INSIGHT

Trial Exam Paper

2007

FURTHER MATHEMATICS

Written examination 2

Worked solutions

This book presents:

- worked solutions, giving you a series of points to show you how to work through the questions.
- mark allocations
- tips and guidelines

Core

Question 1

The theme park Rollercoaster World earns revenue from attendance sales, merchandise sales and food sales. One aspect of this revenue is the three ice-cream stalls situated within the park.

In order to analyse the performance of the three ice-cream stalls the owner of Rollercoaster World records how many ice-creams each stall sells each day for one week. This data is presented in Table 1 below.

Table 1

	Number of ice-creams sold		
Day of Week	Stall A	Stall B	Stall C
Monday	562	532	362
Tuesday	388	481	305
Wednesday	575	599	416
Thursday	492	63	365
Friday	549	587	425
Saturday	641	683	488
Sunday	617	721	457

1a. Complete Table 2 by calculating the mean number of ice-creams sold by Stall A. Write your answer correct to one decimal place.

Table 2

Number of ice-creams sold	Stall A	Stall B	Stall C
mean	**546.3**	523.7	402.6
standard deviation	84.7	219.1	62.6

1 mark

Worked solution

Either use the appropriate function on your calculator OR

$$\bar{x} = \frac{562 + 388 + 575 + 492 + 549 + 641 + 617}{7}$$

$= 546.285.....$

$= 546.3(1dp)$

Mark Allocation

- 1 mark for correct answer

CORE – continued

1b. Use only the data in Table 2 to state which stall was the most successful in terms of the number of ice-creams sold. Justify your answer.

Solution

Stall A is the most successful as it sold more ice-creams, as evidenced by the largest mean.

1 mark

Mark allocation

- 1 mark only if both the correct answer and justification are given

Stall B appears to have a relatively high standard deviation. The owner believes the main reason for this is because on one particular day a major ride located near to stall B was closed for repairs, and so the stall had far less customers than normal.

1c. Identify which day the ride near stall B was closed and prove that the number of ice-creams sold from stall B on this day could be considered to be an outlier.

Worked solution

Thursday is the day and the sales for that day are below the lower value to identify outliers

$\bar{x} - 2s = 523.7 - 2 \times 219.1$
$= 85.5$

OR

$Q_1 - 1.5 \times IQR = 481 - 1.5 \times 202$
$= 178$

2 marks

Mark allocation

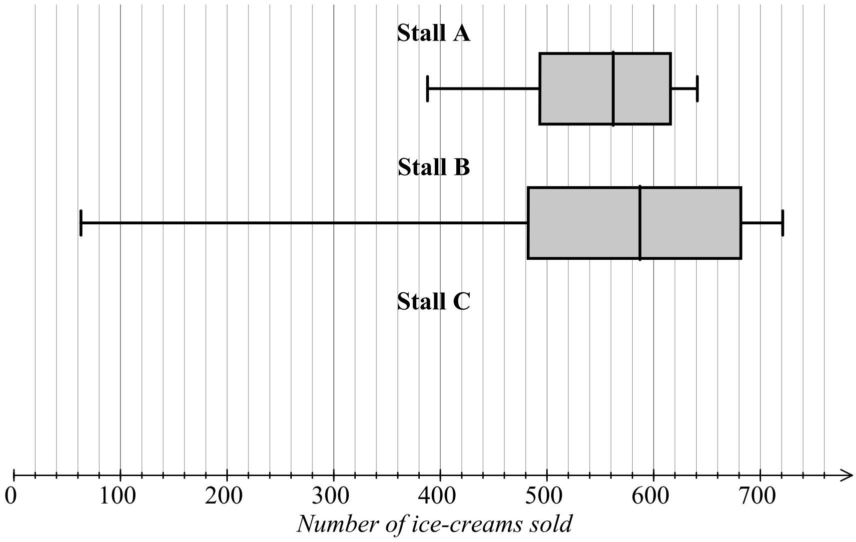
- 1 mark for identifying Thursday
- 1 mark for carrying out outlier calculation and comparing it with the value of ice-creams sold that day (63)

Tip

- *You would probably use the mean and standard deviation to prove the value is an outlier as you have already been given all the required values in previous questions, but there is no reason why the alternative calculation using the median and interquartile range cannot be used.*
- *The outlier calculation alone is not enough. You must conclude the value of 63 is an outlier as it lies below the lower limit for outliers in order to gain the second mark.*

Using the data from Table 1, boxplots are constructed to display the distributions of ice-cream sales for stalls A and B as shown in Boxplot 1.

