FURTHER MATHEMATICS EXAM 1 SOLUTIONS

Section A (Core)

Question 1 **D**

Mode: The most frequent score was 60 as it occurred three times.

Mean:

$$mean = \frac{\sum scores}{n}$$

$$= \frac{8+18+19+22....}{27}$$

$$= \frac{1134}{27}$$

$$= 42$$

Median: the middle score

Middle score

$$= \frac{n+1}{2}$$

$$= \frac{27+1}{2} = 14th\,score$$

14^{th} score is 45

Alternatively, use graphic calculator with data in list L_1.

```
1-Var Stats
 x̄=42
 Σx=1134
 Σx²=53644
 Sx=15.21133181
 σx=14.92698278
↓n=27
```

```
1-Var Stats
↑Sx=15.21133181
 σx=14.92698278
 n=27
 minX=8
 Q1=30
↓Med=45
```

Thus mean, median and mode in that order is 42, 45 and 60.

Question 2 **B**

Using graphics calculator

```
1-Var Stats
 x̄=56
 Σx=560
 Σx²=32094
 Sx=9.030811456
 σx=8.567379996
↓n=10
```

The standard deviation is 9.0308.. and to one decimal place it becomes 9.0. Response C is more accurate however accuracy was only wanted to one decimal place as in response B 9.0.

Question 3 **B**

The upper and lower medians are

$$(x_U, y_U) = (6,20)$$

$$(x_L, y_L) = (2,10)$$

The gradient of the 3-median regression line is:

$$\frac{y_U - y_L}{x_U - x_L} = \frac{20-10}{6-2}$$

$$= \frac{10}{4}$$

$$= 2.5$$

Question 4 **C**

95% limit is

mean plus or minus two standard deviations

or

$$95\%\,\text{limit} = \bar{x} \pm 2s$$

$$= 12 \pm 2 \times 0.1$$

$$= 12 - 0.2 \;\; and \;\; 12 + 0.2$$

$$= 11.8 \;\; to \;\; 12.2$$

Question 5 **E**

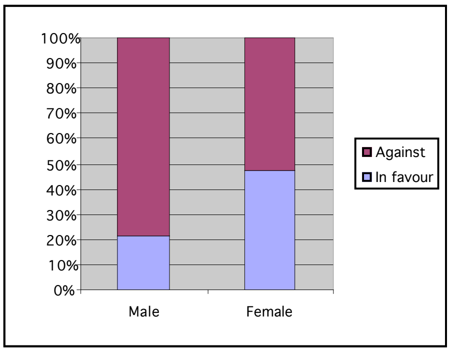
Back-to-back stem plot are for Categorical vs Numerical Bivariate analysis.

Response E *best bench press weight and age of gymnast* are both numerical variables.

Question 6 **D**

From this table, the independent variables are gender; male and female. Percentaging by columns, we obtain

	Male	Female
In favour of change	$= \frac{123}{575} \times \frac{100}{1}$ $= 21.4\%$	$= \frac{230}{485} \times \frac{100}{1}$ $= 47.4\%$
Against change	$= \frac{452}{575} \times \frac{100}{1}$ $= 78.6\%$	=100% - 47.4% =52.6%
Total	100%	100%

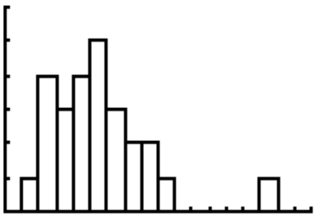


Question 7 **D**

Median and mean are affected by outliers is a false statement as only mean is affected by an outlier.

Question 8 **A**

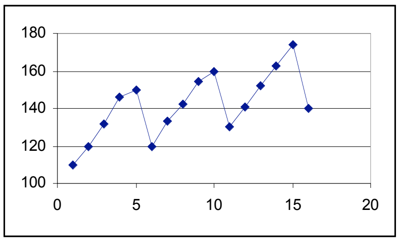
The data set contains one outlier with a value of 15 and thus histogram A is correct as it shows the outlier. Option E is not to be considered as the minimum number of groupings is usually five.



