The Mathematical Association of Victoria

FURTHER MATHEMATICS

Trial written examination 2
(Analysis task)

2005

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

Student's Name: ____

QUESTION AND ANSWER BOOK

Structure of book

Core		
Number of questions	*Number of questions to be answered*	*Number of marks*
2	2	15
Module		
Number of modules	*Number of modules to be answered*	*Number of marks*
5	3	45 Total 60

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

These questions have been written and published to assist students in their preparations for the 2005 Further Mathematics Examination 2. The questions and associated answers and solutions do not necessarily reflect the views of the Victorian Curriculum and Assessment Authority. The Association gratefully acknowledges the permission of the Authority to reproduce the formula sheet.

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Working Space

Instructions

This task paper consists of a core and five modules. Students should answer all the questions in the core and then select three modules and answer all questions within the modules selected.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π, e, surds or fractions.

Core

From the table of earnings for AFL football players in years 2003 and 2004 are summarised in the frequency table below

Salary	2003 Players	2004 Players
$0 to <$100 000	153	146
$100 000 to <$200 000	184	188
$200 000 to <$300 000	102	107
$300 000 to <$400 000	47	57
$400 000 to <$500 000	31	24
$500 000 to <$600 000	8	12
$600 000 to <$700 000	4	4
$700 000 to <$800 000	0	0
$800 000 to <$900 000	1	4
Total	530	542

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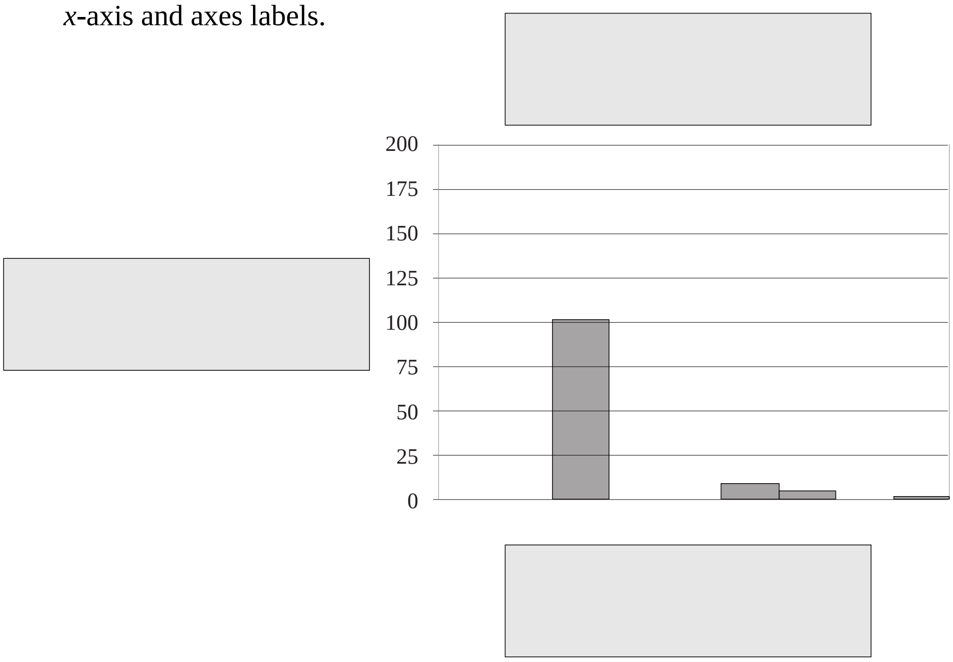
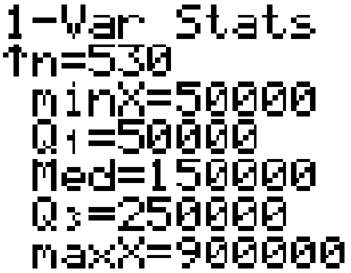
Question 1

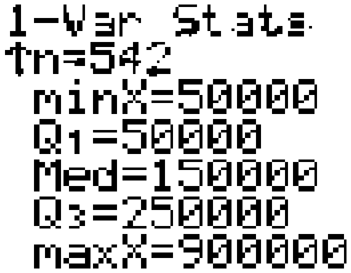
a) Complete the following histogram from the above frequency table for earnings of the 2003 players, including title, *x*-axis and axes labels.



