

The Mathematical Association of Victoria

2000

MATHEMATICS: FURTHER

Trial Examination 2

Reading time: 15 minutes Writing time: 1 hour 30 minutes

Student's Name: ____

Directions to students

This examination consists of Core material and five modules.

Answer all of the two Core questions.

Select three modules and answer **all** questions in each of your selected modules.

All working and answers should be written in the spaces provided.

The marks allotted to each part of each question appear at the end of each part.

There are **60 marks** available for this task.

A formula sheet is attached.

These questions have been written and published to assist students in their preparations for the 2000 Further Mathematics Examination 2. The questions and associated answers and solutions do not necessarily reflect the views of the Board of Studies Assessing Panels. The Association gratefully acknowledges the permission of the Board to reproduce the formula sheet.

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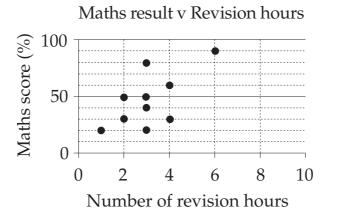
Core: Data analysis

A group of 12 students was offered extra revision classes in an effort to improve their prospects in Mathematics. The following table shows, for each student, the number of revision hours undertaken and their percentage result on the next test.

Student	Revision hours	Maths result
1	2	50
2	5	100
3	4	60
4	3	40
5	1	20
6	2	30
7	6	90
8	3	50
9	4	30
10	1	70
11	5	80
12	3	20

Question 1

The following scatterplot shows the maths score and number of revision hours for 10 of the 12 students



a. The points corresponding to students numbered 2 and 10 are missing. Plot these 2 points on the scatterplot.

[1 mark]

b. Complete the following sentence.

A scatterplot has been used to investigate the	relationship between maths score and	
revision hours as the two variables are both 🛛		
l	[1 mark	(]

c. Briefly explain why revision hours has been selected as the independent variable and maths score as the dependent variable

		[1 mark]
On the given	catterplot draw in a line of best fit 'by eye'	[1 mark]
The equation	f the line you have drawn can be written in the form	
Maths score =	$a \times revision hours + b$	
Find the value	s of a and b for your line	
		[2 marks]
The equation	f the least squares regression line has been calculated to be	
Maths score =	$10.62 \times revision hours + 18.82$	
Complete the	ollowing sentence	
-	es equation suggests that for every increase of rs, maths score will increase by approximately 10.62 %	
		[1 mark]
Use a calculat places	r to determine Pearson's correlation coefficient (r) correct to 4	decimal
		[1 mark]
By considerin	your result from g. complete the following	
	% of the variation in maths score can be explained by the	variation in
revision hours	studied (answer correct to 1 decimal place)	[1 mark]
	de from these statistics that increasing revision time causes stud naths scores ? Explain your answer briefly	

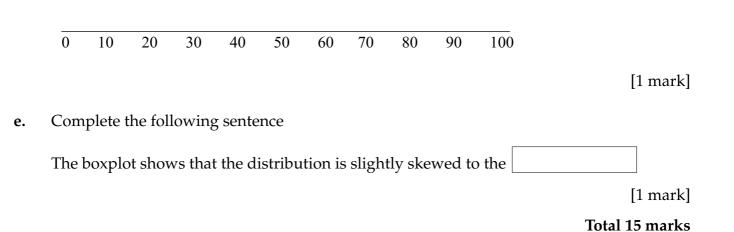
The class teacher prepared a 5 number summary of the maths results as follows

Minimum	20
Lower quartile	30
Median	
Upper quartile	75
Maximum	100

- **a.** Fill in the missing median value
- **b.** Calculate the interquartile range (IQR)

[1 mark]

- c. Use your result from ii) to briefly explain why there are no outliers
- d. Draw a boxplot of the mathematics scores



Module 1 : Number Patterns and Applications

Question 1

Matthew is a research student studying a particular type of wallaby in a national park. He has been given a map of the national park which has a scale of 1:50,000.

a. On the map the straight-line distance, from the carpark to the location of a colony of these wallabies, is 7.8 centimetres. What is the actual straight-line distance from the carpark to the location ? Assume the terrain is flat and give your answer in kilometres correct to one decimal place.

