 & 0 \end{bmatrix} \end{array}$$

(2 marks)

Question 2

a. The activities which immediately preceed activity G are C, E and F. **(1 mark)**

b. The earliest start time for activity G is 32. The latest start time for activity for activity D is 14 and for activity H is 25. **(3 marks)**

c. The critical path is ADFGJ **(2 marks)**

d. 14 + 10 + 8 + 12 + 4 = 48 minutes **(1 mark)**

e. A or D or F or G or J **(1 mark)**

f. Slack time is the difference between the latest finish time and the earliest start time plus the duration of the activity, that is, 32 - (0 + 5) = 27 minutes **(1 mark)**

Total 15 marks