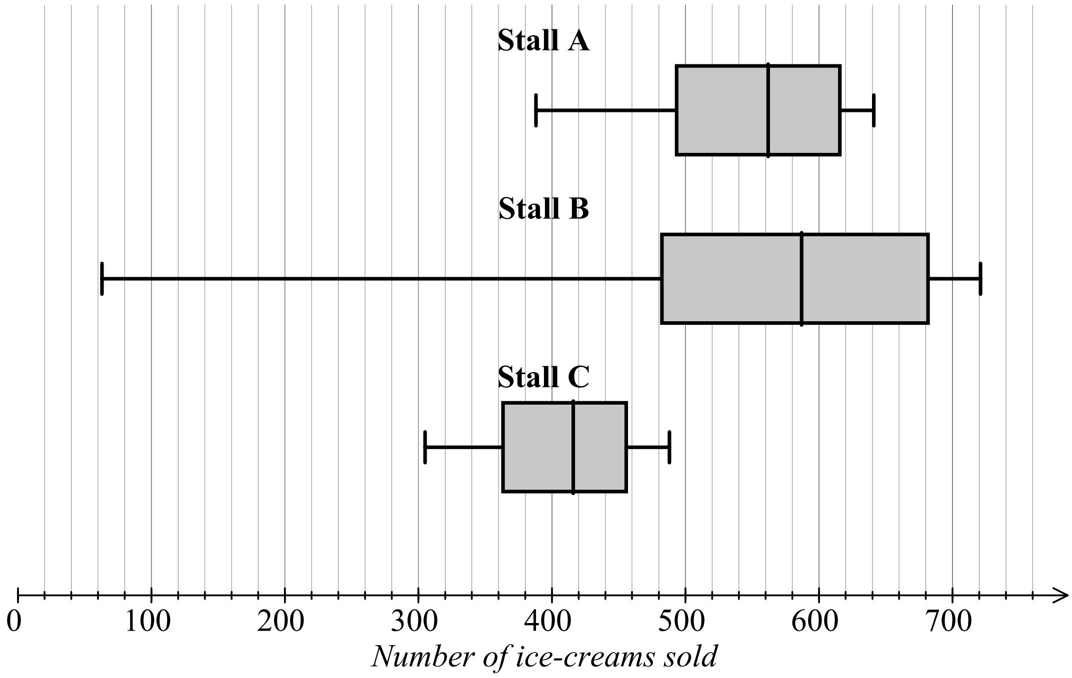
Boxplot 1



1d. Complete Boxplot 1 by constructing and drawing a boxplot that shows the distribution of ice-cream sales for stall C.

Worked solution

Boxplot 1



Five figure summary is Min = 305, Q_1 = 362, Median = 416, Q_3 = 457, Max = 488

1 mark

Mark allocation

- 1 mark if only one value incorrect, 0 marks if more than one value incorrect

Tip

- *Care should be taken to draw any graph as accurately as possible in order to ensure full marks. Correct equipment, such as a ruler, should be used for the same reason.*

1e. Does Boxplot 1, in particular the median values, support your statement in part (b), which stated which of the three stalls was the most successful in terms of most ice-creams sold? Explain your answer.

Solution

Stall B has the highest median (587) therefore it can be concluded that this stall is the most successful. This contradicts the answer given in part (b) which claimed stall A was the most successful.

1 mark

Mark allocation

- 1 mark for concluding that stall B is the correct choice and relating it back to the answer given in part (b)

Tip

- *Questions such as "explain your answer" or "justify your answer" will not gain full marks, or indeed any marks, without an explanation to accompany the correct answer.*

The owner of Rollercoaster World correctly concludes that the median is more appropriate than the mean as a measure of central tendency for this data

1f. Explain why the owner's conclusion is correct.

Solution

The mean is affected by outliers and so can be unreliable when outliers are present. Outliers are present in the data for stall B, hence the median is a more reliable measure of central tendency.

1 mark

Mark allocation

- 1 mark for correct explanation

Question 2

The attendance figures for Rollercoaster World from 2004 to 2006 are given in Table 3.

Table 3

Year	2004				2005				2006			
Season	Aut	Win	Spr	Sum	Aut	Win	Spr	Sum	Aut	Win	Spr	Sum
Attendance (000s)	382	242	351	465	416	309	366	515	429	347	413	562

This data will be used to predict future attendance at the theme park.