[2 marks]

b. From previous experience walking in this national park Matthew has found that it takes him 3 hours to walk 10 kilometres. How long will it take him to walk from the carpark to the location of the wallabies. Give your answer in hours correct to one decimal place.

The size of a colony of this type of wallaby is thought to increase by 40% each year under ideal conditions.

Matthew's initial count of the colony is 25 and he has calculated that there will be 35 wallabies in the colony at the start of the second year.

Assuming that the size of the colony continues to increase by 40% each year :-

a. Calculate the size of the colony at the beginning of the third year.

[1 mark]

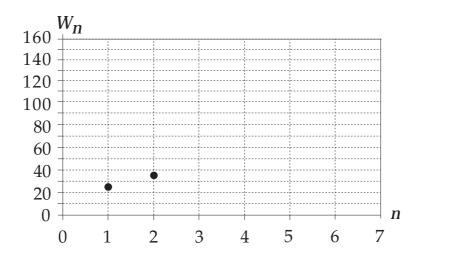
b. Write down an expression for W_n , in terms of n, the size of the colony at the beginning of the nth year.

[2 marks]

c. In which year will the size of the colony first reach 150?

[1 mark]

d. On the axes below complete the graph of the sequence W_n for $1 \le n \le 6$



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Matthew has observed that, although the colony is still increasing naturally, there are other factors (predators, availability of food etc.) that are affecting the size of the colony and he has done an actual count of the colony at the beginning of each of three years. He has found the following numbers :-

Start of year	1	2	3
Count	25	30	37

and has concluded that the size of the colony at the beginning of the nth year, W_n , would be better modelled by a difference equation of the form :-

 $W_{n+1} = aW_n + b$; where *a* and *b* are constants and $W_1 = 25$

a. Use the table above, or otherwise, to find the values of **a** and **b**

[2 marks]

b. Using your difference equation calculate the size of the colony at the beginning of the fourth year. Give your answer to the nearest whole number.

[2 marks]

c. In which year will the size of the colony first reach 150, assuming that the size of the colony continues to increase according to the difference equation.

[1 mark] Total 15 marks

Module 2: Geometry and Trigonometry

Question 1

Johnny, the orienteering ace of Mt. Hotham, sets off from the competition starting hut on a bearing of N 48°E for a distance of 3 kilometres reaching the first checkpoint. Unfortunately, Johnny has to return to the starting hut to collect his backpack he had left behind.

a. What is the bearing for his trip back to the starting hut?

[1 mark]

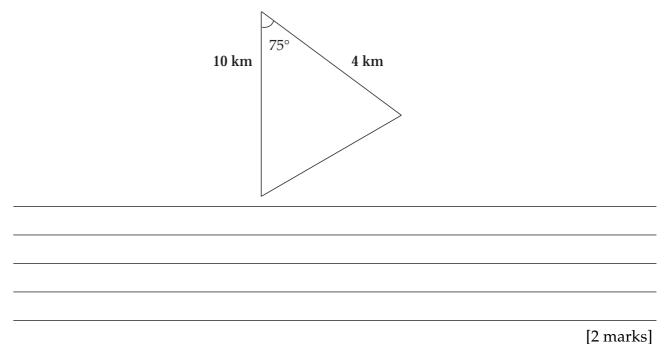
Returning to the first checkpoint, Johnny changes direction and heads on a bearing of N 45° W walking at a steady pace of 6 km per hour for $1\frac{1}{2}$ hours

b. Draw an appropriate diagram for the first two legs of the journey taken by Johnny.

[2 marks]

c. Calculate the distance Johnny is from the starting hut at the completion of first two legs to one decimal place.

d. All other competitors at any one time could be found anywhere within the following region shown below. Calculate the area of the region bounded by this triangular course, correct to 1 decimal place. (Diagram not to scale.)



Question 2

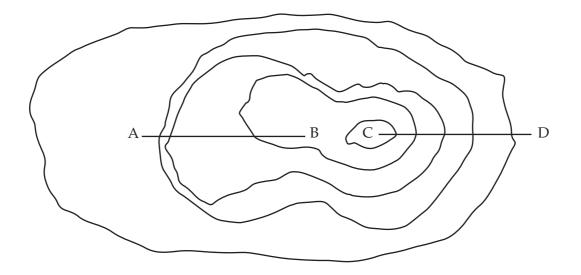
Johnny is ready for the next leg, a climb up a gradual but steep hill. Instructions provided states the hill has a gradient of 1:60.

a. Calculate the angle of elevation of this hill to the nearest degree.