3 marks

The five figure summary for earnings for 2003 and 2004 were the same as shown by the screen dumps from a graphics calculator.





b) State the Interquartile range for AFL player earnings.

c) Draw a boxplot to represent the above five-figure summary. Does your boxplot suggest there are outliers? Justify

Boxplot of AFL player Earnings for either 2003 or 2004

Annual Salary ($)

3 marks

Statistics	2003	2004
Mean	$186 000	$195 000
Standard deviation	$132 000	$143 000

However, the mean and standard deviation for 2003 and 2004 were different as summarised in the table above.

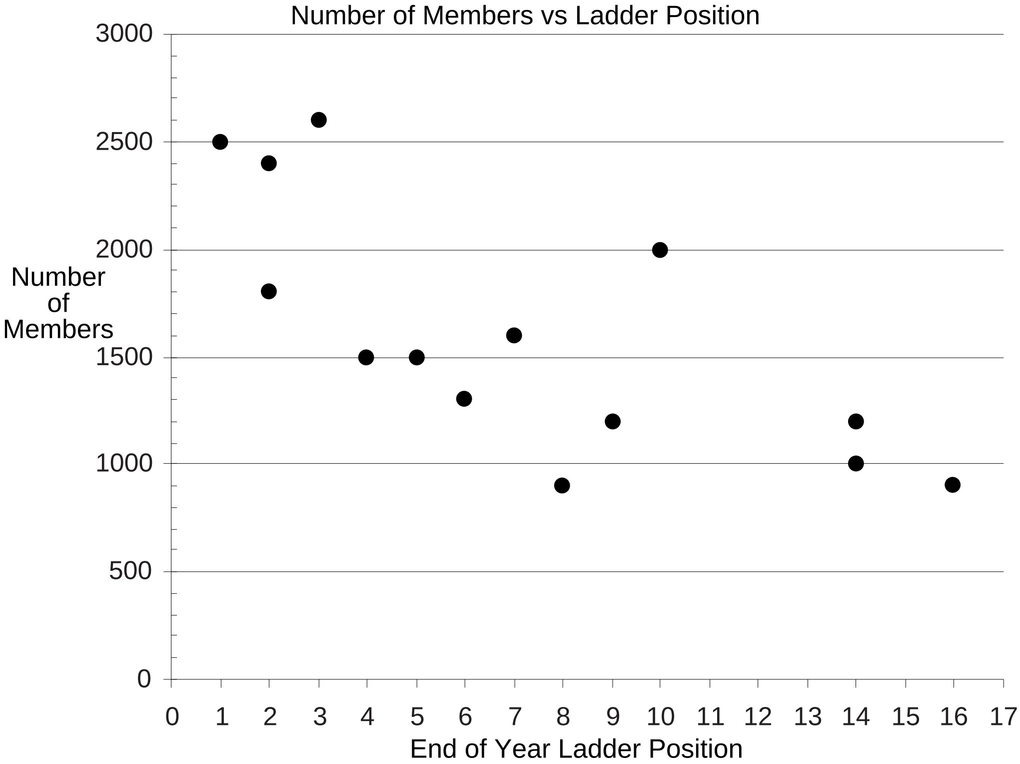
d) Explain why there is a noticeable difference in the mean and standard deviation.

2 marks

Question 2

A football club in the local league had analysed the on field performance (end of year ladder position) and club membership (number of members to the nearest 100's) over the past 14 years and represented this on a table and a scatterplot.