2a. In this analysis, the **dependent** variable is [ ]

Worked solution

Attendance

1 mark

Mark allocation

- 1 mark for using the word attendance

Tip

- *Ensure you read this type of question carefully as you may be asked for either the dependent or independent variable*
- *You must state what the dependent variable is. A common mistake is to answer "on the y-axis" as the dependent variable is always the y-axis. This is not an acceptable answer.*
- *If one of the variables is time, this is always the independent variable.*

To start the analysis, the time values have been rescaled as $x = 1$ to $x = 12$ (autumn 2004 = 1, winter 2004 = 2, and so on). This data is displayed in Table 4.

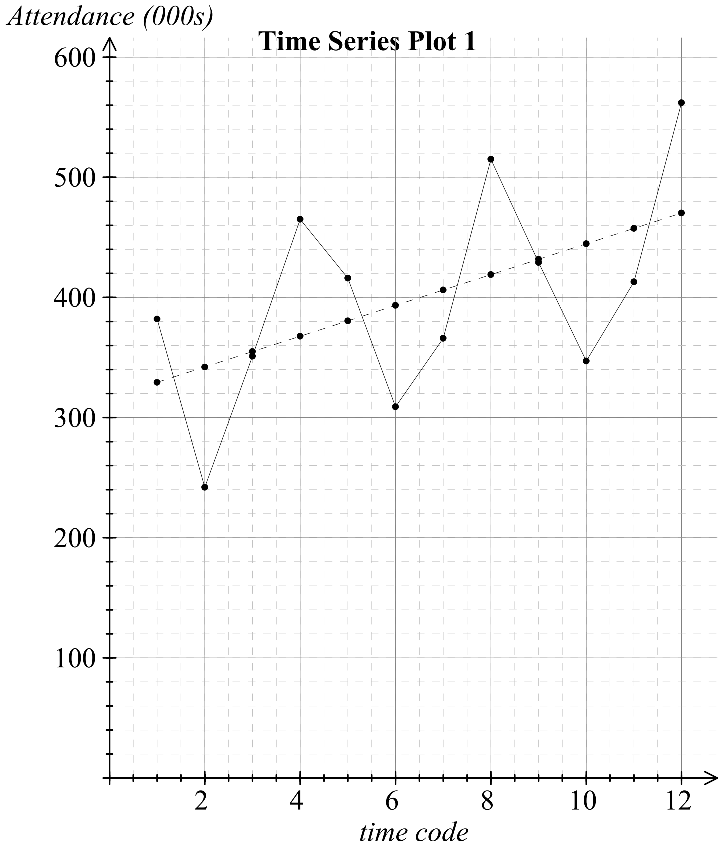
Table 4

Time code	1	2	3	4	5	6	7	8	9	10	11	12
Attendance (000s)	382	242	351	465	416	309	366	515	429	347	413	562

This rescaled data is displayed in Time Series Plot 1 below. Also displayed is the least squares regression line for the rescaled data.

The equation for the least squares regression line is

$$attendance(000s) = 316.5 + 12.8 \times timecode$$



CORE – continued

2b. Use the information in Time Series Plot 1 to describe the relationship between *attendance* and *time* in terms of **form** and **direction**.

Worked solution

The data is seasonal with an increasing trend.

2 marks

Mark allocation

- 1 mark for seasonal
- 1 mark for increasing trend

Tip

- *Note words in the question which are highlighted in* ***bold*** *indicate specific key points which must be addressed in your answer.*

2c. Using the regression equation $attendance(000s) = 316.5 + 12.8 \times timecode$ to model attendances at Rollercoaster World, we would say that the average increase in attendances was $\boxed{}$ people per season.

Worked solution

12 800 people

1 mark

Mark allocation

- 1 mark for correct value provided the value has been adjusted according to attendances being quoted in 1000s. 12.8 is not acceptable

2d. Complete Table 5 by calculating the seasonal index for winter.

Table 5

Season	Index
Autumn	1.026
Winter	**0.745**
Spring	0.943
Summer	1.286

Worked solution

$4 - (1.026 + 0.943 + 1.286) = 0.745$

1 mark

Mark allocation

- 1 mark for correct answer

Tip

- *The sum of the seasonal indices equals the number of seasons*

Using the seasonal indices in Table 5 the attendances from Table 3 are deseasonalised.

