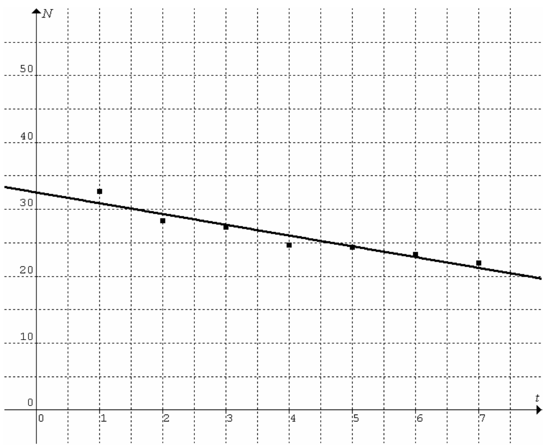
**2006 Further Mathematics Trial Exam 2 Solutions**


**Core – Data analysis**

Q1a Three-year moving average $= \dfrac{26+25+22}{3} = 24.3$

Q1b



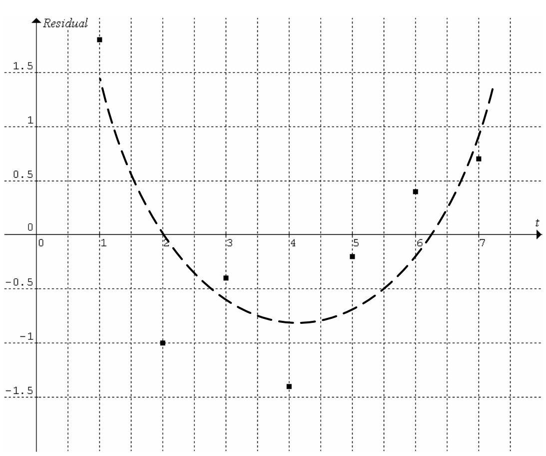

Q1ci $r = -0.955$ (By graphics calculator)

Q1cii $r^2 = 0.912$, $\therefore 91\%$

Q1ciii $N = -1.611t + 32.529$

Q1civ See Q1b

Q1d Residual = actual value – predicted value
When $t = 5$, the actual value $N = 24.3$,
the predicted value $N = -1.611(5) + 32.529 = 24.474$
$\therefore$ residual $= 24.3 - 24.474 = -0.2$




Q1e The pattern is non-random and appears to follow the trend as indicated by the dotted curve.

Q1f log(5) = 0.70, $N = 24.3$

Q1g $N = -12.166\log(t) + 32.521$

Q1h
$r = -0.993$ is closer to $-1$ than $r = -0.955$ found in Q1ci.

Q1i Year 2006, $t = 9$, $N = -12.166\log(9) + 32.521 \approx 21$

Q1j $N \to \log(N)$
$\log(N)$ compresses high *N* values more than low *N* values, thus it helps to linearise the data. The effect is not as great as $\log(t)$ because the variation in *N* is small percentagewise.

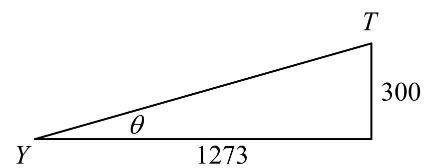
**Module 2: Geometry and trigonometry**

Q1a She could see her home from *X* but not from *Y*. There is a hill between *Y* and *H*.

Q1b $\overline{XY} = \sqrt{0.7^2 + 1.5^2} = 1.7$ km

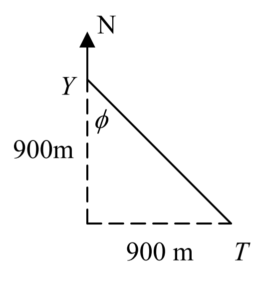
Q1c The angle of elevation is greater at location *Y* because *T* is closer to *Y* than to *X*.