Question 9 **B**

Sum of all seasonal index = number of seasons
Spring + Autumn + 0.78+ 0.92 = 4
Spring + Autumn = 4 – (0.78+0.92)
Spring + Autumn = 2.3
Spring = 1.15 as spring and autumn seasonal indices are the same.

Question 10 **E**



It is **seasonal** as it repeats low (and high) values every 5th period namely 1st ,6th ,11th and 16th time period and **increasing** as these values show an upward trend.

Question 11 **B**

The median of the three values (37.1, 37.0, 37.5) centred about t = 10 is 37.1

t	8	10	12
Temperature	37.1	37.0	37.5

Question 12 **C**

If June 2005 has a timecode of 1 then June 2006 has a timecode of 13.

$$\text{Deseasonalised Monthly sales} = 1200 \times \text{timecode} + 10000$$
$$= 1200 \times 13 + 10000$$
$$= 25600$$

Actual expected sales where June seasonal index is 0.8,

$$\text{Actual figure} = \text{deseaonalised} \times \text{seasonal index}$$
$$= 25600 \times 0.8$$
$$= 20480$$
$$= \$20\ 480$$

Question 13 **B**

The above graph can be either compressed in the x-axis or stretched in the y-axis
y axis transformation is y^2
x axis transformation can be either $\log x$ or $\frac{1}{x}$

The only option with y^2 and $\log x$ or $\frac{1}{x}$ is B

Section B (Modules)

Module 1: Number patterns and applications

Question 1 **C**

The common ratio, $r = \frac{t_2}{t_1} = \frac{1.5}{1} = 1.5$

The 4th term $= t_3 \times 1.5$
$= 2.25 \times 1.5$
$= 3.375$

Question 2 **B**

An arithmetic sequence with first term a = 5 and common difference, d = 5
The sum of ten terms:

$S_{10} = \frac{10}{2}\left[2\times 5 + (10-1)\times 5\right]$
$= 5[10 + 9\times5]$
$= 5[55]$
$= 275$

Question 3 **E**

The sequence is arithmetic with a = 41 and d = -3
Solving $t_n < 0$
$a + (n-1)d < 0$
$41 + (n-1)\times -3 < 0$
$44 - 3n < 0$
$44 < 3n$
$14.667 < n$
n is greater than 14.667,
so $n = 15$

Question 4 **D**

$t_4 = 9$: a + 3d = 9 … .(1)
$t_8 = -11$: a + 7d = -11 ……(2)
Subtract equation (1) from equation (2)
4d = -11 -9
= -20
d = -5 substituting in (1) gives
a + 3×-5 = 9
a = 24

Question 5 **C**

Substituting in the difference equation:
$t_2 = 2\times t_1 - 1 = 2\times3 - 1 = 5$
$t_3 = 2\times5 - 1 = 9$
$t_4 = 2\times9 - 1 = 17$

Question 6 **D**

This difference equation generates a sequence that is neither arithmetic nor geometric. You could 'solve' the difference equation by substituting 'a' = 2 and 'b' = -1 and $t_1 = 3$ in :

$t_n = a^{n-1}t_1 + \frac{b(a^{n-1}-1)}{a-1}$

$t_n = 2^{n-1}\times 3 + \frac{-1(2^{n-1}-1)}{2-1} = 3\times 2^{n-1} - (2^{n-1}-1) =$
$2\times 2^{n-1} - 1 = 2^n - 1$

Alternatively test the solutions to give the sequence 3, 5, 9, 17,….
A gives 3, 5, 7,…
B gives 3, 6, …
C gives 3, 5, 7…
D gives 3, 5, 9, 17 correct
E gives 3, 9, …

