Year 2004

VCE Further Mathematics Trial Examination 1

Suggested Solutions

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Question 1 D

Use your graphics calculator.

Press stat \rightarrow edit and type in the values in the given table. Points scored in L1 and number of students in L2.

Press stat \rightarrow calc \rightarrow 1 : 1 – Var Stats \rightarrow Enter Type L1, L2 ENTER

Read off the mean as $\bar{x} = 6$

Question 2 E

Use the same screen from the calculator. The sample standard deviation $S_x = 2.25$ $\overline{x} \pm S_x$

= 6 + 2.25

 $= 3.75 \rightarrow 8.25$

the number of scores that lie within 3.75 and 8.25 is the number of students who got a score of 4, 5, 6, 7 or 8. That is, 3 + 7 + 7 + 6 + 5 = 28

Question 3 D

From the same screen on the calculator, the upper quartile is $Q_3 = 8$

Question 4 D

This is bivariate data where time is the dependent variable. The relationship between the two variables is most easily seen in a scatter plot.

Ouestion 5 C

95% of scores lie within \pm 2 standard deviations of the mean. That is, between $45 \pm 2 \times 13$. That is, from 19 to 71. 5% of results lie outside this range. 2.5% less than 19 and 2.5% greater than 71. Therefore, 2.5% lie above 71.

Question 6 D

 $r^2 = 0.49$

$$\Rightarrow r = \pm \sqrt{0.49} = \pm 0.7$$

But from the graph we can see that the correlation is negative.

Therefore, r = -0.7

Question 7 E

This box plot is negatively skewed because the median is up towards the right hand end of the box plot. 25% of the results lie within each whisker. The percentage of results in the box = 50% and the percentage of results in both whiskers together = 50%. 12 is the upper quartile so 25% + 50% = 75% of results are less than this value.

Question 8 D

Use your graphics calculator and enter the data using stat – edit.

Then use stat – calc – linear regression.

L2, L1 if you entered number of units sold (*y*) in list 2. Press ENTER.

This gives y = 15.66 + 0.56x

Question 9 A	Question 10 E
Using the same screen, read $r = 0.51$	The given graph is $y = \frac{1}{x}$
	If we plot y against $\frac{1}{x}$, we will get a straight line.
Question 11 A	Question 12 A
Taking the values for 2000, 2001 and 2002 gives	The sum of the seasonal indices $= 4$.
23, 21 and 38. The median of 21, 23, 38 is 23	Therefore, seasonal index for winter
Therefore, sales = $23 \times 100,000 = \$2,300,000$	=4-(1.3+1.5+0.7)=0.5
	Question 13 D
	Seasonally adjusted value
	1.2
	$=\frac{1}{0.7}$
	= \$1.7 million

2004 Further Mathematics Trial Examination 1 Module 1 Number patterns and applications. Suggested solutions.

Question 1 B

Arithmetic sequence

$$a = -9$$

$$d = 4$$

$$n = 20$$

$$t_n = a + (n-1)d$$

$$t_n = -9 + (20 - 1) \times 4$$

$$t_n = -9 + 19 \times 4$$

$$t_n = 67$$

Question 2 B

 S_{∞} of geometric sequence = $\frac{a}{1-r}$

$$a = 12$$

$$r = \frac{1}{3}$$

$$S_{\infty} = \frac{12}{1 - \frac{1}{3}} = \frac{12}{\frac{2}{3}} = 18$$

Question 3 D

Geometric sequence

$$a = 100$$

$$r = 0.8$$

$$n = 12$$

$$t_n = ar^{n-1}$$

$$t_{12} = 100 \times 0.8^{11}$$

$$t_{12} = 8.6$$

Question 4 E

A straight line graph represents an arithmetic sequence. Because the graph is decreasing, the common difference is negative.

Question 5 E

$$ar^2 = 27$$
 (1)

$$ar^5 = 8$$
 (2)

$$(2) \div (1) \rightarrow r^3 = \frac{8}{27}$$

$$\Rightarrow r = \sqrt[3]{\frac{8}{27}} = \frac{\sqrt[3]{8}}{\sqrt[3]{27}} = \frac{2}{3}$$

Substituting in (1)

$$\Rightarrow a \times \frac{4}{9} = 27$$

$$\Rightarrow a = 27 \div \frac{4}{9} = 60 \frac{3}{4}$$

Question 6 D

Bananas Oranges Apples 5 : 4

4 3 : 2

Make number of oranges = 12 for both ratios.

