2003 Further Mathematics
Written Examination 1 (facts, skills and applications)
Suggested answers and solutions

Answers – Multiple Choice

SECTION A

Core: Data analysis

1. **C**	2. **B**	3. **E**	4. **C**	5. **B**
6. **C**	7. **A**	8. **D**	9. **C**	10. **E**
11. **B**	12. **D**	13. **C**		

SECTION B

Module 1: Number patterns and relations

1. **D**	2. **C**	3. **A**	4. **B**	5. **A**
6. **E**	7. **E**	8. **B**	9. **C**	

Module 2: Geometry and trigonometry

1. **C**	2. **D**	3. **E**	4. **C**	5. **D**
6. **A**	7. **A**	8. **B**	9. **A**	

Module 3: Graphs and relations

1. **A**	2. **C**	3. **A**	4. **C**	5. **B**
6. **C**	7. **B**	8. **E**	9. **D**	

Module 4: Business related mathematics

1. **C**	2. **B**	3. **D**	4. **B**	5. **A**
6. **E**	7. **D**	8. **C**	9. **E**	

Module 5: Networks and decision mathematics

1. **D**	2. **B**	3. **C**	4. **C**	5. **D**
6. **C**	7. **A**	8. **E**	9. **C**	

Core: Data analysis

Question 1 [C]

The median (the position of the middle mark in the 'box') of the *median house prices* for 2001 is approximately \$225 000, and \$275 000 for 2002; a difference of \$50 000.

Question 2 [B]

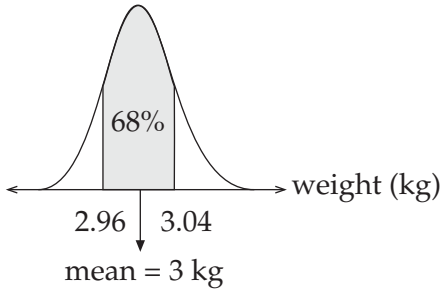
The interquartile range is represented by the length of the 'box'; this is the same for both 2001 and 2002.

Question 3 [E]

Time is a continuous variable and hence the most appropriate type of graph is a histogram.

Question 4 [C]

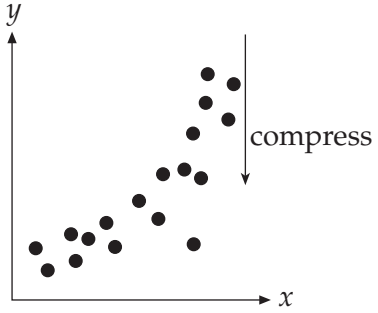
The weight of 68% of the bags of apples fall within **one** standard deviation of the mean.



3.04 = mean + 1 × standard deviation
= 3 + 0.04
Standard deviation = 0.04 kg = 40 gram

Question 5 [B]

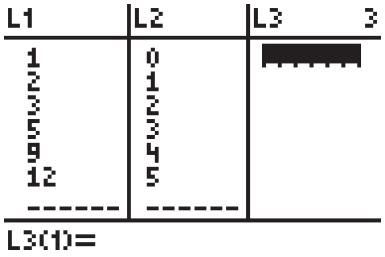
A $\frac{1}{y}$ transformation will compress the y-scale linearising the points:



Question 6 **[C]**

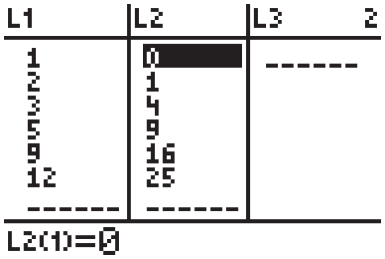
To apply a y^2 transformation to the given data :

Enter the data into the lists using STAT

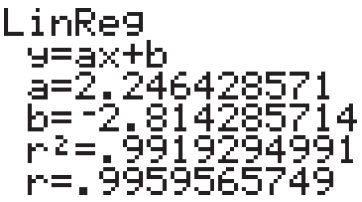


Transform L2 by typing in the label

$L2 = L2^2$



Find the least-squares regression line equation of L1,L2



The dependent variable is now y^2 hence the equation:

$y^2 = 2.25x - 2.81$

Question 7 **[A]**

Using 3-median smoothing:-

Time period	Data	3-median smoothing
1	205	
2	274	median is 274
3	305	median is 288
4	288	median is 305
5	327	median is 312
6	312	

Question 8 **[D]**

The graph is based on percentages. There is no way of estimating the number of people surveyed in each of the suburbs.

