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FURTHER MATHEMATICS

TRIAL EXAMINATION 2

(ANALYSIS TASK)

2001

Reading Time: 15 minutes
Writing time: 90 minutes

Instructions to students

This exam consists of section A and Section B.
Section A contains a set of extended answer questions from the core, "Data Analysis".
Section A is compulsory and is worth 15 marks.
Section B consists of 5 modules. You should choose 3 of these modules and answer every question in each of your chosen modules. Each of the modules is worth 15 marks.
There is a total of 60 marks available for this exam.
The marks allocated to each of the four questions are indicated throughout.
Students may bring up to two A4 pages of pre-written notes into the exam.

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Section A

Answer every question in Section A.

Question 1

An analysis was undertaken into cordial sales ($'000) to a group of supermarkets during two seasons (summer/winter).
The data collected is displayed in the back-to-back stemplot shown in Figure 1 below.

summer				winter				
			0					
			1	1	1	3	7	9
			2	4	6	6		
	9	7	3	2	7			
7	5	0	4	5				
8	5	3	5					
	3	1	6	3				
	5	0	7					

Figure 1

a. Write down the dependent variable in this analysis.

1 mark

b. For the winter season, write down the

i. mean of the cordial sales

1 mark

ii. median of the cordial sales

1 mark

iii. the interquartile range of the cordial sales

1 mark

Question 2

Cordial may contain some fruit juice. The fruit juice content (%), compared to price ($'s), for one litre bottles of cordial is shown in Table 1 below for 12 different brands.

Fruit juice content (%)	Price ($'s)
25	2.05
10	1.65
5	1.60
32	2.05
42	2.30
5	1.70
40	2.30
15	1.90
20	2.00
18	1.85
30	2.20
35	2.15

Table 1

a. **i.** Before any regression analysis can be conducted on this data, what needs to be established?

____

____

1 mark

ii. How is this best achieved?

____

____

1 mark

The scatterplot showing fruit juice content (%) plotted against price ($'s) is shown in Figure 2 below.

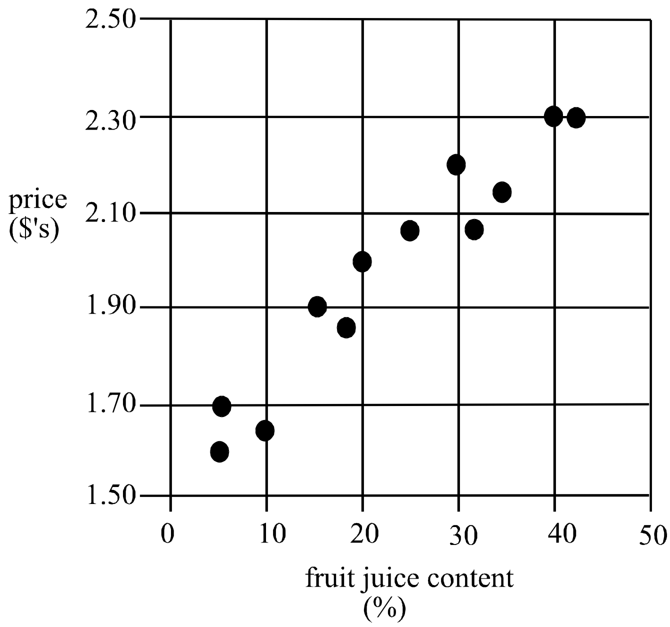

Figure 2

b. Use this scatterplot to describe the association between fruit juice content and price in terms of form only.

____

____

____

1 mark

c. Calculate r, Pearson's correlation coefficient. Express your answer correct to 2 decimal places.

____

____

2 marks

d. Using a calculator, find the coefficients, (correct to 2 decimal places) for the equation below which gives the equation of the least squares regression line for the data.

price = [ ] + [ ] × fruit juice content (%)

2 marks

e. Using this equation, predict the price of a 1 litre bottle of cordial with 35% fruit juice content.

____

____

1 mark

f. From the regression equation we can conclude that the price of 1 litre bottles of cordial increased on average by [ ] dollars for each 1 % increase in fruit juice content.

____

____

1 mark

g. Write down the percentage of the variation in the price of 1 litre bottles of cordial that can be explained by the variation in the fruit juice content of the cordial correct to 2 decimal places.