[1 mark]

b. On the map it indicates he has to travel 800 metres (horizontal distance). If the altitude at the start of the hill climb was 30 metres, what will be the altitude after the climb?

c. From the contour map shown, which line \overline{AB} or \overline{CD} , would have been the most likely journey taken by Johnny when climbing the steep, but gradual hill.



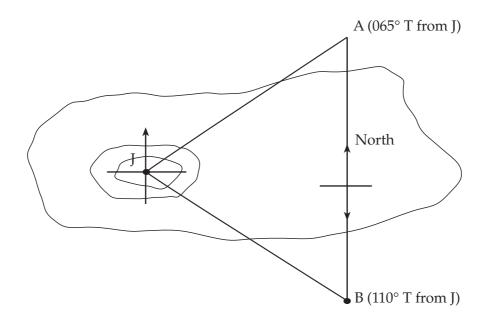
[1 mark]

d. The map scale is stated as 1 cm = 500 metres. Express this as a map scale ratio eg 1:20

[1 mark]

e. On the map, Johnny travelled a total of 32 cm for the entire journey. What was this distance in kilometres to one decimal place.

During the stage at the top of the hill Johnny sees two checkpoints. On the map it indicates they are 6.4 km apart on a North-South line.



Using the provided map and bearings to the checkpoints taken by Johnny, as shown above, calculate the distance to the nearest checkpoint (to 1 decimal place).



[2 marks] **Total 15 marks**

Module 3 Graphs and Relations

Question 1

The Yallambie Football League (YFL) began playing matches at Legend Stadium in 2000. Members of the YFL who book tickets over the phone pay a booking fee of \$3 per ticket. An additional once only fee of \$5 is charged by the ticket distributor for the phone purchase.

A function that represents the total cost incurred during a phone booking can be written as C = aN + b where *C* represents the cost and *N* the number of tickets purchased.

a. Write down the values	of	a and	b
---------------------------------	----	-------	---

b.	Find the cost incurred by a member	er who books	
υ.	This the cost incurred by a memory	er who books	

i. 1 ticket

ii. 5 tickets

[2 marks]

[1 mark]

c. Using your results from b) or otherwise, sketch a graph of the cost function on the axes provided

-		
	▶	

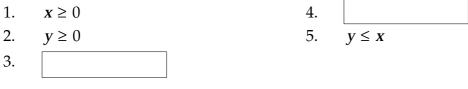
d.	One particular member, Joe, paid \$41 when booking an allocation of seats. How many	Į
	seats did Joe purchase ?	

11 mark	
[1 mark]	

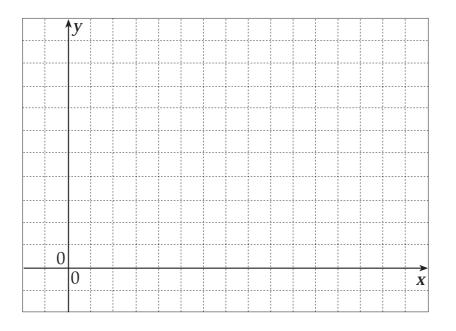
2 categories of seating are available for members at Legend Stadium – Premium and Standard. The league may vary the total of these from week to week under the following conditions

- The total number of seats available to members will not exceed 5000
- No more than 3000 Premium seats will be made available
- The number of Standard seats cannot exceed the number of Premium seats

If *x* represents Premium seats and *y* represents Standard seats then the following inequations can be formed



- **a.** Write down the remaining constraints in the spaces provided
- b. Sketch these constraints and indicate the feasible region



[3 marks]

c. If Premium seats sell for \$25 dollars and Standard seats sell for \$15 , write down an expression for the revenue obtained by the league on any given day

[1 mark]

d. Determine the number of seats of each type that must be sold in order to maximise revenue

[2 marks]

e. Hence determine the maximum revenue that the league can expect on any given day

[1 mark] **Total 15 marks**

Module 4: Business Related Mathematics

INSTYLE Limo Service has a limousine car bought for \$58 400 which is to depreciate at a rate of 40 cents for every kilometre travelled.

