

**THE
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GROUP**

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**FURTHER MATHEMATICS
TRIAL EXAMINATION 1
2001
SOLUTIONS**

Section A

Section B : Answers

Answers	Module 1	Module 2	Module 3	Module 4	Module 5
Core					
1. E	Arithmetic	Geometry	Graphs	Business	Networks
2. A	and	and	and	related	and
3. C	applications	trigonometry	relations	mathematics	decision
4. C					maths
5. A	1. C	1. B	1. E	1. D	1. C
6. C	2. C	2. D	2. C	2. B	2. C
7. C	3. D	3. B	3. B	3. B	3. B
8. E	4. A	4. A	4. B	4. C	4. D
9. B	5. B	5. E	5. D	5. C	5. D
10. D	6. C	6. B	6. C	6. E	6. B
11. E	7. D	7. B	7. C	7. B	7. A
12. D	8. A	8. C	8. D	8. A	8. E
13. B	9. D	9. C	9. B	9. E	9. D

Section A: Core – solutions

Question 1

There were 9 jobs which gave the contractor a profit of less than \$400. There were a total of 50 jobs, so the percentage is given by $\left(\frac{9}{50} \times \frac{100}{1}\right)\% = 18\%$. The answer is E.

Question 2

The distribution is negatively skewed since the data tails off to the left and is clustered towards the right hand side. The answer is A.

Question 3

For Sunday, the interquartile range is $60 - 30 = 30$ and for Saturday the interquartile range is $65 - 50 = 15$. So the difference between the 2 interquartile ranges is 15. The answer is C.

Question 4

The box representing the Sunday transactions is larger than that for Saturday, that is, the data is more spread out or variable. Also, each of the minimum value, the first quartile, the median, the third quartile and the maximum value of the Sunday data is less than that of Saturday. So the number of sales on a Sunday is generally lower than on a Saturday. The answer is C.

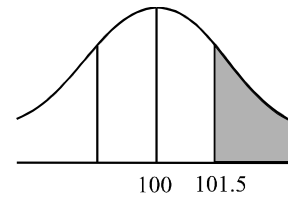
Question 5

We know that 68% of packets of the biscuits will weigh between 1 standard deviation either side of the mean, that is, $100 - 1.5 = 98.5\text{g}$ and $100 + 1.5 = 101.5\text{g}$

So, $100\% - 68\% = 32\%$ lie outside this range.

From the diagram we see that the region we require which is shaded must be half 32% because of the symmetry of the bell-shaped curve. So, 16% of packets of “Sonice” rice crackers weigh more than 101.5g.

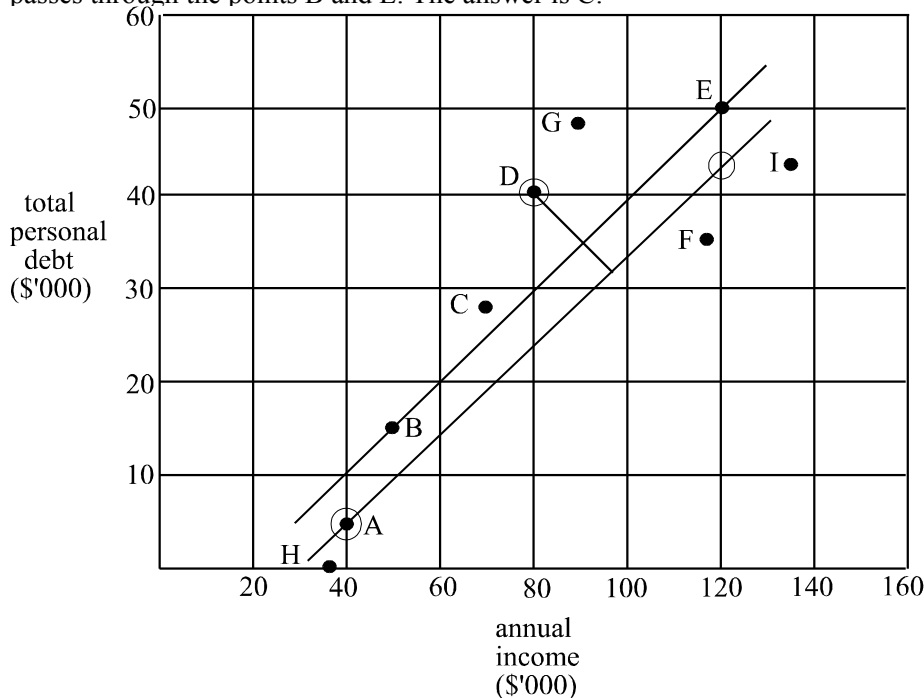
The answer is A.