End of the Year ladder Position	Number of members
5	1500
6	1600
9	1200
14	1200
14	1000
16	900
6	1300
4	1500
2	1800
1	2500
3	2600
2	2400
10	2000
8	900



a) Calculate the strength (to 2 decimal places) and direction of the relationship between end of year premiership ladder and membership numbers.

strength of the relationship [ ] direction of relationship [ ]

1 mark

b) Using your answer from above complete the following statement that can be made about the relationship between on field performance and membership numbers.

"We can conclude from this that [ ] % of the variation in the number of members can be explained by the variation in [ ] .

The other 46% variation in number of members is due to other factors.

2 marks

c) On the scatterplot, draw the three-median line and circle the two points that were used to determine the gradient of the line.

2 marks

The line of best fit for the above data was calculated using the three median technique where
Number of Members = – 100 × position on the ladder + 2417.

d) Use the above relationship to predict the number of members if the club finishes 12th on the ladder.

____

____

1 mark

Total 15 marks

END OF CORE

Module 1: Number patterns and applications

Megan has opened a sandwich shop in the busy part of the city. She is interested to see the trend in sales so for the first week and the third week she has counted the number of sandwiches that are sold in her shop. The results are recorded in the table below:

Week 1	Week 2	Week 3
192	—	432

Megan has several models to consider that could predict her future sales of sandwiches :-

Question 1

Assuming that the number of sandwiches sold follows an arithmetic sequence.

a. Find the number of sandwiches sold in the second week.

b Write a rule for A_n that would give you the number of sandwiches sold in the *n*th week of operation.

c. Find the number of sandwiches sold in the 5th week.

d. Megan has calculated that the maximum number of sandwiches that she can make and sell in a week is 800. Using this model in which week will she first exceed 800 sandwiches?

1 + 2 + 1 + 1 = 5 marks

Question 2

Assuming the number of sandwiches sold follows a geometric sequence.

a. Show that the number of sandwiches sold in the second week is 288.

b. State the common ratio of this geometric sequence

c. In which week will the number of sandwiches sold in a week first exceed 800?

1 + 1 + 1 = 3 marks

Megan is not satisfied with these two models proposed, so in the 5th and 6th weeks she also counts her sales and finds her sandwich sales for these weeks are 680 and 716 respectively.

It is now proposed that a difference equation of the form $C_{n+1} = 1.2C_n - k$ would be a suitable model.

Question 3

a. Use the figures given for weeks 5 and 6 to find the value of k

b. Using this model find the week when her predicted sales will be more than 800.

2 + 2 = 4 marks

Megan realises that none of the three models already proposed is ideal as she cannot increase her sales indefinitely and so in the 7th and 8th week she also counts the number of sandwiches sold and finds that these values are increasing **but that this increase is decreasing**. The figures for the number of sandwiches sold are given in the table below:

Week 5	**Week 6**	**Week 7**	**Week 8**
680	716	740	756

Question 4

a. Find the sequence that represents the **increase in sales** over these weeks.

b. What is the maximum number of sandwiches that are sold in a week if this sequence continues indefinitely.

1 + 2 = 3 marks

Total 15 marks

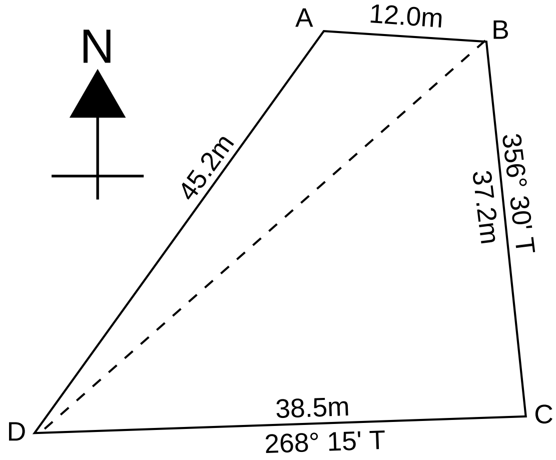
END OF MODULE 1

TURN OVER

Module 2: Geometry & trigonometry

There are many land developments sprouting throughout Melbourne. Many of the blocks made available are of irregular shape. This has resulted in complicating the task of a surveyor when calculating size of blocks and directions and lengths of boundaries.