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____

____

____

2 marks

Total 15 marks

Section B

Section B consists of 5 modules. Choose 3 of those modules and answer every question in each of those modules.

Module 1 : Number patterns and applications

If you choose this module, **all** questions are to be answered.

Question 1

The Fairy Port folk festival is held each year and performers entertain audiences at various venues during the festival.
In 1981, the first year of the festival, there were a total of 7600 tickets sold for performances at the festival and the number increased by 1500 each year after that.

a. **i.** How many tickets were sold in 1987?

1 mark

ii. If, in the *n*th year of the festival, the number of tickets sold is given by t_n, write down a formula for t_n in terms of *n*.

1 mark

iii. During which year did the number of tickets sold first exceed 30 000?

1 mark

b. In 2000, the ratio of the number of artists performing compared to the number of tickets sold was 1 : 95.

How many artists performed at the Fairy Port folk festival in2000?

2 marks

Question 2

The amount of rubbish generated by those attending the festival was monitored by the organizers of the festival.

In 1981, the first year of the festival, 120 cubic metres of rubbish was generated.

According to the festival organizers, the quantity of rubbish in cubic metres, increased by 5% each year.

a. **i.** How much rubbish was generated at the 1982 festival according to the festival organizers?

1 mark

ii. Write down a formula for the quantity of rubbish generated , R_n, in cubic metres, at the *n*th festival according to the festival organizers.

1 mark

iii. In which year did the quantity of rubbish first exceed 200 cubic metre?

1 mark

iv. What was the total quantity of rubbish generated at the first 20 festivals? Express your answer to the nearest cubic metre.

2 marks

Question 3

According to the local council, the quantity of rubbish R, in cubic metres, generated by those attending the festival, is described by the difference equation

$$R_{n+1} = 1.03R_n + 6 \quad \text{where } R_1 = 120$$

a. How much rubbish was generated at the 1984 festival according to the local council? Express your answer correct to the nearest cubic metre.

1 mark

b. Show that the solution to the difference equation

$$R_{n+1} = 1.03R_n + 6 \quad \text{where } R_1 = 120$$

is given by

$$R_n = 320(1.03)^{n-1} - 200$$

2 marks

c. During which year does the **difference** between the quantity of rubbish generated at the festival according to the council and the quantity of rubbish generated at the festival according to the organizers first exceed 50 cubic metres?

2 marks

Total 15 marks

Module 2 : Geometry and trigonometry

If you choose this module, **all** questions are to be answered.

Two overlapping, triangular sails are to be placed over a play area. A design showing the sails from directly above is shown (not to scale) in Figure 1 below.

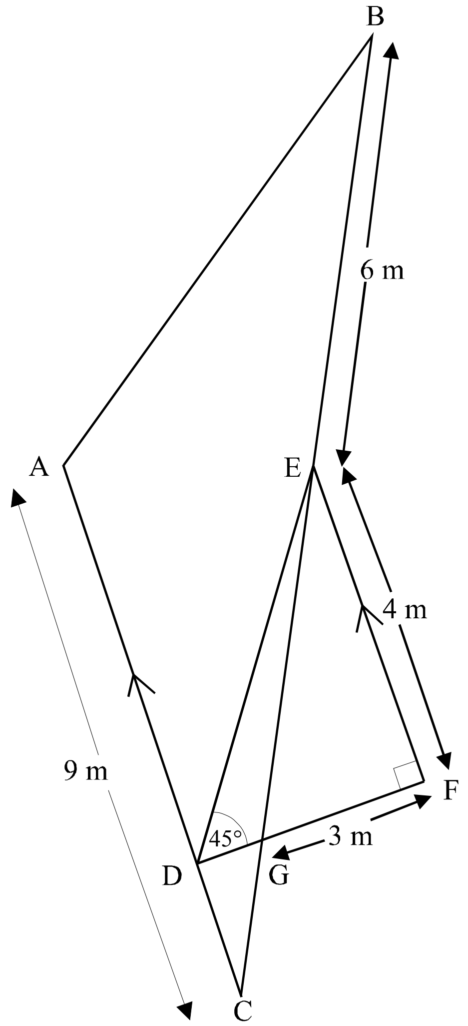

Figure 1

The larger of the two sails, ABC, is overlapped by the smaller sail DEF. This smaller sail is in the shape of a right angled triangle with angle EFG equal to $90°$. The side AC of the larger sail runs parallel to the side EF of the smaller sail. The distance AC is 9 metres, the distance EF is 4 metres, the distance FG is 3 metres, and the distance EB is 6 metres. Angle EDF is equal to $45°$.