2e. Complete Table 6 by calculating the deseasonalised attendance value for spring 2006.

Table 6

Year	2004				2005				2006			
Season	Aut	Win	Spr	Sum	Aut	Win	Spr	Sum	Aut	Win	Spr	Sum
Time code	1	2	3	4	5	6	7	8	9	10	11	12
Actual attendance (000s)	382	242	351	465	416	309	366	515	429	347	413	562
Deseasonalised attendance (000s)	372	325	372	362	406	415	388	400	418	466	**438**	437

Worked solution

Actual value divided by season index $= 413 \div 0.943 = 438$

1 mark

Mark allocation

- 1 mark for correct value

The least squares regression line for the deseasonalised data is

$$deseasonalised \quad attendance\,(000s) = 340.3 + 9.2 \times timecode$$

2f. Use the regression equation for the deseasonalised data to predict the actual attendance in summer 2007.

Worked solution

Summer 2007 = time code 16

$deseasonalised \quad attendance(000s) = 340.3 + 9.2 \times 16 = 487.5$

Actual attendance = $487.5 \times 1.286 \times 1000 = 626925$ people

2 marks

Mark allocation

- 1 mark using correct time code of 16
- 1 mark for correct answer (accept rounding to nearest thousand)

Total 15 marks

END OF CORE

Module 1: Number patterns

Question 1

Tree conservation is an important aspect of protecting the world environment. The company Timber Resources Ensuring Environmental Sustainability (or TREES for short) supplies wood for building but replants a tree for every one it cuts down. This not only sustains the tree population but also provides a continuous supply of new trees which can grow and then be cut down.

When initially planted the tree samplings have a diameter of 6 cm, with the diameter increasing by 2.4 cm per year.

1a. What would be the diameter today of a tree planted five years ago?

Worked solution

Diameter = $6+5\times 2.4=18\,\text{cm}$

1 mark

Mark allocation

- 1 mark for correct answer

1b. How many years after planting would a tree have a radius of 18.6 cm?

Worked solution

Diameter = $2\times 18.6=37.2\,\text{cm}$

$t_n=a+d\times n$

$37.2=6+2.4\times n$

$\therefore 2.4\times n=31.2$

$\therefore n=\dfrac{31.2}{2.4}$

$\therefore n=13$

Tree has radius of 18.6 cm after 13 years.

2 marks

Mark allocation

- 1 mark for using correct method of reversing the calculation/equation
- 1 mark for accurate answer

Tip

- *Alternative method: Original radius is 3 cm and radial growth is 1.2 cm per year hence,*
  $t_n=a+d\times n$
  $18.6=3+1.2\times n$
  $\therefore 1.2\times n=15.6$
  $\therefore n=\dfrac{15.6}{1.2}$
  $\therefore n=13$

1c. A difference equation that generates the terms of this sequence of diameters is

$$t_{n+1} = rt_n + d \qquad \text{where} \qquad t_1 = 6$$

What are the values of r and d?

$r =$ $\boxed{}$ $d =$ $\boxed{}$

Worked solution

As the sequence is arithmetic then d = common difference = 2.4

For the same reason then the sequence is not geometric hence $r = 1$

2 marks

Mark allocation

- 1 mark each for correct values for r and d

Tip

- *If the sequence is only arithmetic then r = 1*
- *If the sequence is only geometric then d = 0*

Question 2

The newly planted tree samplings from question 1 begin with a height of 1.23 m.

The height growth for the trees in the first three years after planting is shown in Table 1

Table 1

Year	1	2	3
Height growth (m)	2	1.8	1.62

2a. Show that the terms of this height growth sequence are geometric in nature

Worked solution

$\frac{t_2}{t_1} = \frac{1.8}{2} = 0.9 \qquad \frac{t_3}{t_2} = \frac{1.62}{1.8} = 0.9$

Same common ratio of 0.9 hence the sequence is geometric in nature

1 mark

Mark allocation

- 1 mark awarded only if both ratio calculations are shown and the conclusion is present

2b. What is the percentage decrease in height growth from one year to the next?

Worked solution

The height growth for the next year is 90% ($r = 0.9$) of the previous year, hence the percentage decrease is 10% (100 - 90)

1 mark

Mark allocation

- 1 mark for correct percentage

2c. What would the height growth be in the seventh year in metres correct to 2 decimal places?