Question 1

a. Complete the table below.

Time (years)	Distance travelled (km)	Depreciation (\$)	Book value at end of year (\$)
1	13 290	5 316	
2	15 650		46 824
3	14 175	5 670	41 154
4		4 000	37 154

[3 marks]

b. The limousine is scrapped after it has reached a bookvalue of \$30 000. Calculate the total kilometres travelled.

[1 mark]

To update to the latest model limousine valued at \$120 000, INSTYLE use the \$30 000 as a deposit for a loan. INSTYLE has two options for servicing this loan.

Question 2

The first option is a hire purchase plan with monthly payments over 5 years at 6% per annum. The \$30 000 will be used as a deposit.

a. Calculate the effective interest rate of the hire purchase (to one decimal place).

Calculate the total interest charged on the loan over the 5 year period.
[2 marks]
Calculate the monthly repayment required for the proposed hire purchase.
[2 marks]
Calculate the total cost for the purchase and financing for the \$120 000 limousine?
[1 mark]
stion 3

Option 2 is to apply for a five year bank loan currently offering 9% pa compounded monthly. If INSTYLE Limo service take up this option they will only need to borrow \$90 000.

a. Calculate the compounding factor, *R*, used in the Annuities Formula.

b. Calculate the monthly payment using the Annuities Formula to the nearest dollar.

[3 marks] Total 15 marks

Module 5 : Networks and decision mathematics

A large regional tourism authority wishes to set up their own website which will provide a schedule of events in their region with links to details on each event.

The company contracted to set-up the website have produced **Table 1**, below, which gives the activities involved in setting-up the website, the order in which these activities can proceed and the time taken for completion.

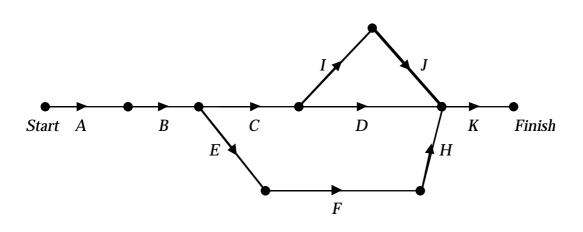
Activity	Description	Predecessor	Duration (weeks)
А	Gather Website requirements	_	5
В	Devise technical solution	Α	3
С	Build	В	5
D	Test	С	2
E	Investigate network providers	В	2
F	Install network infrastructure	E	3
G	Order hardware	С	3
H	Install hardware	<i>F, G</i>	3
Ι	Create On-line Help	С	4
J	Install On-Line Help	Ι	1
K	Install software	D, H, K	1

Table 1

Question 1

A directed graph that can be used to chart the activities involved in setting-up the website is shown in **Figure 1** below.

a. Activity *G* has been omitted from the graph in **Figure 1**. Complete the graph by drawing in activity *G*.





b. Use the information in **Table 1** and **Figure 1** to complete **Table 2** below.

Activity	Earliest start time	Latest start time
A	0	0
В	5	5
С	8	
D	13	
E	8	13
F	10	13
G	13	13
Н		16
Ι	13	14
J	17	
K	19	19

Table 2

[4 marks]

c. What is the earliest completion time for setting-up the website ?

[1 mark]

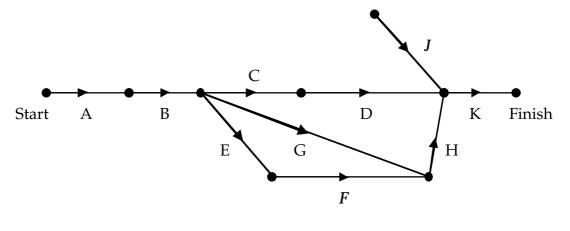
d. Write down the critical path of the network.

[1 mark]

e. Find the slack (float time) for activity *F*

The tourism authority would like the task completed as soon as possible and so the company setting-up the website have reconsidered their schedule and have decided that both activities I and G could be started after the completion of activity B instead of after activity C.

a. Complete the graph in **Figure 2** showing the new position of activity *I*.





[1 mark]

b. What is the earliest completion time for setting-up the website now that activities *I* and *G* have been rescheduled ?

[1 mark]

c. Write down the critical path for this adjusted network.