Question 7 **D**

The original number in the class must be divisible by 2 + 3 = 5. (Only whole numbers are possible for 'people')
Add 2 to the original number and the answer must be divisible by 3 + 4 = 7
The only answer that this is possible for is 40;
$\frac{40}{5} = 8 \quad and \quad \frac{40+2}{7} = 6$
Alternatively you could solve the equations:

$m : f = 2 : 5 \Rightarrow \frac{m}{f} = \frac{2}{5}$

and $m + 2 : f = 3 : 4 \Rightarrow \frac{m+2}{f} = \frac{3}{4}$

Question 8 **A**

$t_1 = a = 4.5\times(0.4)^1 = 1.8$
$t_2 = 4.5\times(0.4)^2 = 0.72$
$r = \frac{0.72}{1.8} = 0.4$

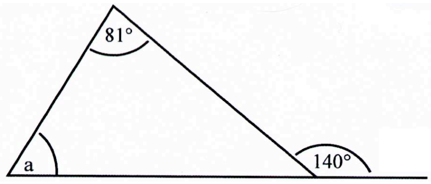
$S_\infty = \frac{a}{1-r} = \frac{1.8}{1-0.4} = \frac{1.8}{0.6} = 3$

Question 9 **E**

Working through the answers:
A: Arithmetic 5, 7, 9, …
B : Geometric but decreasing 100, 90, 81,…
C : Increasing but not geometric $\frac{1}{2}, \frac{3}{4}, \frac{7}{8}, ...$
D : Geometric but alternating positive, negative.
E : $\frac{3}{2}, \frac{9}{4}, \frac{27}{8}, ...$ geometric, positive, increasing

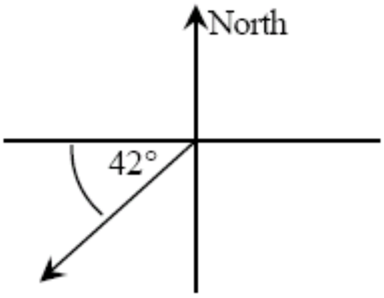
Module 2: Geometry & trigonometry

Question 1 **E**



Exterior angle = sum of the two interior angles
140° = a + 81°
a = 140° - 81°
a = 59°

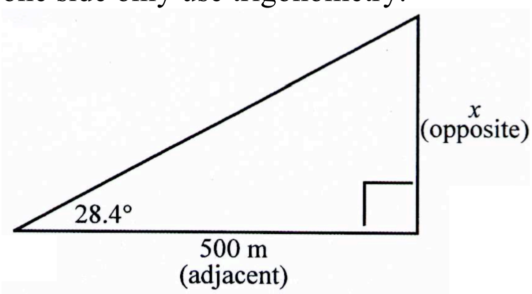
Question 2 C



The angle from south is the complimentary angle of 42° which is 48°.
true bearing = 180 + 48 = 228°T and
compass bearing = S 48° W

Question 3 **A**

For right-angled triangle , given an angle and one side only use trigonometry.



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan 28.4° = \frac{x}{500}$$

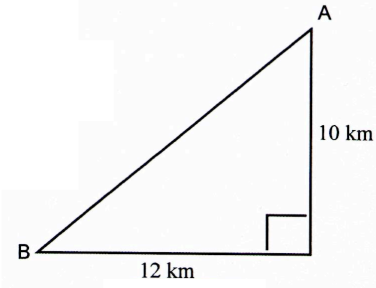
$$x = 500 \times \tan 28.4°$$

$$x = 270.35m$$

A 270 m

Question 4 **D**

Given the two lengths of a right-angled triangle, to find the third length use Pythagoras' Theorem.



$$c^2 = a^2 + b^2$$

$$c^2 = 10^2 + 12^2$$

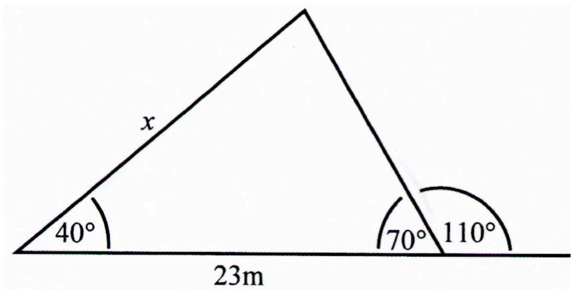
$$c^2 = 100 + 144$$

$$c = \sqrt{244}$$

$$c = 15.62km$$

The distance from A to B is closest to D (15.6 km)