Question 1

a. **i.** Write down the size of angle DEF.

1 mark

ii. Write down the distance DF.

1 mark

iii. Show that DG is equal to 1 metre.

1 mark

b. The two sails have an area of overlap indicated in Figure 1 as triangle DEG.
Show that the distance EG equals 5 metres.

1 mark

c. Use the sine rule to find angle DEG to the nearest degree.

1 mark

d. Use your answer to part **c.** to find the area of overlap of the sails. Express your answer in square metres correct to 1 decimal place.

2 marks

Question 2

Figure 2 below shows triangles CDG and EFG which have been taken from Figure 1.

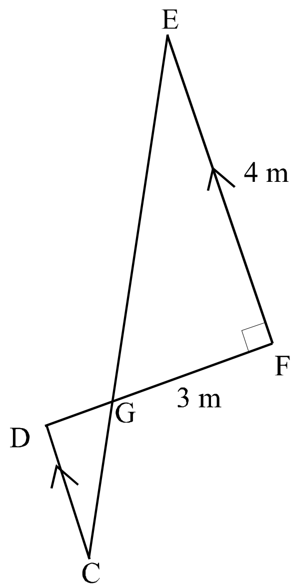

Figure 2

a. Explain why CG is of length $1\frac{2}{3}$ metres.

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2 marks

b. Find the size of angle *FEG* to the nearest degree and hence write down angle *DCG.*

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____

____

2 marks

c. Use your results from parts **a.** and **b.** to find the length of the side *AB* which is shown in Figure 1. Express your answer correct to 1 decimal place.

____

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____

____

2 marks

Question 3

In another project, the designer of the sails does a set of drawings of a sail and the total area covered by the sail on paper is 1.2 square metres.
In actual size, the total area covered by the sail is to be 1920 square metres.

Write down the **distance** scale used by this designer on her set of drawings.

2 marks

Total 15 marks

Module 3 : Graphs and relations

If you choose this module, **all** questions are to be answered.

A dance school offers private lessons to junior and senior students in tap dance and jazz ballet.
The lesson times, in minutes, for the different classes at each level are shown in the table below.

dance type	junior	senior
tap dance	30	45
jazz ballet	60	60

The dance school can allocate up to 45 hours of teaching per week to tap lessons and up to 70 hours per week to jazz ballet lessons.

Let x represent the number of private junior lessons held by the dance school in a week.
Let y represent the number of private senior lessons held by the dance school in a week.

Question 1

a. How many minutes of teaching per week are allocated by the dance school for private lessons in

i. tap dance?

____

ii. jazz ballet?

____

1 mark

b. If no senior students enrolled in private lessons in tap dance, what is the maximum number of private junior lessons in tap dance which could be offered by the dance school in a week?

____

____

1 mark

c. The number of private junior lessons offered by the dance school is described by the inequation $x \geq 0$.
The restriction brought about by the number of teaching hours available for tap dance classes is described by the inequation $30x + 45y \leq 2700$.
Write down the inequation which describes