The company employed to set-up the website has partitioned the job into four tasks and asked four contractors to quote their price for each of these tasks.

Task A : Overall project supervisorTask B : Build and test websiteTask C : Supervise network and hardware installationTask D : Create On-line help

The contractor quotes (in \$100's) on each of the tasks are given in **Table 3**.

	Contractor			
Task	Bryan	Erryn	Petra	Cuong
Α	20	25	30	20
В	200	240	250	240
С	60	65	70	50
D	120	100	110	90

Table 3

a. Using the Hungarian algorithm, or otherwise, complete the following table. Allocate one task to each of the contractors so that the total cost is a minimum.

Task	Assigned to contractor
А	
В	Bryan
C	
D	

[3 marks]

b. What is the minimum total cost of employing the contractors ?

[1 mark] **Total 15 marks**

Trial Examination 2 Solutions

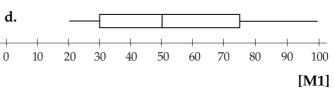
Solutions Core : Data analysis

Question 1

- a. Both points must be plotted accurately. [M1]
- b. Numerical. [A1]
- c. It is likely that revision time will influence results therefore we would select revision hours as the independent variable and maths score as the dependent variable. [A1]
- d. A line that is reasonably representative of the points must be drawn. [M1]
- e. The results will vary according to the line drawn using two points off the line to calculate the gradient. [M1]
 Substitution or reading off the graph to determine the intercept. [M1]
 Values should be somewhat near the least squares values.
- f. One. [A1]
- g. Pearson's correlation coefficient = 0.6284 [A1]
- h. 39.5 %. [A1]
- i. No, since correlation does not imply cause. [A1]

Question 2

- **a.** Median = 50. [A1]
- **b.** IQR = upper quartile lower quartile = 75-30 = 45. [A1]
- c. The maximum and minimum values are less than 1.5 IQR from the upper quartile and lower quartile respectively. [A1]



e. Slightly skewed to the right [A1]

Module 1 : Number patterns and Applications

Question 1

 $1:50000 = 7.8 \times 1:7.8 \times 50000$ a = 7.8 : 390000 cm [M1] 390,000 centimetres = 3900 metres = 3.9 kilometres [A1] b. 10 kilomtres in 3 hours is equivalent 1 kilometre in 3/10 = 0.3 hours. [A1] Matthew walks 3.9 km so it will take him $3.9 \times 0.3 = 1.17$ hours = 1.2 hours; correct to one decimal place. [H1]

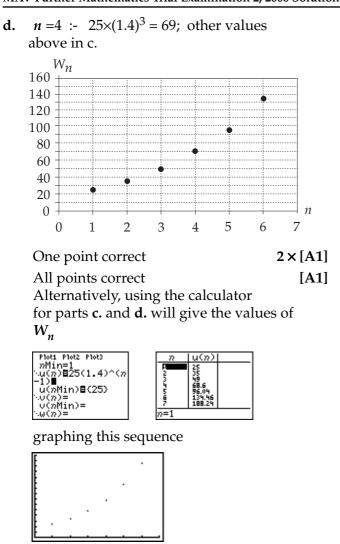
Question 2

Hence
$$W_n = 25 \times (1.4)^{n-1}$$
 [A1]

c. Solve
$$150 = 25 \times (1.4)^{n-1}$$
 by trial and error
 $n = 5 := 25 \times (1.4)^4 = 96$
 $n = 6 := 25 \times (1.4)^5 = 134$
 $n = 7 := 25 \times (1.4)^6 = 188$

These are figures for the colony's size at the beginning of the year hence the colony size will reach 150 during the 6th year. [A1]

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Question 3

a. Substituting the values for Years 1
and 2 into
$$W_{n+1} = a \ W_n + b$$

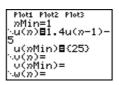
 $n = 1 : W_2 = a \ W_1 + b$
 $30 = a \times 25 + b \dots i$
 $n = 2 : W_3 = a \ W_2 + b$
 $37 = a \times 30 + b \dots i$ [M1]
 $ii - i$ 7 = 5a
 $a = 7/5$
 $a = 1.4$ subst. in i
 $30 = 1.4 \times 25 + b$
 $b = 30 - 35$
 $b = -5$
So $W_{n+1} = 1.4 \ W_n - 5$ [A1]
It would be reasonable to guess that
 $a = 1.4$, as for Question 2, and that $b = -5$, the