**Question 6**

In the 51-65 age group, that is, the column on the right, the percentage of people who drink 3-4 days in a week is given by $65\% - 25\% = 40\%$. The answer is C.

Question 7

The three medians are indicated by an open circle on the diagram below. Two of these medians happen to pass through the points D and A. A line is drawn through the two medians from the left and right hand groups of points. Move this line one third of the way towards the median which happens to be located at point D. This line that has been drawn is the three median line and it passes through the points B and E. The answer is C.

**Question 8**

The scatterplot shows a moderately strong positive linear relationship between the two variables. A reasonable estimate of the product moment correlation coefficient would therefore be 0.8. The answer is E.

Question 9

The residual value equals the actual value minus the predicted value.

On the line of best fit, when $x = 2$, $y = 2$ and so the predicted value is 2.

So, for the data point $(2, 2.8)$, the residual value $= 2.8 - 2 = 0.8$.

The answer is B.

Question 10

Calculating r is a nonsensical answer.

Drawing a line of best fit is a nonsensical answer.

An x^2 transformation expands the upper end of the x scale.

A y^2 transformation expands the upper end of the y scale.

A $\log x$ transformation compresses the upper end of the x scale which is what is required.

The answer is D.

Question 11

Now, deseasonalised sales $= \frac{\text{actual sales}}{\text{seasonal index}}$

So, $242 = \frac{\text{actual sales}}{1.54}$

So actual sales are 372.68 tones. This is closest to 373.

The answer is E.

Question 12

There is a repetitive fluctuation over each year which represents seasonal variation. There is also an overall trend downwards. So we can describe the time series as seasonal and trending downwards. The answer is D.

Question 13

		4-moving median	4-moving median with centring
Jan	22		
Feb	16		
Mar	20	18	17
Apr	14	16	15.5
May	16	15	
Jun	10		

Using smoothing with a 4-moving median with centring, the smoothed number of new enrolments for April is 15.5.

The answer is B.

Section B – solutions**Module 1 : Number patterns and applications****Question 1**

The ratio of boys to girls is 2:3. There are a total of 5 parts, 2 of which account for the girls at the

centre. So, $\frac{2}{5}$ or $\left(\frac{2}{5} \times \frac{100}{1}\right)\% = 40\%$

So, 40% of children at the centre are girls.

The answer is C.

Question 2

$$\begin{aligned} \text{Now, } S_n &= \frac{15}{2}(2a + (n-1)d) \\ &= \frac{15}{2}\left(2 \times \frac{1}{2} + 14 \times \frac{3}{2}\right) \\ &= \frac{15}{2}(1 + 21) \\ &= \frac{15}{2} \times 22 \\ &= 165 \end{aligned}$$

The answer is C.

Question 3

$$\begin{aligned} \text{Now, } S_n &= \frac{a}{1-r} \\ &= \frac{6}{1-\frac{1}{3}} \\ &= 6 \div \frac{2}{3} \\ &= 6 \times \frac{3}{2} \\ &= 9 \end{aligned}$$

The answer is D.

Question 4

The sequence generated by the difference equation is 5, 2, -1, ...