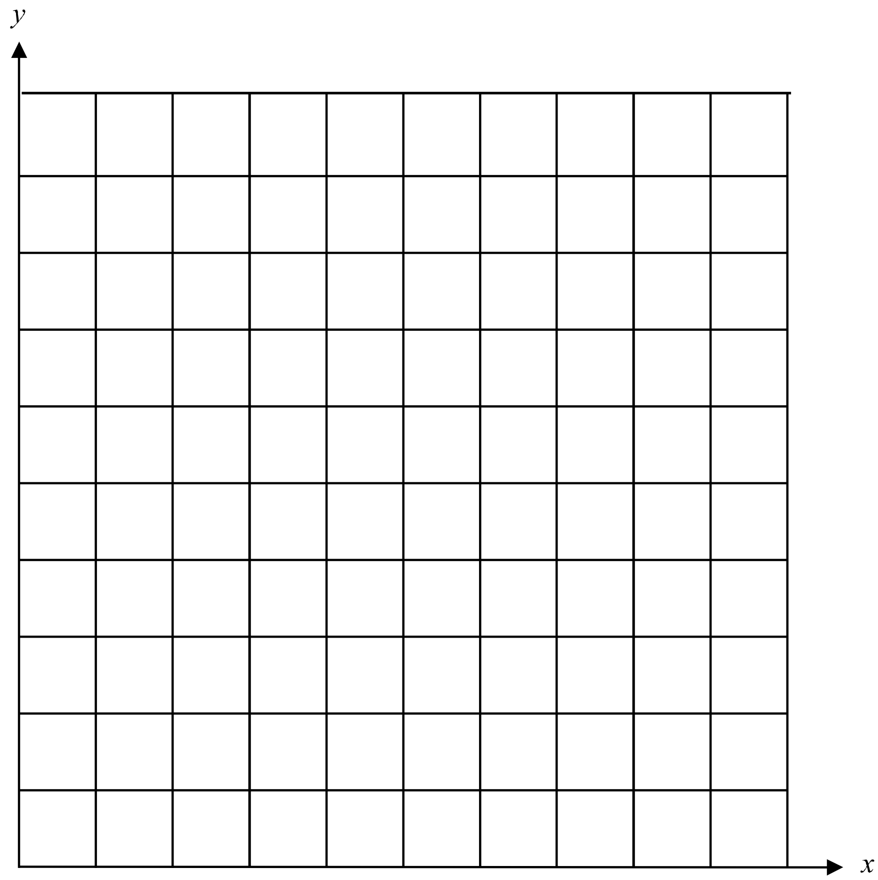
i. the number of private senior lessons offered by the dance school.

1 mark

ii. the restriction brought about by the number of teaching hours available for jazz ballet classes.

1 mark

d. Sketch the four inequations in part **c.** on the axes below.



4 marks

e. Mark clearly on the graph above, the feasible region defined by the four inequations.
1 mark

Question 2

The dance school makes a profit of \$15 for each private junior lesson and \$12 for each private senior lesson.

a. Write down an expression for the weekly profit P, in dollars, of the dance school in its running of private lessons in terms of x and y.

2 marks

b. Using the graph in **Question 1** part **d.**, calculate the number of junior and senior private classes which should be conducted each week for the dance school to make the largest profit.

3 marks

c. Write down the maximum profit that the dance school could make in a week from its private lessons.

1 mark

Total 15 marks

Module 4 : Business related mathematics

If you choose this module, **all** questions are to be answered.

Mick runs a garden supply business and rents the yard where his business is based. He paid a bond of $1500 which has been invested by the landlord while Mick remains his tenant.

The bond of $1500 is invested by the landlord and attracts simple interest of 4% per annum.

a. **i.** If Mick was to rent the yard for 10 years, how much interest would his bond have earned?

1 mark

ii. How long would it take for the bond money, including interest to double in this account?

1 mark

b. If the landlord had invested the bond in an account attracting compound interest of 4% per annum, compounding annually,

i. how much interest would Mick have received after 10 years?