 5×3 : 4×3

 3×4 : 2×4

15 : 12 : 8

so the ratio of Bananas to Apples is 15:8

Question 7 C

$$S_n = \frac{n}{2} \left[2a + (n-1)d \right]$$

$$S_5 = \frac{5}{2} [2a + 4d] = 75$$

$$\Rightarrow 5a + 10d = 75$$
 (1)

$$S_{10} = \frac{10}{2} [2a + 9d] = 175$$

$$\Rightarrow 10a + 45d = 175 \quad (2)$$

$$(1) \times 2 \rightarrow 10a + 20d = 150$$
 (3)

$$(2) - (3) \rightarrow 25d = 25$$
$$\Rightarrow d = 1$$

Substituting in (1)

$$5a = 65$$

$$\Rightarrow a = 13$$

$$S_{15} = \frac{15}{2} [2 \times 13 + 14 \times 1]$$

$$\Rightarrow S_{15} = \frac{15}{2} \times 40 = 300$$

Question 8 C

This is a geometric sequence with

$$a = 2, r = -3$$

$$t_n = ar^{n-1} = 13122$$

$$\Rightarrow 2 \times (-3)^{n-1} = 13122$$

$$\Rightarrow (-3)^{n-1} = 6561$$

Use y = on the graphics calculator

and enter
$$(-3) \wedge X$$

Press second table and read the value of X

that gives
$$y = 6561$$

This is
$$X = 8$$

$$\Rightarrow n-1 = 8$$

$$\Rightarrow n = 9$$

Question 9 D

Geometric sequence

$$a = 6$$

$$r = 1 - 0.05 = 0.95$$

$$t_n = ar^{n-1}$$

$$t_6 = 6 \times (0.95)^5$$

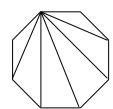
$$t_6 = 4.6$$

2004 Further Mathematics Trial Examination 1 Module 2 Geometry and trigonometry. Suggested solutions.

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Question 1 D

The sum of the angles in each triangle = 180°



Therefore, the sum of the angles in 6 triangles

$$= 6 \times 180 = 1080^{0}$$

In the regular octagon, there are 8 equal angles. Therefore, the size of each angle

$$=\frac{1080}{8}=135^{\circ}$$

Question 2 E

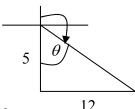
Volume of cylinder $A = \pi r^2 h$

Volume of cylinder B = $\pi (3r)^2 \times 2h = 18\pi r^2 h$

Ratio
$$V_A : V_B = \pi r^2 h : 18 \pi r^2 h$$

$$\Rightarrow$$
 Ratio is 1:18

Question 3 B



 $\tan \theta = \frac{12}{5}$

$$\theta = \tan^{-1}\left(\frac{12}{5}\right) = 67.38^{\circ}$$

True bearing = $90^{\circ} + (90 - 67.38)^{\circ}$

 $=112.62^{\circ}$

which is closest to 113°

Question 4 C

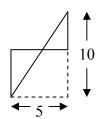
Remaining angle of triangle

$$=180-(80+38)$$

Using Sine Rule

$$\frac{x}{\sin 62^{\circ}} = \frac{12}{\sin 38^{\circ}}$$
$$\Rightarrow x = \frac{12\sin 62^{\circ}}{\sin 38^{\circ}}$$
$$\Rightarrow x = 17.2$$

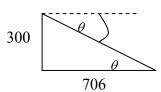
Question 5 B



$$x^2 = 5^2 + 10^2 = 125$$

$$x = \sqrt{125} = 11.18$$

Question 6 A



 θ is the angle of depression.

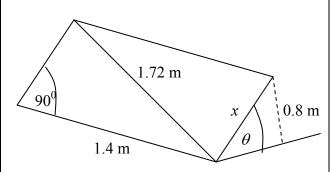
$$\tan \theta = \frac{300}{706}$$

$$\theta = \tan^{-1}\left(\frac{300}{706}\right) = 23^{\circ}$$

2004 Further Mathematics Trial Examination 1 Module 2 Geometry and trigonometry. Suggested solutions.

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Question 7 C



Using Pythagoras' Theorem

$$x = \sqrt{1.72^2 - 1.4^2} = 0.999$$

$$\sin\theta = \frac{0.8}{0.999}$$

$$\Rightarrow \theta = \sin^{-1}\left(\frac{0.8}{0.999}\right)$$

$$\Rightarrow \theta = 53^{\circ}$$

Question 8 B

Using the Cosine Rule

$$2^{2} = 5^{2} + 4^{2} - 2 \times 5 \times 4 \cos \theta$$

$$4 = 25 + 16 - 40 \cos \theta$$

$$\Rightarrow 40\cos\theta = 41 - 4$$

$$\Rightarrow 40\cos\theta = 37$$

$$\Rightarrow \cos \theta = \frac{37}{40}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{37}{40}\right)$$