Question 9 **[C]**

The three suburbs were chosen to represent different socio-economic groups so the graph is being used to compare the categorical variables: *newspaper preference* and *socio-economic group*.

Question 10 **[E]**

From observation the gradient is negative and using any two points on the line will confirm that the gradient is –2.6.

The intercept cannot be read from the Y-axis because the X-axis does not start at zero.

Substitution of a point on the line into

$y = -2.6x + c$ will give the intercept, $c = 184$

Question 11 **[B]**

Residual = Actual value – Predicted value

The predicted values are the y-values of the points on the regression line.

The regression line is above the point (40, 72) so the residual will be negative and below the point (63, 30), so the residual will be positive.

Predicted values are approx. (40, 80) and (63, 20) so residuals are –8 and 10 respectively.

Question 12 **[D]**

The time-series graph shows a regular, repeated pattern so could be described as seasonal. Although there are differences from year to year in the pattern there is no definite increasing or decreasing trend overall.

Question 13 **[C]**

In each year there are peaks in the June and December quarters. When the seasonal indices are calculated the two highest values will be in June and December.

Module 1: Number patterns and applications

Question 1 [D]

1 : 15000 or = 1 cm : 15 000 cm
= 30 cm : 450 000 cm
= 30 cm : 4 500 m
= 30 cm : 4.5 km

Therefore 30 cm represents 4.5 km between the two shopping malls.

Question 2 [C]

There are a total of 20 minor sections or distances (1+12+7 = 20) so each section is worth 5% of the total distance.

swim : cycle : run
1 : 12 : 7
5% : 60% : 35%

Therefore the run segment is 35% of the triathlon distance.

Question 3 [A]

The ratio of heights are

original : photo
175 cm : 25 cm
7 : 1

Question 4 [B]

Arithmetic, $a = -9$, $d = +3$

$$S_{10} = \frac{10}{2}\left[2 \times -9 + (10-1) \times 3\right]$$

$$= 45$$

Question 5 [A]

$$t = ar^{n-1}$$

$$256 = ar^7$$

$$4 = ar^4$$

$$\therefore \frac{256}{4} = \frac{ar^7}{ar^4}$$

$$64 = r^3$$

$$r = 4, \text{ as } 4^3 = 64$$

Question 6 [E]

$$S_\infty = \frac{a}{1-r} \text{ where } a = 32 \text{ and } r = \frac{1}{4}$$

$$S_\infty = \frac{32}{1-\frac{1}{4}} = \frac{32}{\frac{3}{4}} = \frac{128}{3} = 42\frac{2}{3}$$

Question 7 [E]

For $t_{n+1} = t_n - 2$, $t_1 = 6$

$t_1 = 6$
$t_2 = 6 - 2 = 4$
$t_3 = 4 - 2 = 2$
$t_4 = 2 - 2 = 0$
$t_5 = 0 - 2 = -2$

Question 8 [B]

For
$t_{n+1} = 2t_n + 8$, $t_1 = -6$ where $a = 2$ and $b = 8$

$$t_n = a^{n-1}t_1 + \frac{b\left(a^{n-1}-1\right)}{a-1}$$

$$t_n = 2^{n-1} \times -6 + \frac{8\left(2^{n-1}-1\right)}{2-1}$$

$$t_n = 2 \times 2^{n-1} - 8$$

$$t_n = (2)^n - 8$$

Question 9 [C]

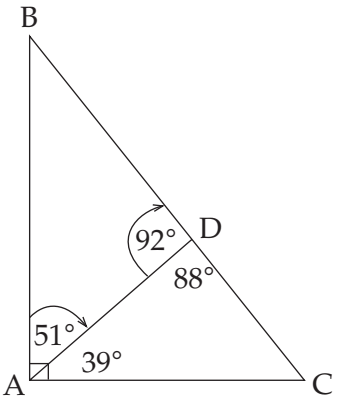
The first term is 4 so A and B are incorrect.
Difference equation in C is the only one that gives the required sequence.