Q1d Horizontal distance from *Y* to *T* $= \sqrt{900^2 + 900^2} = 1273$ m
Vertical distance from *Y* to *T* $= 400 - 100 = 300$ m




$\tan\theta = \dfrac{300}{1273}$, $\theta = \tan^{-1}\left(\dfrac{300}{1273}\right) = 13^o$

Q1e




$\tan\phi = \dfrac{900}{900} = 1$, $\phi = 45^o$
$\therefore$ *T* is *SE* of *Y*

Q1f Horizontal distance from *X* to *T* $= \sqrt{600^2 + 1600^2} = 1709$
Vertical distance from *X* to *T* $= 400 - 100 = 300$
$\overline{XT} = \sqrt{1709^2 + 300^2} = 1735$
$\overline{YT} = \sqrt{1273^2 + 300^2} = 1308$ (See Q1d)
$\overline{XY} = \sqrt{700^2 + 1500^2} = 1655$
Use the cosine rule:
$\cos\angle XTY = \dfrac{1735^2 + 1308^2 - 1655^2}{2(1735)(1308)}$, $\angle XTY = 64^o$

Q1g Horizontal distance from $X$ to $T$ = 1709 m
Horizontal distance from $Y$ to $T$ = 1273 m
Horizontal distance from $X$ to $Y$ = 1655 m

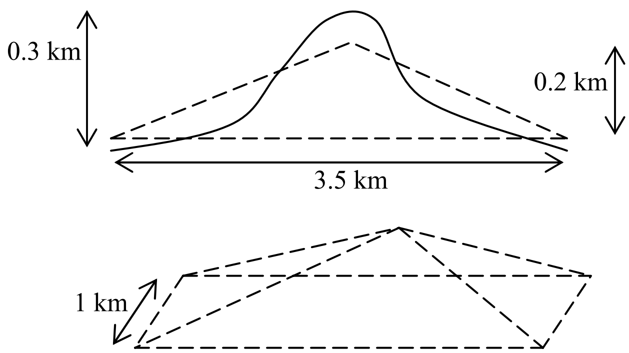
$s=\dfrac{1709+1273+1655}{2}=2318.5$

Land area $=\sqrt{s(s-a)(s-b)(s-c)}$

$=\sqrt{2318.5(2318.5-1709)(2318.5-1273)(2318.5-1655)}$

$=990000\text{ m}^2$

Q1h Approximate the terrain as a pyramid with a rectangular base.



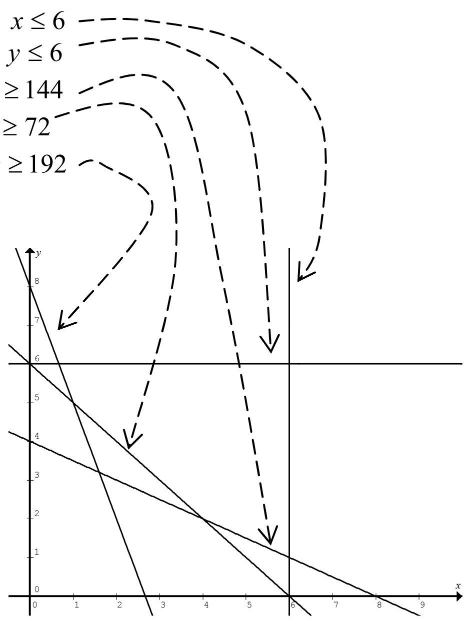

Scale 10 cm : 1 km

Volume of clay $=\dfrac{1}{3}(35\times10\times2)\approx230\text{ cm}^3$

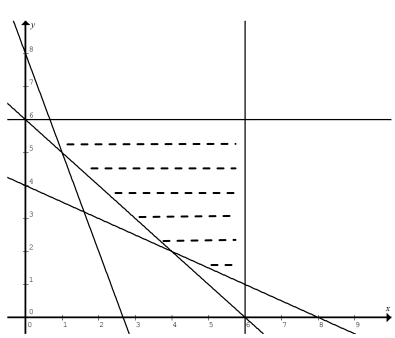
**Module 3: Graphs and relations**

Q1a
$x\le6$
$y\le6$
$18x+36y\ge144$
$12x+12y\ge72$
$72x+24y\ge192$

Q1b



Q1c



Q1d $72x+24y=192$ and $y=6$, $\therefore x=\dfrac{2}{3}$ $\left(\dfrac{2}{3},6\right)$

$72x+24y=192$ and $12x+12y=72$, solve simultaneously by elimination to obtain $x=1$, $y=5$ $(1,5)$