$$\Rightarrow \theta = 22.3^{\circ}$$

Question 9 C

Triangles ABE and CDE are similar (AAA)

Therefore, their sides are in the same ratio.

Let DE = x cm

Therefore, AE = 25 - x

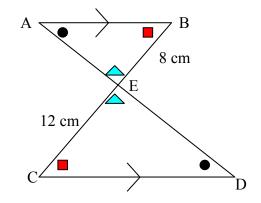
$$\Rightarrow \frac{8}{12} = \frac{25 - x}{x}$$

$$\Rightarrow 8x = 12(25 - x)$$

$$\Rightarrow 8x = 300 - 12x$$

$$\Rightarrow 20x = 300$$

$$\Rightarrow x = \frac{300}{20} = 15$$



Question 1 L	Q	uestion	1	\mathbf{E}
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Highest value of the graph is 600. This is the greatest distance from his house. Therefore, the shop is 600 m from his house.

Question 2 D

It takes 18 minutes to reach the shop. He returns from the shop in half this time. That is, he takes 9 minutes to return home. He leaves the shop after 20 minutes.

So time to reach home = 20 + 9 = 29 minutes.

Ouestion 3 A

He travelled 100 m in 6 minutes.

Therefore, speed = $\frac{100}{6}$ = $16\frac{2}{3}$ m/min

Question 4 B

The break even point is where the two graphs intersect.

Question 5 C

 $60 < 8x + 4 \le 120$

$$\Rightarrow$$
 60 - 4 < 8x + 4 - 4 \le 120 - 4

$$\Rightarrow 56 < 8x \le 116$$

$$\Rightarrow \frac{56}{8} < \frac{8x}{8} \le \frac{116}{8}$$

$$\Rightarrow$$
 7 < $x \le 14.5$

Question 6 A

A straight line graph has the form y = mx + cc is the y intercept = 1 so y = mx + 1m is the gradient of the line joining the points

$$(0,1)$$
 and $(-\frac{1}{2},0)$

$$m = \frac{1-0}{0--\frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2$$

$$\Rightarrow$$
 $y = 2x + 1$

Question 7 A

$$x \ge 0$$

$$y \ge 0$$
 : not E

$$x + 2y \le 8$$

$$2y \le -x + 8$$

$$y \le -\frac{1}{2}x + 4$$

This line joins the points (0,4) and (8,0) Less than this line. Therefore, not C or D.

$$2x - y \le 6$$

$$-y \le -2x + 6$$

$$y \ge 2x - 6$$

This line joins the points (0,-6) and (3,0) Greater than this line. Therefore, A

Question 8 D

$$x + 2y = 8$$
 (1)

$$2x - y = 6$$
 (2)

$$(1) \times 2$$

$$\Rightarrow 2x + 4y = 16$$
 (1a)

$$2x - y = 6 \qquad (2)$$

$$(1a) - (2)$$

$$\Rightarrow 5y = 10$$

$$\Rightarrow$$
 $y = 2$

Question 9 A

x is at least double y

$$x \ge 2y$$

z is no greater than one third x

$$z \le \frac{1}{3}x$$

If $x \ge 2y$

then $2y \le x$

so
$$y \le \frac{1}{2}x$$

If
$$z \le \frac{1}{3}x$$

then
$$\frac{1}{3}x \ge z$$

so
$$x \ge 3z$$

Therefore, A is not true.