Question 5 **C**



From the figure above, the value of the angle opposite side x can be found using the supplementary rule.

$$Angle\ X = 180° - 110° = 70°$$

The angle opposite the side of length 23 metres can be found using sums of angles in a triangle rule.

$$180° = 40° + 70° + third\ angle$$

$$third\ angle = 70°$$

To find the value of x use the Sine Rule

$$\frac{a}{\sin A°} = \frac{b}{\sin B°}$$

$$\frac{x}{\sin(70°)} = \frac{23}{\sin(70°)}$$

Question 6 **E**

Volume of a pyramid is

$V_{pyramid} = \frac{1}{3}AH$ where area of a square base is

$A_{square} = l^2$.

Therefore given $V_{pyramid} = 4500$ cm^3

and $H = 10$ cm

$V_{pyramid} = \frac{1}{3}AH = \frac{1}{3} \times l^2 \times H$

$4500 = \frac{1}{3} \times l^2 \times 10$

$l^2 = 1350$

$l = 36.7423$

Perimeter of a square is

$Perimeter = 4l$

$= 4 \times 36.7423$

$= 146.969$

$= 147$

Then the perimeter of the base is closest to E (147 m)

Question 7 **A**

Need one other side before the area can be found.

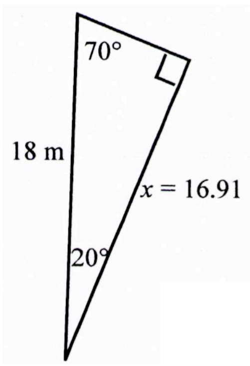
For side x, using trigonometry

$\cos\theta = \frac{adjacent}{hypotenuse}$

$\cos 20° = \frac{x}{18}$

$x = 18 \times \cos 20°$

$x = 16.9145m$



The area of the triangle at right is

$A = \frac{1}{2}ab\sin C$

$= \frac{1}{2} \times 18 \times 16.9145 \times \sin 20°$

$= 52.065$

$= 52.1cm^2$

A 52.1 cm^2

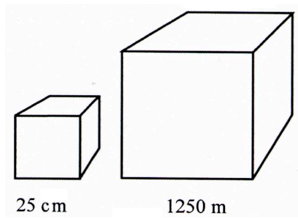
Question 8 **D**

Map scale = Linear scale factor, $k = 1 : 50$ or $\frac{1}{50}$

$k = \frac{1}{50} = \frac{Length\ of\ image}{Length\ of\ original}$

$\frac{1}{50} = \frac{25mm}{Length\ of\ original}$

$Length\ of\ original = 50 \times 25mm = 1250mm$



1 : 50

1250 mm = 1.25 m

$V_{prism} = AH$

$= l^3$

$= (1.25m)^3$

$= 1.953m^3$

Conversion is

1 m^3 = 1000 litres

1.953 m^3 = 1953 litres

The volume of the real refrigerator is closest to D (1950 litres)

Question 9 **D**

1:250 000

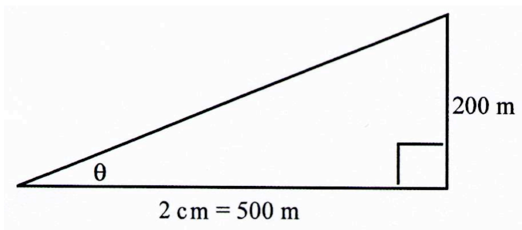
1 cm = 250000 cm

1 cm = 2500 m

Now from the contour map distance from A to B is 2 cm

2cm = 2 x 2500 m

From A to B is 5000 m



Gradient $= \frac{\text{rise}}{\text{run}} = \frac{\text{altitude}}{\text{horizontal distance}}$

$= \frac{200m}{5000m}$

$= \frac{1}{25}$ D (0.04)

Module 3 : Graphs and relations.