This is an arithmetic sequence with $a = 5$ and $d = -3$

$$\text{Now, } t_n = a + (n-1)d$$

$$\begin{aligned} \text{So we have, } t_n &= 5 + (n-1) \times -3 \\ &= 5 - 3n + 3 \end{aligned}$$

$$\text{So, } t_n = -3n + 8$$

$$\text{Check that } t_1 = 5. \text{ Now } t_1 = -3 \times 1 + 8 = 5$$

The answer is A.

Question 5

The sequence shown on the graph is 10, 20, 40, 80.... This is a geometric sequence with $a = 10$ and $r = 2$. The difference equation $t_{n+1} = 2t_n, t_1 = 10$ will generate this. Note that $t_n = 2n$ will generate the sequence 2, 4, 6, 8....

The answer is B.

Question 6

The sequence generated by the difference equation $t_{n+1} = 3t_n + 1$ where $t_1 = -4$ is -4, -11, -32, -95,....

Since $-11 - (-4) \neq -32 - (-11)$ the sequence is not arithmetic.

Also, $\frac{-11}{-4} \neq \frac{-32}{-11}$ and hence the sequence is not geometric.

The only possible option is to say that the sequence is decreasing.

The answer is C.

Question 7

The scale on the map is 1 : 15 000

So, we have 40 : 600 000

So the actual distance covered is 600 000 mm which is 600 m or 0.6 km

The answer is D.

Question 8

Now, $A_1 = 20\,000$, $A_2 = 1.04 \times 20\,000 + 3000$, $A_3 = 1.04 \times A_2 + 3000$

The difference equation which generates this sequence is

$$A_{n+1} = 1.04A_n + 3000, \quad A_1 = 20\,000.$$

The answer is A.

Question 9

The annual pension of Mrs. Davidson forms a geometric sequence with $a = 12\,000$ and $r = 1.03$

$$\begin{aligned} \text{So, } S_{10} &= \frac{a(r^n - 1)}{r - 1} \quad r > 1 \\ &= \frac{12000((1.03)^{10} - 1)}{0.03} \\ &= 137566.55 \end{aligned}$$

The answer is D.

Module 2 : Geometry and trigonometry**Question 1**

$\angle ABC = 80^\circ$ (alternate angles in parallel lines)

$\triangle ABC$ is an isosceles triangle and therefore $\angle BAC = \angle ACB$

So, since the angles in a triangle add to give 180° , we have $\angle BAC = 50^\circ$.

The answer is B.

Question 2

$$\begin{aligned} \text{In } \triangle PQR, \quad \sin(\angle PRQ) &= \frac{\text{opposite}}{\text{hypotenuse}} \\ &= \frac{5}{7.5} \end{aligned}$$

$$\text{So,} \quad \angle PRQ = 41^\circ 48'$$

The closest answer is 42°

The answer is D.

Question 3

$$\begin{aligned} \text{Area} &= \frac{1}{2}bc\sin A \\ &= \frac{1}{2} \times 10 \times 4\sin 30^\circ \\ &= 10 \end{aligned}$$

The answer is B.

Question 4

Using the cosine rule, we have $c^2 = a^2 + b^2 - 2ab\cos C$

$$4 = 16 + 25 - 40\cos C$$

$$\cos C = \frac{37}{40}$$

$$C = 22^\circ 19'$$

The closest answer is 22° .

The answer is A.

Question 5

The scale factor is 4, that is, the radius of the larger sphere is 4 times the radius of the smaller sphere. Hence the volume of the larger sphere is 4^3 times the volume of the smaller sphere.

$$\text{So, } 2304\pi \times 64 = 147456\pi$$

The answer is E.

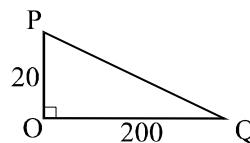
Question 6

The scale on the map is 1 : 20 000. The distance between points P and Q on the map is 10 mm and therefore the actual horizontal distance between point P and point Q is $20\,000 \times 10$ mm or 200 m.

So, the average slope of the line joining points

$$\text{P and Q is } \frac{20}{200} = 0.1$$

The answer is B.