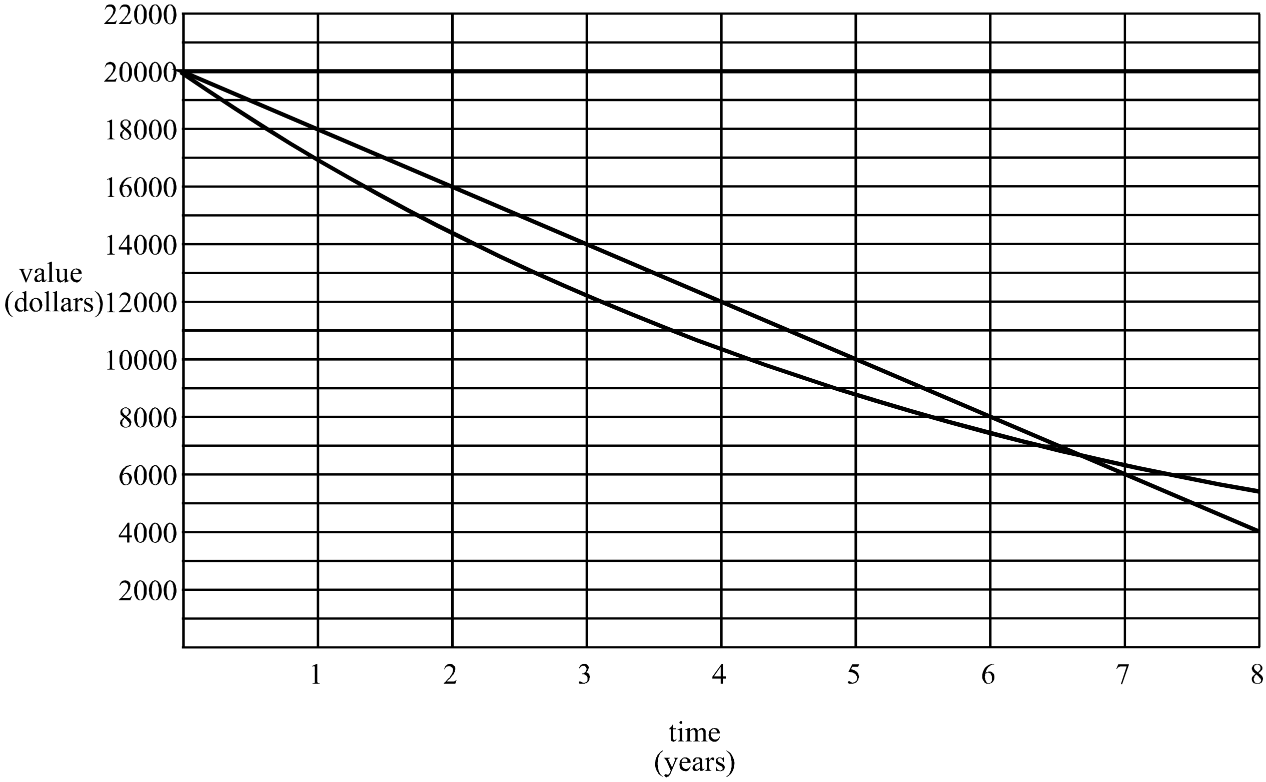
2 marks

ii. how long would it take for the bond money, including interest, to double in this account. Express your answer to the nearest whole year.

2 marks

Question 2

Mick looks at the depreciation of his new digger. He looks at a graph constructed by his accountant, which shows how his digger will depreciate in value using either flat rate depreciation or reducing value depreciation.



a. Write down the value of the digger when it was new.

1 mark

b. Label clearly the graph which shows the value of the digger using flat rate depreciation.

1 mark

c. Using the graph, estimate the annual flat rate of depreciation used by Mick's accountant.

1 mark

d. The formula used by the accountant to find the value of the digger using reducing balance depreciation is $V = PR^n$ where V represents the value of the digger n years after it has been purchased for P dollars.
Given that 1 year after it's purchase, the value of the digger, assuming reducing balance depreciation, is $17 000, write down the value of R.

2 marks

Question 3

Mick had to take out a reducing balance loan to upgrade his truck and digger. He borrowed \$40 000 at an interest rate of 8% per annum compounded quarterly. He repays \$2000 each quarter.

The annuities formula $A = PR^n - \frac{Q(R^n - 1)}{R - 1}$ where *A, P* and *Q* are expressed in dollars, is used to calculate how much Mick owes at a given time.

a. Write down the value of *n*, 4 years after Mick starts the loan.

1 mark

b. How much will Mick owe at this time?

1 mark

c. How long will it take for Mick to completely repay this loan? Express your answer to the nearest quarter.

2 marks

Total 15 marks

Module 5 : Networks and decision mathematics

If you choose this module, **all** questions are to be answered.

A tour of 8 wineries has been organized. Figure 1 below shows the network of roads joining the wineries.

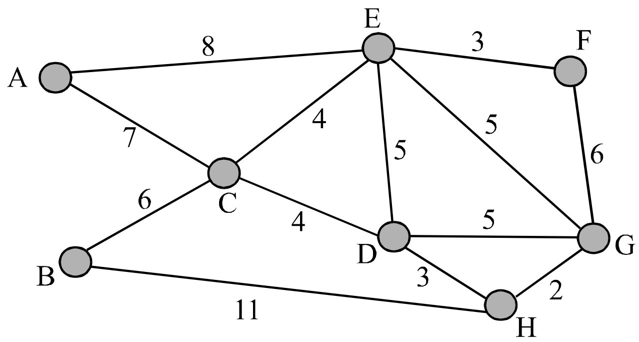


Figure 1

The distances indicated in Figure 1 are in kilometres.