Worked solution

$t_n = ar^{n-1}$

$t_7 = 2 \times 0.9^{7-1}$

$t_7 = 1.062882$

Hence the tree height would increase by 1.06 m in the seventh year

1 mark

Mark allocation

- 1 mark for correct answer

2d. If a tree continues to grow according to this pattern, what is the maximum possible height of the tree?

Worked solution

$S_\infty = \dfrac{a}{1-r}$

$S_\infty = \dfrac{2}{1-0.9}$

$S_\infty = 20$

Initial height was 1.23 m, hence maximum possible height = 1.23 + 20 = 21.23 m

2 marks

Mark allocation

- 1 mark for sum to infinity
- 1 mark for correct final answer

2e. The ‘TREES’ company requires the trees to reach a minimum height of 18 m and a minimum diameter of 50 cm before they can be harvested for timber. What is the minimum number of years after planting that the trees would satisfy these conditions for harvest?

Worked solution

Compare both sequences using your graphics calculator. The equation representing the diameter has probably already been used in question 1 and is $Y_1 = 6 + 2.4X$. The equation for the height requires you to add the initial height of 1.23 m to the growth which is the sum of a geometric series hence $Y_2 = 1.23 + \frac{2(1-0.9^X)}{1-0.9}$. Enter these equations into your graphics calculator and use the table function to compare:

X	Y_1	Y_2
15	42	17.112
16	44.4	17.524
17	46.8	17.895
18	49.2	18.228
19	51.6	18.528
20	54	18.798
21	56.4	19.042

From the table, the minimum height of 18 cm is reached during the 18th year and the minimum diameter is reached during the 19th year so the trees are ready for harvest 19 years after planting.

2 marks

Mark allocation

- 1 mark for comparing the two sequences
- 1 mark for correct answer

Question 3

The ‘TREES’ company decides to change its policy on harvesting and replanting. In its new policy the company commits cutting down only 5% of its tree population during the year and then planting 30 000 new trees just before the end or each year. The initial number of trees planted was 200 000 trees.

3a. Write a difference equation that specifies the number of planted trees after *n* years.

Worked solution

$T_1 = 200000$ (this was how many trees we started with)

$r = 0.95$ (a 5% decrease means 95% of trees remain, hence geometric component is 0.95)

$d = 30000$ (planting 30 000 new trees each year is arithmetic in nature)

$$T_{n+1} = 0.95T_n + 30000 \qquad \text{where} \qquad t_1 = 200000$$

1 mark

Mark allocation

- 1 mark for correct answer

3b. How many trees will there be after three years of this policy?

Worked solution

$T_2 = 0.95 \times 200000 + 30000$ $T_3 = 0.95 \times 220000 + 30000$ $T_4 = 0.95 \times 239000 + 30000$

$T_2 = 220000$ $T_3 = 239000$ $T_4 = 257050$

Hence there will be 257050 trees after 3 years.

1 mark

Mark allocation

- 1 mark for correct answer

3c. Explain why the number of trees will never exceed 600 000 trees.

Worked solution

5% of 600 000 is 30 000, hence when the number of trees planted is 600 000 then the number of trees cut down would equal the number of trees planted each year and so the number of trees would never exceed 600 000.