And if $t_{n+1} = 0.5t_n + 2$, $t_1 = 4$

$t_1 = 4$
$t_2 = 0.5 \times 4 + 2 = 4$
$t_3 = 0.5 \times 4 + 2 = 4$
$t_4 = 0.5 \times 4 + 2 = 4$
$t_5 = \text{etc.}$

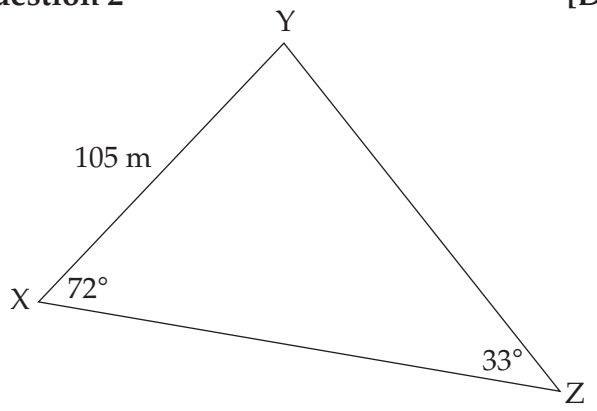
Module 2: Geometry and trigonometry

Question 1 [C]



In ADC : $\angle DAC = 90° - 51°$
$= 39°$
$\angle ADC = 180° - 92°$
$= 88°$
$\angle BCA = 180° - 39° - 88° = 53°$

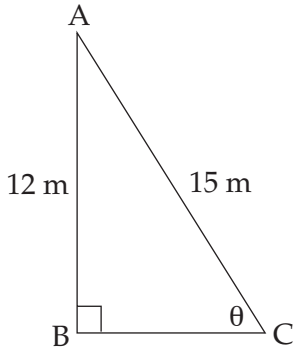
Question 2 [D]



Not a right-angled triangle; use the sine rule.

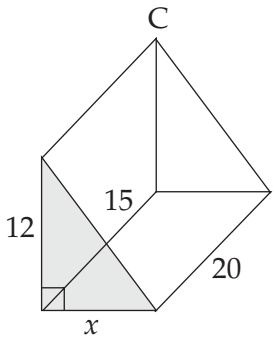
$$\frac{YZ}{\sin 72°} = \frac{105}{\sin 33°}$$
$$YZ = \frac{105 \sin 72°}{\sin 33°}$$
$$= 183.35 \text{ m}$$

Question 3 [E]



$$\sin\theta = \frac{12}{15}$$
$$\theta = \sin^{-1}\frac{12}{15}$$
$$= 53.13°$$

Question 4 [C]