Question 1

The tour starts at winery A and follows a Hamiltonian circuit.

a. Write down a possible order in which the winerys might be visited.

____

____

1 mark

b. Write down the length of the shortest route from winery A to winery H.

____

____

1 mark

c. The 8 wineries share a water supply system. The water pipes connecting the wineries run along the roads and form a minimal-length spanning-tree.
On Figure 2 below, draw this minimal-length spanning-tree.

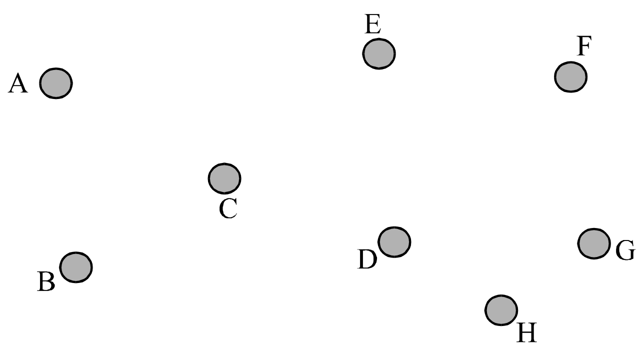


Figure 2

2 marks

d. What is the minimum length of pipes required to connect each of the wineries to at least one other winery?

___

1 mark

e. In order to take in the beautiful vistas around the wineries, the tour leader wondered whether it would be possible to start at winery A and travel on each of the roads joining the wineries just once before finishing at winery A.
To make this possible, a new road needs to be constructed between which two towns?

___

___

___

___

1 mark

Question 2

The construction of the underground cellars at winery D was a project involving 10 major activities. Those activities, the time they took to complete (in months),and the activities which were their immediate predecessors in the project, are indicated in Table 1 below.

Activity	Immediate predecessor	Time taken to complete (months)
A	-	2
B	-	3
C	A	2
D	A	5
E	B	3
F	B	8
G	C	6
H	C	3
I	D, E, G	5
J	H, I, F	2

Table 1

A directed graph that can be used to show the activities involved in the construction of the cellars is shown in Figure 3 below.

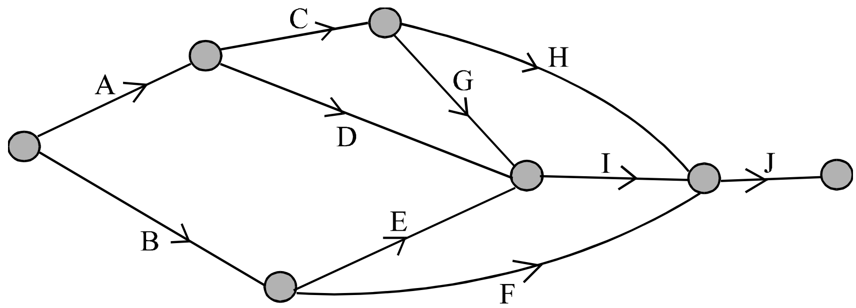


Figure 3

a. Complete Table 2 below using the information From Figure 3 and Table 1.

Activity	Earliest start time	Latest start time
A	0	0
B	0	4
C	2	2
D	2	5
E	3	7
F	3	7
G	4	4
H	4	
I		10
J		15

Table 2

3 marks

b. Write down the critical path of the network.

1 mark

c. Write down the completion time for the project.

1 mark

d. Write down the slack (float time) for activity E.

1 mark

Question 3

Each of the four largest wineries; A, B, C and D, advertise for a part time worker in their wineries. Four friends, Emily, Trent, Jo and Sally, decide to apply for the jobs.
The distance from each of the friends homes to each of the wineries is shown in Table 3 below.

	A	B	C	D
Emily	6	3	2	5
Trent	4	2	3	7
Jo	6	3	3	3
Sally	8	2	9	7

Table 3

Using the Hungarian algorithm or otherwise, assign each of the 4 friends to one of the wineries so that the distance they have to travel is a minimum.

3 marks

Total 15 marks