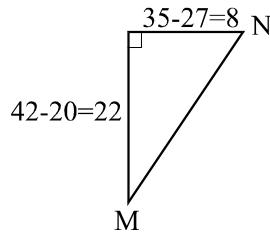
Question 7

From the diagram, we have

$$MN = \sqrt{22^2 + 8^2}$$

$$= 23.4 \text{ m (correct to 1 decimal place)}$$

The answer is B.

**Question 8**

From the diagram, in $\triangle BCD$,

$$BC = \sqrt{6^2 + 6^2}$$

$$= \sqrt{72}$$

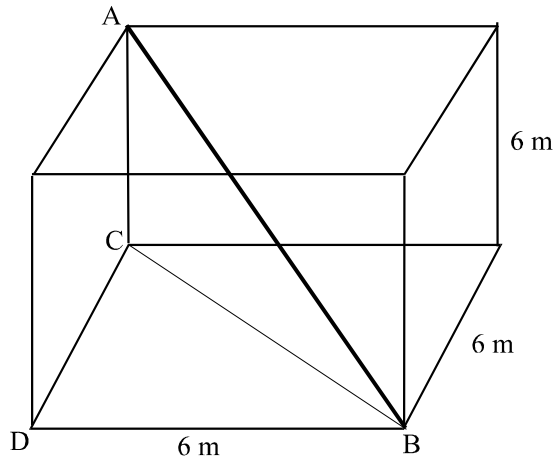
So, in $\triangle ABC$,

$$AB = \sqrt{6^2 + (\sqrt{72})^2}$$

$$= \sqrt{108}$$

$$= 10.4 \text{ (correct to 1 decimal place)}$$

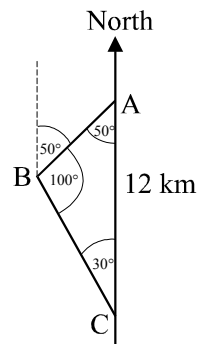
The answer is C.

**Question 9**

In $\triangle ABC$, $\frac{BC}{\sin 50^\circ} = \frac{12}{\sin 100^\circ}$

$$BC = 9.3 \text{ (correct to 1 decimal place)}$$

The answer is C.



Module 3 : Graphs and relations**Question 1**

To find the gradient of the line, choose any two points which lie on the line, say, (0,1) and (6,5).

$$\begin{aligned}\text{Gradient} &= \frac{5-1}{6-0} \\ &= \frac{2}{3}\end{aligned}$$

The y -intercept of the line is 1. Now, the equation of a straight line with a gradient of m and a y -intercept of c is given by $y = mx + c$.

So the equation of our line is $y = \frac{2}{3}x + 1$

$$\begin{aligned}\text{So,} \qquad \qquad \qquad 3y &= 2x + 3 \\ 3y - 2x &= 3\end{aligned}$$

The answer is E.

Question 2

The times when the temperature was zero correspond to when the graph crosses or touches the horizontal axis. This occurs on two occasions. The answer is C.

Question 3

All of the options are true except for option B. The longest period that enrolments remained constant for was 3 terms. This occurred in Terms 1, 2 and 3 in the year 2000.

The answer is B.

Question 4

The POQ courier company will break even when $R = C$, given that $R = 15x$ and $C = 5x + 3000$.

So, we require that $15x = 5x + 3000$

$$10x = 3000$$

$$x = 300$$

So, the POQ courier company needs to make 300 deliveries to break even. The answer is B.

Question 5

All of the options are untrue except for option D. The value of the Australian dollar was on 52 US cents for 3 hours and on 51 US cents for 1 hour. The rest of the time it was on or below 50 US cents. The answer is D.

Question 6

The region above the horizontal line is given by $y \geq 3$.

The region to the right of the vertical line is given by $x \geq 4$.

The region below the diagonal line sloping up to the right is given by $y \leq x$.

The region below the diagonal line sloping up to the left is given by $x + y \leq 10$.