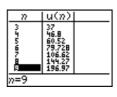
a = 1.4, as for Question 2, and that b = -5, the difference between the colony numbers for Year 2 in Questions 2 and 3.

b.
$$W_{n+1} = 1.4 \ W_n - 5$$

 $n = 3 \implies W_4 = 1.4W_3 - 5$ [M1]
 $W_4 = 1.4 \times 37 - 5$
 $= 46.8$
 $= 47$ to the nearest
whole number. [A1]

c. Using the calculator,





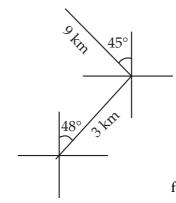
The size of the colony at the beginning of the 8th year is 144 and at the beginning of the 9th year is 197. Hence the colony must reach 150 wallabies during the 8th year. **[A1]**

Module 2: Geometry and Trigonometry

Question 1

b.

a. This is a 180° turn around or **S** 48° **W** [A1]



for each leg [A1]

c. Using cosine rule

$$c^{2} = a^{2} + b^{2} - 2 \times a \times b \times \cos C^{\circ}$$

$$c^{2} = 9^{2} + 3^{2} - 2 \times 9 \times 3 \times \cos 87^{\circ}$$

$$(87^{\circ} = 180^{\circ} - 45^{\circ} - 48^{\circ})$$

$$c = \sqrt{87.17385.}$$

$$c = 9.3366.. \approx 9.3 \text{ kilometres}$$
[M1]

d. Using the sine rule

$$Area = \frac{1}{2} ab \sin C^{\circ}$$

$$Area = \frac{1}{2} \times 4 \times 10 \times \sin 75^{\circ}$$
[M1]
$$= 19.3 \text{ km}^{2}$$
[A1]

Question 2

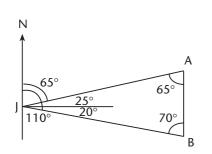
a.
$$\tan \theta = \frac{1}{60}$$
 $\theta = \tan^{-1} \left(\frac{1}{60} \right) = 0.955^{\circ} \approx 1^{\circ}$ [A1]

b. $\tan 0.955^\circ = \frac{x}{800}$ $x = 800 \times \frac{1}{60} = 13\frac{1}{3}$ metres [M1]

New altitude = $13\frac{1}{3} + 30 = 43\frac{1}{3}$ metres [A1]

- *CD* is the climb taken by Johnny as the contour lines are consistantly spaced and close together to indicate a gradual steep climb
 [A1]
- d. 1 cm = 500 m $1 \text{ cm} = 500 \times 100 \text{ cm}$ 1 : 50 000 [A1]
- e. 1 cm = 500 m $32 \text{ cm} = 32 \times 500 \text{ m} = 16\,000 \text{ m} = 16.0 \text{ km}$ [A1]

Question 3



Closest ckeckpoint is B. The side \overline{JA} is opposite the smaller of the two angles.

$$\frac{x}{\sin 65^{\circ}} = \frac{6.4}{\sin 45^{\circ}}$$
 [M1]

$$x = \frac{6.4 \times \sin 65^{\circ}}{\sin 45^{\circ}} = 8.2 \text{ km}$$
 [A1]

Module 3: Graphs and relations

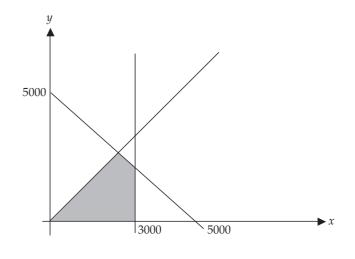
Question 1

- b. i. \$8 [A1]
 - ii. \$20 [A1]
- Axes labelled and 2 correct points plotted [M1]
 Straight line drawn for positive N values only [M1]
- d. 12 seats [A1]

Question 2

a.	$x + y \le 5000$	[A1]
	$x \le 3000$	[A1]

b.



c.
$$R = 25x + 15y$$
 [A1]

- d. maximum when selling 3000 Premium and 2000 Standard [A2]
- **e.** maximum revenue = \$105000 [A1]