2004 Further Mathematics Trial Examination 1 Module 4 Business-related mathematics. Suggested solutions.

= 23.35%

Question 1 E	Question 2 D
	Depreciation = 2400
$I = \frac{PRT}{100}$	
	Fraction depreciation = $\frac{2400}{10500}$
$I = \frac{5000 \times 10 \times 4}{100} = \2000	
	% depreciation = $\frac{2400}{10500} \times 100 = 22.86\%$
Value of investment	10500
= 5000 + 2000	
= \$7000	
Question 3 C	Question 4 C
	Amount depreciation
$(1.0075)^4$	20,000 20 5
$A = 23460 \left(1 + \frac{0.075}{2} \right)^4$	$=20,000 \times \frac{20}{100} \times 5$
$\Rightarrow A = \$27,181.94$	= \$20,000
, , , , , , , , , , , , , , , , , , , ,	Value of car
	=30,000-20,000
	= \$10,000
	. ,
Question 5 C	Question 6 D
Effective interest rate	$18,463 = P\left(1 + \frac{0.1}{4}\right)^{20}$
$=\frac{2n}{1}\times \text{ flat rate}$	$\left[18,403 = F \left(1 + \frac{4}{4} \right) \right]$
$=\frac{1}{n+1}\times$ flat rate	18 463
	$\Rightarrow P = \frac{10,700}{(10,1)^{20}}$
$= \frac{2 \times 36}{36 + 1} \times 12$	$\Rightarrow P = \frac{18,463}{\left(1 + \frac{0.1}{4}\right)^{20}}$
- 22 25%	4)

 \Rightarrow P = \$11,267.43

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Module 4 Business-related mathematics. Suggested solutions.

Question 7 E

Minimum amount for February is the amount in the account from 1 Feb to 19 Feb 19 inclusive. This equals the amount in the account after 24 Jan = \$15,631.46

Ouestion 8 A

Minimum amount in the account in December = \$14,247.19

Interest =
$$\frac{0.05}{12} \times 14247.19 = $59.36$$

Question 9 B

Cash deposit =
$$\frac{10}{100} \times 3500 = $350$$

Interest paid on balance of 3500 - 350 = 3150

Interest =
$$3150 \times \frac{8.5}{100} \times 3 = 803.25$$

Amount to be paid back = 3150 + 803.25 = 3953.25

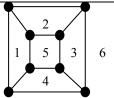
Number of repayments = $3 \times 12 = 36$

Value of each repayment = $3953.25 \div 36 = 109.82

Question 1 D

A Hamiltonian Path passes through each vertex once only, starting and finishing at different vertices. This cannot be done with diagram D.

Question 2 C



Number of faces = 6

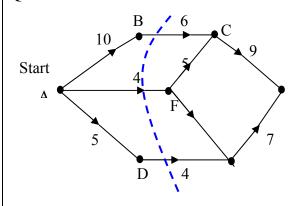
Question 3 B

2 + 2 + 6 + 3 = 13

Ouestion 4 E

For an Eulerian circuit to exist, all vertices must be of even degree. Since D and A are of odd degree, adding an edge joining A to D would make both these vertices even.

Question 5 B



Maximum flow = minimum cut Minimum cut = 6 + 4 + 4 = 14Therefore, maximum flow = 14

Ouestion 6 D

DE can hold 4 and the maximum amount of 4 can flow along here when maximum flow is occurring.

Oraștian 7 C	Owestion 9 E
Question 7 C	Question 8 E
From A to $A = 0$	Circle the smallest value in each row
From A to $B = 2$	5 4 (3) 6
From A to $C = 0$	9 10 6 9
So first row is 0 2 0 therefore, not B	10 13 8 7
From B to $A = 1$	6 9 10 10
From B to $B = 0$	Subtract this circled value from each value in its
From B to $C = 1$	row.
So second row is 1 0 1 therefore, C	2 ① ① 3
	3 4 0 3
	3 6 1 ①
	0 3 4 4
	Circle the smallest value in each column.
	Subtract this value from each value in its
	column.
	D
	3 3 0 3
	3 5 1 0
	0 2 4 4
	Take a row and column where only one zero
	occurs. (See the lines drawn above)
	⇒Jake makes the Racing Bike
	⇒Ida makes the Super Athlete Bike
	⇒Greg makes the Mountain bike
	Therefore, Helen makes the Non-geared Bike.
	Therefore, E
Question 9 C	THOUSION, L
Earliest time G can start is when C and F are	
both finished. C finishes in 8 hours. F finishes in	
both finished. C finishes in 8 hours. F finishes in $4 + 3 + 2 = 9$ hours. Therefore G must wait to	
start until F finishes. So, the earliest G can start is 9 hours from the beginning.	

End of suggested solutions 2004 Further Mathematics Trial Examination 1

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