One such block is given below as taken from Land Titles Certificate.



Question 1

a. The fence line BC and CD are not at right angles. Calculate the angle between these two boundaries of the block (in degrees and minutes), given the true bearings of the two boundaries as 356°30'T and 268°15'T respectively.

1 mark

b. The surveyor has forgotten his tape measure and needs the length and direction of the traverse line from D to B. Using trigonometry calculate the length (to one decimal place) and direction as true bearing (nearest degree) of the traverse line from B to D. Show an appropriate diagram.

2 + 2 + 1 = 5 marks

c. Calculate the triangular area given by BCD (to nearest square metre)

2 marks

The front boundary was calculated to be 12.0 m.

d. Now calculate the triangular area bounded by ABD (to nearest square metre)

2 marks

e. Hence calculate the area of the block to the nearest square metre.

1 mark

The surveyor wishes to prepare a Surveyor's traverse summary notes. This is so he can show to his boss when he gets back to his office and confirm the dimensions and directions of the boundaries and the area of the block.

Question 2

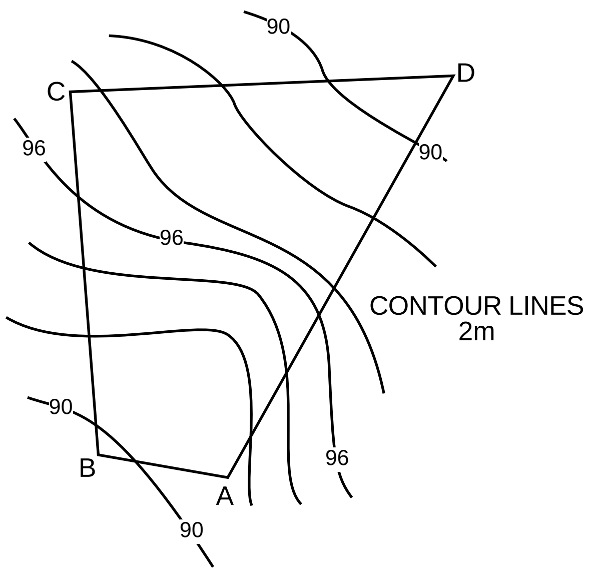
a. Complete the missing parts of the following surveyor's notes. Some of the notes have been already done.

	B	
	0	
C 26.6	25.2	
	43.6	10.0 ☐
	☐	
	D	

2 marks

To prepare the block for building a new house the land site is to be levelled.

b. On the plan on this page indicate the region on this block that has the steepest slope where most of the soil will need to be removed.



1 mark

c. On this plan, join with a straight line any two places on the block where the difference in altitude is 4 metres.

1 mark

Total 15 marks

END OF MODULE 2

Module 3: Graphs and relations

A football manufacturer makes two sizes of football: the *Standard* and the *Junior*.

The footballs are made from cowhide and each cowhide yields enough leather to make either six *Standard* footballs or seven *Junior* footballs.

The manufacturer has found that he needs to make at least 100 Standard footballs per day and also at least twice as many *Standard* footballs as *Junior* footballs.

On a particular day he has 50 cowhides available to make footballs. Let x be the number of *Standard* footballs and y the number of *Junior* footballs made on this day.

Question 1

a. The constraint associated with the 50 cowhides available is $\frac{x}{6} + \frac{y}{7} \leq 50$

Rearrange this constraint so that it is in the form $ax + by \leq c$ where a, b and c are constants.

Fill-in the the values for a and c below.