Volume of a prism = Area of cross-section × Length

Area of cross-section $= 12 \times x \times \frac{1}{2}$

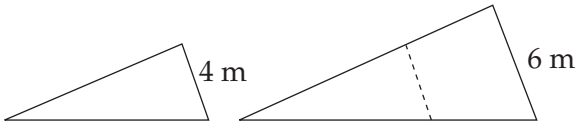
$x^2 = 15^2 - 12^2$
$= 81$
$x = 9$ m

Area of cross-section $= \frac{1}{2} \times 12 \times 9$
$= 54 \text{ m}^2$

Volume $= 54 \times 20$
$= 1080 \text{ m}^3$

Question 5 **[D]**

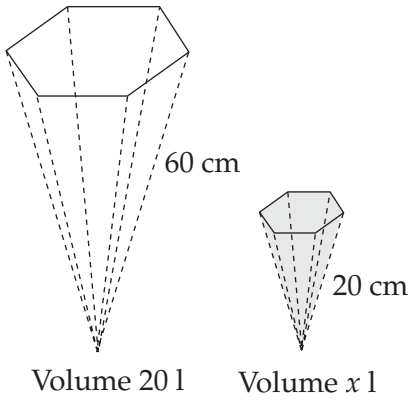
Similar triangles



Ratio of lengths = 4 : 6 = 2 : 3
Ratio of areas = $2^2 : 3^2 = 4 : 9$
Shaded area = 4 units
Unshaded area = 9 – 4 = 5 units
Ratio of shaded : unshaded = 4 : 5

Question 6 **[A]**

Similar figures



Ratio of lengths = 60 : 20 = 3 : 1
Ratio of volumes = $3^3 : 1^3 = 27 : 1$
Using
Volume of large figure : Volume of small figure
$20 : x = 27 : 1$

$$\frac{x}{20} = \frac{1}{27}$$

$$x = \frac{20}{27} = 0.7407 \text{ l}$$

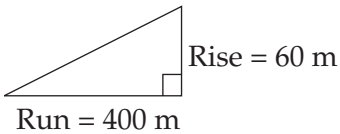
Question 7 **[A]**

4 cm on the map is equivalent to 40 000 cm in actual horizontal distance.

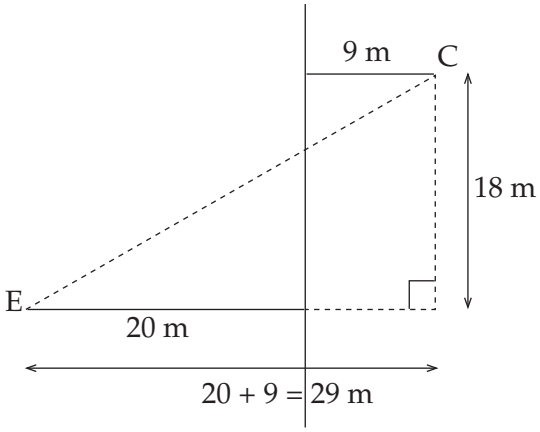
40 000 cm = 400 metres

C is on the 40 metre contour line and D is on the 100 m contour line. So D is 60 metres vertically above C.

$$\text{Average slope} = \frac{\text{rise}}{\text{run}} = \frac{60}{400} = 0.15$$

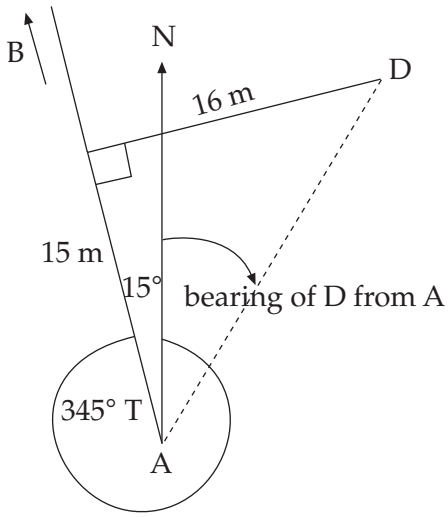


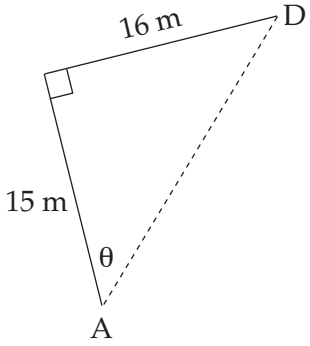
Question 8 **[B]**



$EC^2 = 29^2 + 18^2$
EC = 34.13 metres
= 34.1 m correct to one decimal place

Question 9 **[A]**





$$\tan\theta = \frac{16}{15}$$

$\theta = 46.848°$

Bearing of D from A = 46.848° – 15°
= 31.848°

∴ bearing = 31.8° correct to one decimal place

Module 3: Graphs and relations

Question 1 [A]

In the equation $y = 5 - 3x$, when $y = 11$ the value of x is:

$y = 5 - 3x$

$11 = 5 - 3x$

$6 = -3x$

$x = -2$

Question 2 [C]

\$133 – \$25 = \$108 worth of half hourly sessions at \$36 each allows for in excess of ONE hour and LESS than 90 minutes. With a start time of 1:15 pm then the computer technician was on premises after 2:15 pm until 2:45 pm.