Note that to find the equation of the boundary of this last region, you use the fact that the gradient

is negative and is $\frac{-10}{10} = -1$. Also, the y -intercept is 10. The equation of the line is therefore

$$y = -x + 10 \text{ which becomes } x + y = 10.$$

The only option which describes all these regions is option C.

The answer is C.

Question 7

The smallest value in the feasible region will occur at one of the corner points.

The corner points are (4,3), (4,4) (7,3) and the point of intersection of the lines with equations $y = x$ and $x + y = 10$. Substituting $y = x$ into $x + y = 10$ gives us $2x = 10$ and so $x = 5$ and $y = 5$. The fourth corner point is the point (5,5).

$$\text{At (4,3), } 2x - 3y = -1$$

$$\text{At (4,4), } 2x - 3y = -4$$

$$\text{At (7,3), } 2x - 3y = 5$$

$$\text{At (5,5), } 2x - 3y = -5$$

The smallest value of $2x - 3y$ is -5

The answer is C.

Question 8

The gradient of the line which passes through the points (0,2) and (2,4) is $\frac{4-2}{2-0} = 1$

The y -intercept is 2. The graph of y against x^3 is given by $y = x^3 + 2$.

The answer is D.

Question 9

For Jack, the cost of hiring a marquee for 3 days was \$430. So, $430 = 3a + b$ (A)

For Sam, the cost of hiring a marquee for 5 days was \$670. So, $670 = 5a + b$ (B)

Now, (B) – (A) gives $240 = 2a$ and so $a = 120$.

So, the cost per day of hiring a marquee is \$120.

The answer is B.

Module 4 : Business related mathematics**Question 1**

$$\text{Now, } I = \frac{PrT}{100}$$

$$\text{So, } 840 = \frac{8000 \times r \times 3}{100}$$

$$\begin{aligned} \text{So, } r &= \frac{840 \times 100}{8000 \times 3} \\ &= 3.5 \end{aligned}$$

The answer is D.

Question 2

The value of the collection in 10 years time would be $75\,000(1.04)^{10} = 111\,018.32$

The answer is B

Question 3

When the car has depreciated by $\$45\,000 - \$20\,000 = \$25\,000$, it must be traded. Now, $25\,000 \div 0.34 = 73\,529.41$. So, Miranda's car can be driven 73 529 km before it must be traded.

The answer is B.

Question 4

$$\text{Now, } A = PR^n \quad \text{where } R = 1 + \frac{r}{100}$$

$$\text{So, } A = 5000 \times (1.004)^{48} \quad \text{since } R = 1 + \frac{0.4}{100}$$

The answer is C.

Question 5

The minimum balance for the month of May is \$4582.79

$$\begin{aligned} \text{Now, } I &= \frac{PrT}{100} \\ &= \frac{4582.79 \times 0.2 \times 1}{100} \\ &= 9.17 \end{aligned}$$

The answer is C.

Question 6

$$\begin{aligned} \text{Total repayments} &= 400 + 50 \times 60 \\ &= 400 + 3000 \\ &= 3400 \end{aligned}$$

$$\begin{aligned} \text{Total interest} &= 3400 - 2800 \\ &= 600 \end{aligned}$$

$$\begin{aligned} \text{Flat interest rate} &= \frac{600}{2400} \times 100 \\ &= 25 \end{aligned}$$

The answer is E.

Question 7

The quantity of walnuts produced, increases each year by an increasing amount. Graph A is not appropriate since the quantity increased by the same amount each year. Graph D is not appropriate since the quantity increased by a decreasing amount each year. Graphs C and E are not appropriate since in the first year the walnut production was 0. Graph B best represents the annual walnut production at this plantation. The answer is B.