Question 1 **C**

The line goes through (0, 2) and (6, 10)

Gradient $m = \frac{10-2}{6-0} = \frac{8}{6} = \frac{4}{3}$

y-intercept : $c = 2$

Equation : $y = \frac{4}{3}x + 2$

$3y = 4x + 6$

$3y - 4x = 6$

Question 2 **E**

Substitute 3y = x + 2 into equation

3y – 5x = 6 to give

$(x + 2) - 5x = 6$

$x + 2 - 5x = 6$

$-4x = 4$

$x = -1$

Substitute x = – 1 into 3y = x + 2

$3y = -1 + 2$

$3y = 1$

$y = \frac{1}{3}$

Question 3 **C**

The pulse rate is between 100 and 140 beats/minute from time 7 minutes to 10 minutes (3 minutes) and from time 32 minutes to 40 minutes (8 minutes).

A total of 11 minutes.

Question 4 **D**

The average rate of change between two points

= the gradient of the straight line joining the two points.

= the gradient of the line joining (10, 140) and (40, 100)

$= \frac{100-140}{40-10}$

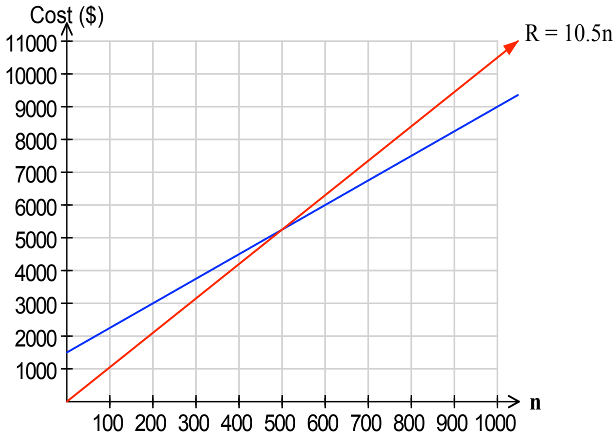
$= \frac{-40}{30}$

$= \frac{-4}{3}$

≈ -1.33

Question 5 **C**

Graphing R = 10.5n on the set of axes: the line goes through the origin (0, 0) and the point (1000, 10500)



The break-even point is the point where the graphs intersect ie. at n = 500

Question 6 **E**

Photocopying costs:

A lot of 50 costs $50 \times 20^c = \$10$

A lot of 250 costs $250 \times 10^c = \$25$

A lot of 500 costs $500 \times 10^c = \$50$

Total cost $85

One lot of 800 costs 800 × 5 = $40

$85 - $40 = a saving of $45

Question 7 **E**

Considering the four boundary lines of the feasible region:

The lines $x = 0$ and $x = 8$ mean x is between 0 and 8 giving the constraint $0 \le x \le 8$

A line goes though the points (0, 20) and (20, 0). This line has the equation $x + y = 20$ and since the point (0, 0) is in the region the constraint is $x + y \le 20$

The line going through the points (0, 0) and (10, 10) has the equation $y = x$ and the feasible region contains the points where the y values are greater than the x values so the constraint is $y \ge x$

Question 8

The maximum value will occur at one of the **extreme points** (vertices of the feasible region)

Extreme point	$V = 3x + 4y$
(0, 0)	3×0 + 4×0 = 0
(0, 20)	3×0 + 4×20 = 80
(8, 12)	3×8 + 4×12=72
(8, 8)	3×8 + 4×8 = 56

The maximum value for $V = 3x + 4y$ is 80.