Question 3 [A]

$x + 3y - 3 = 0$

Find the x and y intercepts

When $x = 0$	When $y = 0$
$0 + 3y - 3 = 0$	$x + 3 \times 0 - 3 = 0$
$3y = 3$	$x - 3 = 0$
$y = 1$	$x = 3$
y intercept (0,1)	x intercept (3,0)

Question 4 [C]

$y = kx - 7$ passes through the point $(-5, 3)$
substitute $x = -5$ and $y = 3$

$3 = k \times -5 - 7$

$10 = -5k$

$k = -2$

Question 5 [B]

The gradient of the straight line of the d versus v^2 graph is

$$m = \frac{30}{3000} = \frac{1}{100} \text{ and so } d = \frac{v^2}{100}$$

Question 6 [C]

Given $I = \frac{k}{r^2}$ then transpose the formula and substitute $I = 0.04$ and $r = 5$. The value of k is

$0.04 \times 5^2 = k$

$0.04 \times 25 = k$

$1.0 = k$

Question 7 [B]

For the objective function $Z = 3x + 4y$,
the maximum value of Z would be given by the vertex with the co-ordinates of :

A	(0, 10)	$Z = 3 \times 0 + 4 \times 10 = 40$
B	(15, 10)	$Z = 3 \times 15 + 4 \times 10 = 85$
C	(10, 10)	$Z = 3 \times 10 + 4 \times 10 = 70$
D	(0, 20)	$Z = 3 \times 0 + 4 \times 20 = 80$
E	(5, 15)	$Z = 3 \times 5 + 4 \times 15 = 75$

So therefore B (15,10) maximizes the function.

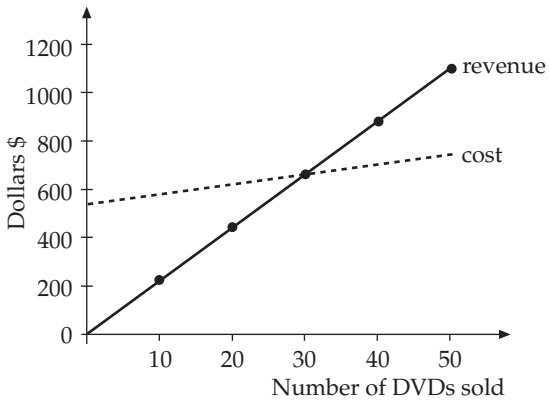
Question 8 [E]

$x + 3y \leq 30$	$5x + 15y \geq 300$
$3y \leq -x + 30$	$15y \geq -5x + 300$
$y \leq -\frac{1}{3}x + 10$	$y \geq -\frac{1}{3}x + 20$

The equations have the same gradient and different y-intercept, therefore they are parallel lines and do not intersect.

Question 9 [D]

The break even point is the where the revenue and cost functions intersect. The value was \$660.



DVDs	COST	REVENUE
0	570	0
10	600	220
20	630	440
30	660	660
40	690	880
50	720	1100

Module 4: Business related mathematics

Question 1 [C]

$$I = \frac{\text{Prt}}{100} \Rightarrow 600 = \frac{4000 \times 3 \times t}{100} \Rightarrow 600 = 120t \Rightarrow t = 5$$

Question 2 [B]

For shirt 1 he paid 95% of the price i.e.

$0.95P_1 = 38 \Rightarrow P_1 = 40$

For shirt 2 he paid 90% of the price i.e.