$\square\ x + 6y \leq \square$

2 marks

b. Write a constraint associated with the statement : "The manufacturer has found that he needs to make at least 100 *Standard* footballs per day"

1 mark

c. The constraint associated with the statement : "The manufacturer has found that he needs to make …. at least twice as many *Standard* footballs as *Junior* footballs" is $x \geq 2y$

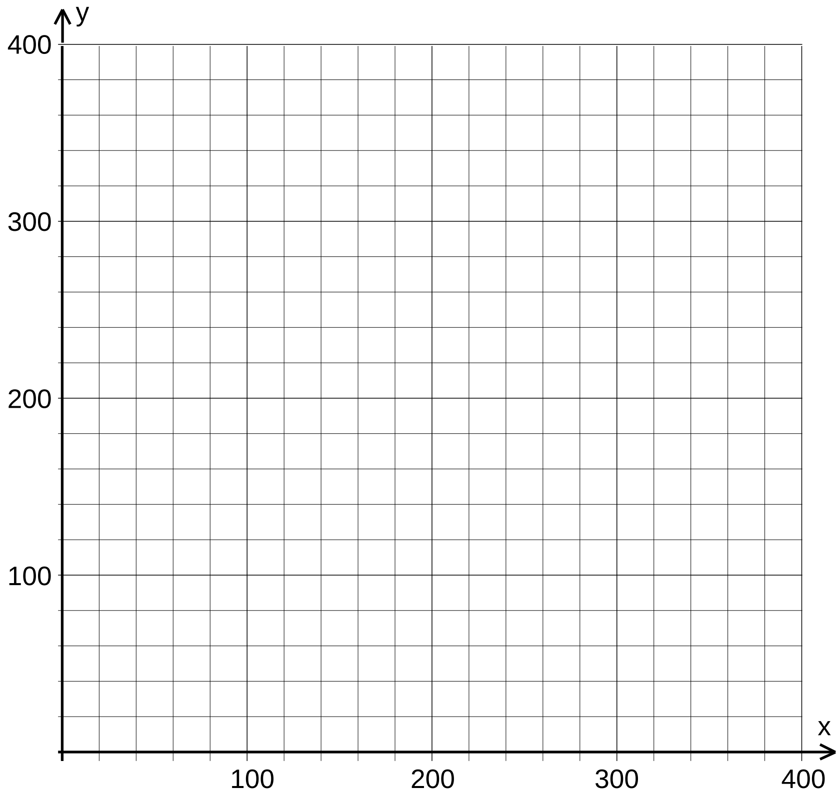
Rearrange this to the form $y \leq kx$ and write your value for the constant k in the space provided below.

$y \leq \square\ x$

1 mark

The fourth constraint is $y \geq 0$

d. On the set of axes below graph the four constraints and show clearly the feasible region defined by the four constraints.



4 marks

e. The manufacturer makes a profit of $24 on each Standard football and $22 on each Junior football.
Write an objective function for the manufacturer's profit, P dollars.

1 mark

f. Find the co-ordinates of all the extreme points for your feasible region from part D.

2 marks

g. Find the number of each type of football that the manufacturer should make on this day so that he satisfies his constraints and makes maximum profit.

2 marks

Question 2

On another day there is no restriction on the number of hides available but there is a maximum of 320 footballs that canbe made (the other constraints remain the same).

Find the number of each type of football that the manufacturer should make on this day so that he satisfies the constraints and makes maximum profit.

2 marks

Total 15 marks

End of Module 3
TURN OVER

Module 4: Business Related Mathematics

John is keen to buy a plasma-screen television and has saved \$4000 towards its purchase. The television that he would like to buy is priced at \$6490 in the local electrical store.

Question 1

a. If the retailer in the electrical store has included the 10% goods and services tax in the price of \$6490, how much goods and services tax will he pay on this television? Give your answer to the nearest dollar.

1 mark

b. John has observed that the price of plasma televisions is decreasing by 8% each year. If he waits a year to purchase his favoured television what can he expect to pay for it if the price decreases by 8%. Give your answer to the nearest dollar.