Question 9 **A**

From the shape the graph is either $y = kx^2$ or $y = kx^3$ (Eliminates answers D and E)

Substituting the point (4, 20) into these equations gives:

$y = kx^2$: $20 = k \times 4^2$

$k = \frac{20}{16} = \frac{5}{4}$

Equation $y = \frac{5}{4}x^2$

$y = kx^3$: $20 = k \times 4^3$

$k = \frac{20}{64} = \frac{5}{16}$

Equation $y = \frac{5}{16}x^3$

The equations from the answers:

A: $y = \frac{5}{4}x^2$

B: $y = \frac{4}{5}x^3$

C: $y = \frac{4}{5}x^2$

The only one that matches is answer A.

Module 4 : Business related Mathematics

Question 1 **C**

If x is the amount that the retailer retains then:-

x + 10% of x = 59.95

$$x + \frac{10}{100}x = 59.95$$

$$x(1 + 0.1) = 59.95$$

$$1.1x = 59.95$$

$$x = \frac{59.95}{1.1}$$

Question 2 **B**

Using the simple interest formula:

$$Interest = \frac{\Pr T}{100}$$

$$= \frac{15000 \times 5.4 \times \frac{1}{4}}{100}$$

$$= 202.5$$

Question 3 **E**

Monthly interest rate

$$= \frac{6.72}{12} = 0.56\%$$

$$R = 1 + \frac{0.56}{100} = 1.0056$$

Using the compound interest formula :

Amount owing = PR^n where n = 6

Amount owing = $12500 \times 1.0056^6 = 12925.92$

Question 4 **D**

The monthly interest rate = $\frac{4.8}{12} = 0.4\%$ so

$$R = 1 + \frac{0.4}{1000} = 1.004$$

4 years = 48 months : n = 48

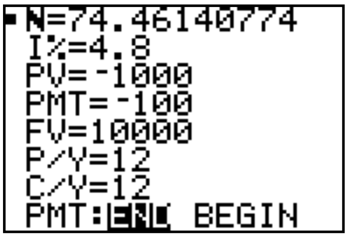
P = 1000; the original investment.

Regular payments of \$100 : Q = 100

Payments are being added to the initial amount in the account hence the '+' sign between the terms.

Question 5 **A**

Using the TVM solver on the calculator:-



Question 6 **A**

The minimum balance for January and February is \$4280

Interest = $\dfrac{4280 \times 1.5 \times \frac{2}{12}}{100} = 10.7$

The minimum balance for March to June (4 months) is \$3420

Interest = $\dfrac{3420 \times 1.5 \times \frac{4}{12}}{100} = 17.1$

Total interest for the 6 months = \$27.80

Question 7 **D**

\$4400 is owed after the deposit is paid.
The repayments total
12×402.60 = \$4831.20
The interest paid is \$4831.20 – \$4400 = \$431.20

Interest rate = $\dfrac{431.20}{4400} \times 100 = 9.8\%$

Question 8 **B**

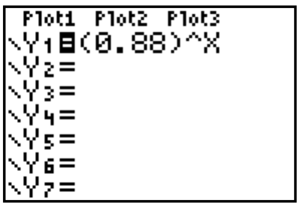
Depreciation of 12% p.a. means a multiplying factor of $\left(1 - \dfrac{12}{100}\right) = 0.88$

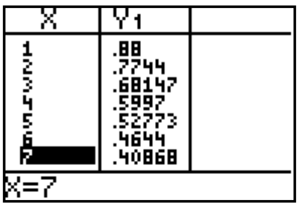
If the fixtures are valued at \$P initially then their value after n years will be $P \times (0.88)^n$

Solving for n when the value is 0.5P:-

$$0.5\,P = P \times (0.88)^n$$
$$0.5 = (0.88)^n$$

Using the calculator to solve:

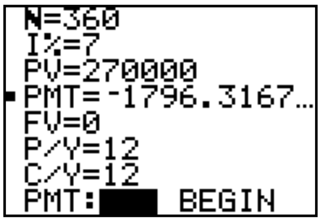




After 6 years the value will be less than half the original value. This will be at the beginning of the year 2005.