$0.9P_2 = 72 \Rightarrow P_2 = 80$

Total paid = 40 + 80 = \$120

Question 3 [D]

Using the formula $A = PR^n$

$$R = 1 + \frac{r}{100} \Rightarrow R = 1 + \frac{5}{100} = 1.05$$

$A = 10\,000 \times 1.05^4$

= \$12155.06

(may also be done using TVM solver)

Question 4 [B]

Reducing balance value after t years =

$$\text{purchase price} \times \left(1 - \frac{\text{depreciation rate}}{100}\right)^t$$

$$1300 = 4000 \times \left(1 - \frac{d}{100}\right)^3$$

$$0.325 = \left(1 - \frac{d}{100}\right)^3$$

$$0.6875 = 1 - \frac{d}{100}$$

$$\frac{d}{100} = 0.3125 \Rightarrow d = 31.25\%$$

Question 5 [A]

The amount that Barb reduces the value by each year decreases **and** by a lesser amount each time.

(After year 1 it reduces by \$1249.86 , after year 2 it reduces by \$859.32 and after year 3 it reduces by \$590.81)

Question 6 [E]

This is correct if Theo is paying 8% per annum therefore 2% per quarter

$$R = 1 + \frac{r}{100} \Rightarrow R = 1 + \frac{2}{100} = 1.02$$

Question 7 [D]

$$\text{Effective rate of interest} \approx \frac{2n}{n+1} \times \text{flat rate}$$

$$= \frac{2 \times 12}{12 + 1} \times 11$$

= 20.31%

Question 8 [C]

Effective interest rate =

$$\frac{\text{interest paid}}{\text{average principal owing}} \times 100$$

Question 9 [E]

Firstly determine Fofi's quarterly repayments (Q)

$$A = PR^n - \frac{Q\left(R^n - 1\right)}{R - 1}$$

$$0 = 120\,000 \times 1.015^{80} - \frac{Q\left(1.015^{80} - 1\right)}{1.015 - 1}$$

Q = \$2585.80

Now determine the amount owing after 40 repayments

$$A = PR^n - \frac{Q\left(R^n - 1\right)}{R - 1}$$

$$A = 120\,000 \times 1.015^{40} - \frac{2585.80\left(1.015^{40} - 1\right)}{1.015 - 1}$$

Q = \$77 356.36

(this question can be more easily solved using the TVM solver)

Module 5: Networks and decision mathematics

Question 1 [D]

The degree of each vertex is the number of edges coming in to it

4+4+3+3=14

Question 2 [B]

To contain an Euler circuit, the graph must be connected and all vertices have an even degree. Only the fourth diagram given satisfies the definition.

Question 3 [C]

Firstly determine the number of faces using Euler's formula

$v - e + f = 2$

$10 - 14 + f = 2$

$f = 6$

Connected planar graphs have 1 infinite region therefore 5 **finite** regions.

Question 4 [C]

A Hamiltonian path will be followed by Mike as he runs through each vertex exactly once, not starting and finishing at the same vertex.

Question 5 [D]

Katrina's graph is complete as all pairs of vertices will be connected by an edge.

Question 6 [C]

For an activity to be critical, the earliest and latest start times will be equal.

Question 7 [A]

A problem like this may be solved using the Hungarian Algorithm. However within a multiple choice format, you are best adding the values for each arrangement.

A 6+9+6+8=29

B 7+9+4+8=28

C 8+9+6+4=27

D 8+9+4+5=26

E 7+5+7+8=27

Question 8 [E]

The capacity of 15 is the sum of the weights of the edges that go from left to right across the cut therefore only b is included and a is ignored.

5+4+b=15 so b=6

Question 9 [C]

Adam can only play drums so this position is filled.

Dinh can only play lead guitar so this position is filled.

Keyboards can only be played by Billy so this position is filled.

This leaves Craig and Ethan to fill the bass guitar and vocals positions and as they are both capable of doing either, they are the only two with a choice of positions.