Module 4: Business Related Mathematics

Question 1

Time (years)	Distance travelled (km)	Depreciation (\$)	Book value at end of year (\$)
1	13 290	5 316	53 084
2	15 650	6 260	46 824
3	14 175	5 670	41 154
4	10 000	4 000	37 154

a. \$58 400 - \$5 316 = \$53 084 [A1] 15 650 km × \$0.40 = \$6 260 [A1]

 $4000 \div 0.40 = 10\ 000\ \text{km}$ [A1]

Question 2

a. Effective interest rate
$$=\frac{2 \times n}{n+1} \times$$
 Flat Rate
 $=\frac{2 \times 60}{60+1} \times 6\%$ [M1]
 $=11.8\%$ [A1]

b. Interest charged =
$$\frac{90000 \times 6 \times 5}{100}$$
 = \$27000 [M1][A1]

c. Total repayments = \$ 90 000 + \$27 000 = \$117 000

Monthly repayments

$$= \frac{\$117000}{60 \text{ payments}} = \$1950.00 \text{ per month}$$
[M1][A1]

d. Total cost = deposit + 60 monthly repayments
 = \$30 000 + 60 × \$1950
 = \$147 000 [A1]

Question 3

a.
$$R = 1 + \frac{9/12}{100} = 1.0075$$
 [A1]

b.
$$Q = \frac{PR^n (R-1)}{R^n - 1}$$
 where $n = 60 P = $90 000$

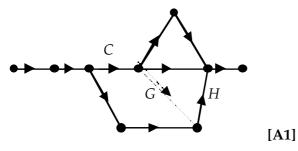
$$Q = \frac{90000 \times 1.0075^{60} (1.0075 - 1)}{1.0075^{60} - 1}$$

= \$1 868.25 [A1]

Module 5 : Networks and decision Mathematics

Question 1

a. Activity G must be after activity C but is a predecessor to activity H.



b. Latest start time for C is 8 weeks; C(5)is on the critical path and G(3), H(3) and K(1) have to be completed after C; a total of 12 weeks. (The earliest completion time for the project is 20 weeks.) [H1]
[H1 if earliest completion time is incorrect]

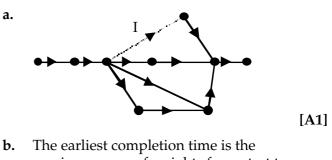
Latest start time for D(2) is 20 - 3 = 17 weeks; only K(I) has to be completed after D making a total of 3 weeks to completion time. [A1]

Earliest start time for H(3) is 16 weeks; A(5), B(3), C(5), G(3) have to be completed first; a total of 16 weeks. [H1]

Latest start time for activity J(1) is 18 weeks as only K(1) has to be completed after J.

- c. Earliest completion time for the project is 20 weeks. Considering the total weight of all the possible paths from start to finish the 'earliest completion time' is the weight of the path that has the largest value. This is the weight of the critical path. [A1]
- d. The critical path is ABCGHK. Any activity on this path, if delayed will delay the completion time of the project. [A1]
- e. Slack (float time) for activity F is: Latest start time – Earliest start time = 13 – 10 = 3 weeks





- The earliest completion time is the maximum sum of weights from start to finish.
 This is now along path ABEFHK and so has a total weight of 17 weeks. [A1]
- c. The critical path is ABEFHK [A1]

Question 3

[H1]

[A1]

a. Using the Hungarian algorithm :-Subtract the minimum from each of the rows

0	5	10	0
0	40	50	40
10	15	20	0
30	10	20	0

Subtract the minimum from each of the columns and cover the zeros with a minimum of lines

0	0	0	0	1
0	0	0	0	
Ο	35	40	40	
0	00	10	-10	
10	10	10	Ø	
30	5	10	0	

Add the minimum uncovered number (5) to the elements at the intersections of the lines and subtract it from the elements not covered by the lines.

0	0	0	5
\bigcirc	35	40	45
5	5	5	\bigcirc
25	\bigcirc	5	0

We now have a an independent set of zeros (circled above)

The allocation is :-

Task	Assigned to contractor
А	Petra
В	Bryan
С	Cuong
D	Erryn

[A1] ×3

b. The minimum cost is

3000 + 20000 + 5000 + 10000 = \$38,000 [A1] (380 acceptable)