Question 9 **E**

If the interest rate rises then either they will need to pay more each month or the term of the loan will increase (or a bit of both). They will also pay more in interest over the term of the loan.
Using the TVM solver on the calculator will confirm answers A to D.
The TVM solver for answer E shows that they will be paying more than \$1739.61 if they increase the loan to 30 years.

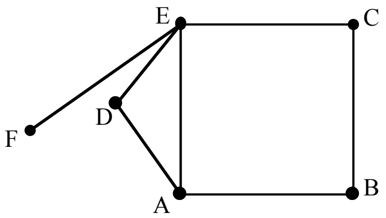


Module 5: Networks & decision mathematics

Question 1 **A**

The number of vertices is 5
The number of edges is 9, then respectively it is A (5 and 9).

Question 2 **C**



A Euler Path uses every edge only once and if there is a pair of vertices with odd degrees then the path must start at either; in this case F (degree of 1) or A (degree of 3)
Answer: C (*A-E-C-B-A-D-E-F*).

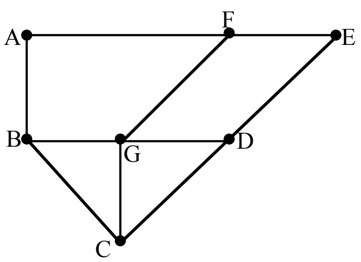
Question 3 **B**

The first row and first column of the matrix represents the number of edges originating from the vertex A, namely,
0 from A, 1 from B, 2 from C or 1 from D or
0 1 2 1
The matrix that represents the network shown above is B

$$\text{B} \quad \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{matrix} A & B & C & D \end{matrix} \\ \begin{bmatrix} 0 & 1 & 2 & 1 \\ 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 2 \\ 1 & 0 & 2 & 0 \end{bmatrix} \end{array}$$

Question 4 **A**

Hamiltonian path or circuit uses each vertex once only and for this example one possible path or circuit is B-C-G-D-E-F-A-B



No Euler path is possible.
The true statement is A:
The planar graph has a Hamiltonian path and circuit, but no Euler path or circuit.

Question 5 **D**

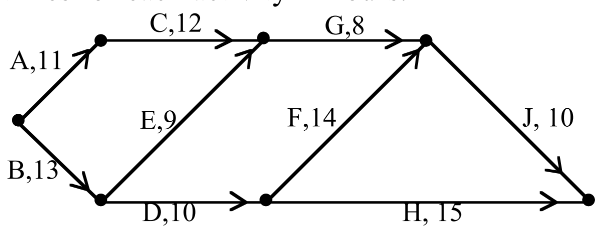
From A to E via B is 7 units
Then from E to H via D is 14 units
And from H to K either path is 2 units,
For a longest path of 23 units.

Question 6 **C**

Jade and Rhiannon have lived in four of the cities while Scott and Alisha have lived in three cities.

Question 7 **C**

The network diagram shown below gives the times for each activity in hours.



The activities that are immediately before predecessors of Activity G are C (*C and E*).

Question 8 **D**

Using Euler's formulae: $V = E - F + 2$
where V = number of vertices, E = number of edges, F = number of faces.
Then the number of faces or regions becomes

$$\begin{aligned} F &= E - V + 2 \\ &= 10 - 6 + 2 \\ &= 6 \end{aligned}$$

Question 9 **B**

Perform a row reduction by subtracting the smallest value from each row.
The smallest value in row A is 11
The smallest value in row B is 12
The smallest value in row C is 11
The smallest value in row D is 12
The reduction matrix becomes

	Cust 1	Cust 2	Cust 3	Cust 4
Driver A	4	2	0	3
Driver B	0	2	1	3
Driver C	2	1	3	0
Driver D	0	0	3	2

Answer is B where
Driver A gets Customer 3,
Driver B gets Customer 1,
Driver C gets Customer 4,
Driver D gets Customer 2.