$12x+12y=72$ and $18x+36y=144$, solve simultaneously by elimination to obtain $x=4$, $y=2$ $(4,2)$

$18x+36y=144$ and $x=6$, $\therefore y=1$ $(6,1)$

$x=6$ and $y=6$ $(6,6)$

Q1e $C=1200x+960y$

Q1f $\left(\dfrac{2}{3},6\right)$, $x=1$, $y=6$, $C=1200(1)+960(6)=6960$

$(1,5)$, $x=1$, $y=5$, $C=1200(1)+960(5)=6000$

$(4,2)$, $x=4$, $y=2$, $C=1200(4)+960(2)=6720$

$(6,1)$, $x=6$, $y=1$, $C=1200(6)+960(1)=8160$

$(6,6)$, $x=6$, $y=6$, $C=1200(6)+960(6)=12960$

$\therefore$ minimum weekly total operating cost is \$6000 when mine 1 operates for 1 day and mine 2 operates for 5 days.

Q1g The minimum weekly total operating cost remains at \$6000 because mine 1 operates for 1 day only per week to keep cost at the minimum.

Q1h The new operating cost for mine 1 after reduction of \$240 is \$960. It is the same operating cost as for mine 2. Hence the number of operating days for each mine can vary and the minimum operating cost remains the same. The following solutions satisfy the production requirements while the operating cost is kept to a minimum.

$(1,5)$, $(2,4)$, $(3,3)$ and $(4,2)$

Q1i $C=960\times6=\$5760$

**Module 4: Business-related mathematics**

Q1a For July, minimum balance = \$700.00,

$I=\dfrac{700\times4.75\times\frac{1}{12}}{100}=\$2.77$

For August, minimum balance = \$1000.00,

$I=\dfrac{1000\times4.75\times\frac{1}{12}}{100}=\$3.96$

Total $=2.77+3.96=\$6.73$

Q2a Interest $=850\times30-20000=\$5500$

Q2b Flat rate in % $=\dfrac{100I}{Pt}=\dfrac{100\times5500}{20000\times2.5}=11$

Q2c Effective rate in % $=\dfrac{2n}{n+1}\times$ flat rate

$=\dfrac{2\times30}{31}\times11=21.3$

Q3a Yes, possibly.
Total monthly repayment $= 850 + 1270 = \$2120$
$\therefore 3800 - 2120 = \$1680$ per month for other expenses.

Q3b $r = 7$, $R = 1 + \frac{7}{100 \times 12} = 1.00583333$, $n = 12 \times 5 = 60$,
$P = 180000$, $Q = 1270$

$$A = PR^n - \frac{Q(R^n - 1)}{R - 1}$$

$$= 180000 \times 1.0058333^{60} - \frac{1270(1.00583333^{60} - 1)}{1.00583333 - 1}$$

$= \$164249.52$

Q3c $r = 8$, $R = 1 + \frac{8}{100 \times 12} = 1.0066667$, $n = 12 \times 20 = 240$,
$P = 164249.52$, $A = 0$

$$A = PR^n - \frac{Q(R^n - 1)}{R - 1},$$

$$0 = 164249.52 \times 1.0066667^{240} - \frac{Q(1.0066667^{240} - 1)}{1.0066667 - 1}$$

New monthly repayment $Q = \$1373.85$

Q4a Rate of depreciation $= \frac{35100}{270000} = \$0.13$ per km

Q4b Number of weeks $= \frac{270000}{1800} = 150$

Q4c Depreciation $= 35100 - 11700 = \$23400$
Number of km travelled $= \frac{23400}{0.13} = 180000$
Number of weeks $\frac{180000}{1800} = 100$

Q4d Book value $= P\left(1 - \frac{r}{100}\right)^n$ where $r$ is the weekly depreciation rate in % and $P$ is the cost of the new taxi.

$$\therefore 11700 = 35100\left(1 - \frac{r}{100}\right)^{100},$$

$$1 - \frac{r}{100} = \sqrt[100]{\frac{11700}{35100}} = 0.989074,$$

$r = 1.0926$

*Please inform mathline@itute.com re conceptual, mathematical and/or typing errors*