Question 8

The annuities equation is given by $A = PR^n - \frac{Q(R^n - 1)}{R - 1}$, where $R = 1 + \frac{r}{100}$ and A represents the amount owing after n periods, in this case, quarters.

$$\begin{aligned} \text{Now, } R &= 1 + \frac{3}{100} \\ &= 1.03 \end{aligned}$$

To work out how much he must pay each quarter, that is, to work out Q , Greg can use the fact that after 5 years, that is, 20 quarters, so $n = 20$, the amount owing will be \$0.

$$\text{So, we have } 0 = 30\,000(1.03)^{20} - \frac{Q(1.03^{20} - 1)}{0.03}$$

The answer is A.

Question 9

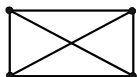
Monthly interest is given by $\left(\frac{39}{7800} \times \frac{100}{1}\right)\% = 0.5\%$

Annual interest is therefore $0.5\% \times 12 = 6\%$

The answer is E.

Module 5 : Networks and decision mathematics**Question 1**

A complete graph is a graph with edges connecting all pairs of vertices. A complete graph with n vertices has $\frac{n(n-1)}{2}$ edges. With 4 vertices, a complete graph will have $\frac{4 \times 3}{2} = 6$ edges. Check this with a diagram.



The answer is C

Question 2

Each vertex is visited once only but does not start and finish at the same point. We have therefore a Hamiltonian path. The answer is C.

Question 3

Using Euler's formula, we have $v - e + f = 2$ where f represents the number of faces or regions.

We have 8 vertices and 13 edges. So, $8 - 13 + f = 2$
 $f = 7$

So we have 7 regions. The answer is B.

Question 4

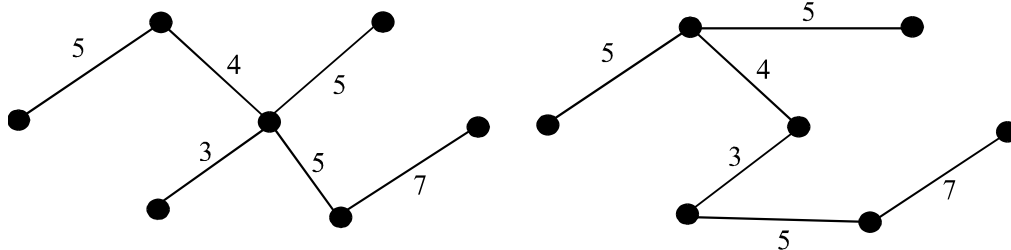
Option D is not true since Anna prefers the same number of flavours as Nicholas. The answer is D.

Question 5

A subgraph contains any subset of the vertices and arcs of a graph. Note that graph A has arc AD redrawn in a different position but it still connects A to D. Note that graph B has vertex D not included, which is fine since we are looking for a subset of vertices and arcs, that is, not all vertices and/or arcs need to be included. Graph D is not a subgraph since it includes the arc CD that does not appear on the original graph. The answer is D.

Question 6

There are a few possible minimal spanning trees. A couple of examples are shown.



The sum for each is the same is $5 + 5 + 5 + 4 + 3 + 7 = 29$
 The answer is B.

Question 7

From the adjacency matrix we see that there are 2 arcs going from A to B, 1 going from A to C, 1 going from B to C and 1 going from C to B. Only graph A shows this correctly.

The answer is A.

$$\begin{array}{c}
 \text{A} \\
 \text{B} \\
 \text{C}
 \end{array}
 \begin{bmatrix}
 & \text{A} & \text{B} & \text{C} \\
 & 0 & 2 & 1 \\
 & 0 & 0 & 1 \\
 & 0 & 1 & 0
 \end{bmatrix}$$

Question 8

The earliest start times for each of the activities are shown in the table below.

Activity	Earliest start time
A	0
B	0
C	3
D	3
E	8
F	8
G	8
H	14
I	14
J	18

The earliest start time for activity J is 18 weeks.

The answer is E.

Question 9

The capacity of the cut is given by the sum of the weights on the arcs flowing from left to right.

In this case, the capacity is $6 + 10 + 8 + 7 = 31$.

Note that the arc with weight 5, which the cut passes through is flowing in the opposite direction and is therefore not counted.

The answer is D.