1 mark

John thinks he will invest his savings for a year and he is considering some options available to him.

Question 2.

a. John can invest his \$4000 savings in a 12-month term deposit paying 5.2% p.a., interest payable at the end of the term. How much interest, to the nearest dollar, will he accumulate in this account for the year?

2 marks

b. Another option is to keep his savings in a cash-management account that pays 4.5%p.a. interest, compounding monthly. How much will he have accumulated in this account at the end of a year? Give your answer to the nearest cent.

2 marks

c. John's target is to have $6000 saved by the end of a year. How much will he need to add each month to the cash-management account, earning 4.5% p.a., compounding monthly, to reach this target?

Give your answer to the nearest cent.

2 marks

The Electrical store where John plans to purchase the television offers a time-payment plan for the purchase of their goods. For his favoured television John would have to pay 20% of the purchase price as a deposit then twelve equal monthly payments of $475.

Question 3

a. For a purchase price of $6490 how much will John have to pay as a deposit? Give your answer to the nearest dollar.

1 mark

b. How much in total will John pay for the television with this time-payment plan? Give your answer to the nearest dollar

1 mark

c. How much interest will he pay over the 12 months? Give your answer to the nearest dollar.

1 mark

d. Calculate the flat rate of interest being charged with this time-payment plan. Give your answer correct to two decimal places.

2 marks

e. Calculate the effective rate of interest being charged with this time-payment plan.

Give your answer correct to one decimal place.

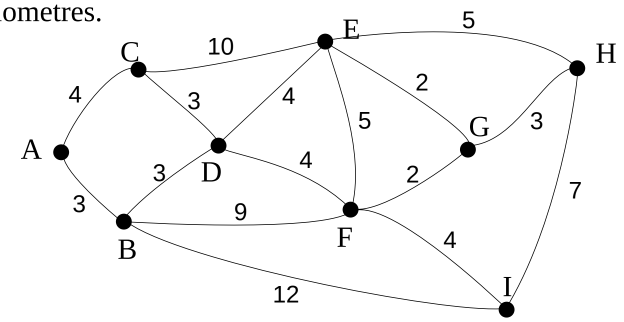
2 marks

Total 15 marks

END OF MODULE 4
TURN OVER

Module 5: Networks & decision mathematics

A cycling tour taking in 9 towns has been organized. The diagram below shows the network of roads joining the towns. The distances indicated are in kilometres.



Question 1

a. The network of roads and towns can be represented as a matrix given below. Complete the four missing elements of the network.

	A	B	C	D	E	F	G	H	I
A	0	1	1	0	0	0	0	0	0
B	1	0	0	1	0	1	0	0	1
C	1	0	0	1	0	0		0	0
D	0	1	1	0	1	1	0	0	0
E	0	0		1	0	1	1	1	0
F	0	1	0		1	0	1	0	1
G	0	0	0	0	1		0	1	0
H	0	0	0	0	1	0	1	0	1
I	0	1	0	0	0	1	0	1	0

2 marks

The tour starts at Town F and follows a Hamiltonian circuit.

b. Given the first two towns are I and B, complete the order in which the towns may be visited.

F – I – B – ______________________

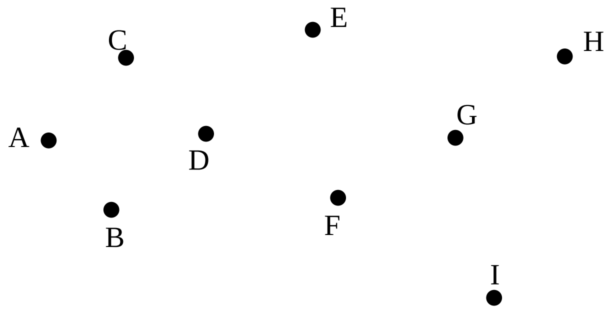
2 marks

c. An urgent call for delivery of supplies from Town A to Town H is required. Write down the length of the shortest route from Town A to Town H.

____

1 mark

d. The 9 towns share an ambulance and country fire brigade service.
The services are located in Town F and the planned emergency route to each of the towns can be represented as a spanning tree.
On the diagram at right, draw this as a minimal-length spanning-tree.



2 marks

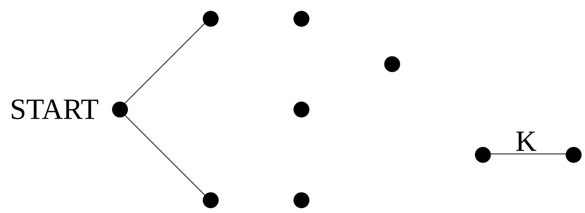
Question 2

The activities that are planned for each day of the cycling tour is managed as a project involving 9 major activities. Those activities, the time they took to complete (in hours),and the activities which were their immediate predecessors in the project, are indicated in the table below.

Activity	Immediate predecessor	Time taken to complete (hours)
A	-	0.5
B	-	1
C	A	0.5
D	B	0.5
E	B	3.5
F	C	3
G	D	3
H	E	0.5
I	E	1
J	F,G,H	1.5
K	I,J	9

A directed graph that can be used to show the activities as summarised in the table above is given below.

a. Complete the graph by drawing in the activities.



3 marks

b. Write down the critical path of the network.

____

1 mark

c. Complete the table below using the information from the table above

Activity	Earliest start time	Latest start time
A	0	1
B	0	0
C	0.5	1.5
D	1	
E	1	1
F	1	2
G		2
H	4.5	4.5
I	4.5	
J	5	5
K	6.5	6.5

2 marks

d. Write down the earliest completion time for the daily activities on a day of a cycle tour.

1 mark

e. Write down the slack (float time) for activity F.

1 mark
Total 15 marks

END OF QUESTION AND ANSWER BOOK

FURTHER MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

This formula sheet is provided for your reference.

Further Mathematics Formulas

Business-related mathematics

simple interest: $I = \dfrac{PrT}{100}$

compound interest: $A = PR^n$ where $R = 1 + \dfrac{r}{100}$

hire purchase: effective rate of interest $\approx \dfrac{2n}{n+1} \times$ flat rate

annuities: $A = PR^n - \dfrac{Q(R^n - 1)}{R - 1}$, *where* $R = 1 + \dfrac{r}{100}$

Geometry and trigonometry

area of a triangle: $\dfrac{1}{2} bc \sin A$

area of a circle: πr^2

volume of a sphere: $\dfrac{4}{3} \pi r^3$

volume of a cone: $\dfrac{1}{3} \pi r^2 h$

Pythagoras' theorem: $c^2 = a^2 + b^2$

sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos C$

Graphs and relations

Straight line graphs

gradient: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

equation: $y - y_1 = m(x - x_1)$ gradient-point form

$y = mx + c$ gradient-intercept form

$\dfrac{y - y_1}{x - x_1} = \dfrac{y_2 - y_1}{x_2 - x_1}$ two-point form

Number patterns and applications

arithmetic series: $a + (a + d) + \ldots + (a + (n - 1)d) = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}(a + l)$

geometric series: $a + ar + ar^2 + \ldots + ar^{n-1} = \frac{a(1-r^n)}{1-r}, \; r \neq 1$

infinite geometric series: $a + ar + ar^2 + ar^3 + \ldots = \frac{a}{1-r}, \; |r| < 1$

linear difference equations:

$$t_n = at_{n-1} + b = a^{n-1}t_1 + b\frac{(a^{n-1} - 1)}{a - 1}, \; a \neq 1$$

$$= a^n t_0 + b\frac{(a^n - 1)}{a - 1}$$

Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Statistics

seasonal index: $\text{seasonal index} = \frac{\text{actual figure}}{\text{deseasonalised figure}}$

END OF FORMULA SHEET