1 mark

Mark allocation

- 1 mark for suitable explanation

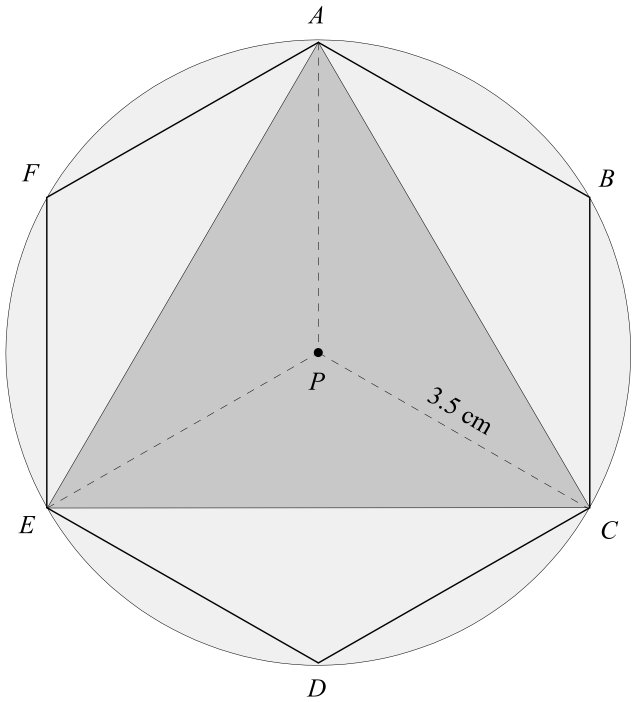
Total 15 marks

END OF MODULE 1

Module 2: Geometry and trigonometry

Question 1

'LOST?' - the orienteering and adventure club, has a logo consisting of an equilateral triangle, inside a regular hexagon, within a circle. All three shapes have the same centre P and the distance from P to where one of the corners of both the triangle and hexagon meet the edge of the circle at C is 3.5 cm.



1a. What is the area of the circle correct to two decimal places?

Worked solution

$A = \pi r^2$

$A = \pi \times 3.5^2$

$A = 38.4845...$

Hence area of circle is 38.48 cm^2

1 mark

Mark allocation

- 1 mark for correct answer

1b. What is the size of angle APC?

Worked solution

Three equal angles at centre P so $\angle APC = 360 \div 3 = 120°$

1 mark

Mark allocation

- 1 mark for correct angle

1c. Determine the area of triangle *ACE* correct to 2 decimal places.

Worked solution

The area of triangle *ACE* is three times bigger than the area of triangle *APC* hence,

$$A = 3 \times \frac{1}{2} ac \sin P$$

$$A = 3 \times \frac{1}{2} \times 3.5 \times 3.5 \times \sin 120$$

$$A = 3 \times 5.3044....$$

$$A = 15.9132...$$

Hence the area of triangle *ACE* is 15.91 cm^2

1 mark

Mark allocation

- 1 mark for the correct answer

Question 2

As well as being a logo on the LOST? uniform, the logo is copied to form a circular landscape design outside the LOST? offices. The landscape feature is an exact enlargement of the design in the diagram with the equivalent distance *PC* being 7 m.

2a. What is the linear scale factor of this enlargement?

Worked solution

$$k = 700 \div 3.5$$

$$k = 200$$

1 mark

Mark allocation

- 1 mark for correct answer

In the landscape design the equilateral triangle is a pond with a depth of 50 cm.

2b. Determine the volume of water required to fill the pond to the nearest cubic metre.

Worked solution

As we are enlarging an area we must square *k* in order to get the scale factor for area.

$k^2 = 200^2$ $\quad A_{POND} = 15.91 \times 40000$ $\quad V_{POND} = 636400 \times 50$

$k^2 = 40000$ $\quad A_{POND} = 636400cm^2$ $\quad V_{POND} = 31820000cm^3$

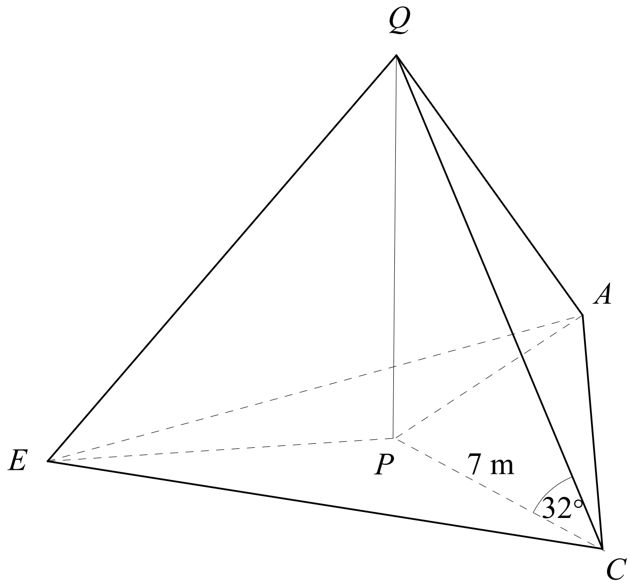
Volume of pond = 31 820 000 / 1 000 000 = 31.82 = 32 m^3

2 marks

Mark allocation

- 1 mark for using area scale factor
- 1 mark for correct volume

A fountain shoots vertically into the air from the centre of the pond. The angle of elevation of the tip of the fountain Q from a corner of the pond is 32˚



2c. Determine the height of the fountain to the nearest centimetre.

Worked solution

$\tan C = \dfrac{PQ}{PC}$

$\tan 32° = \dfrac{PQ}{7}$

$\therefore PQ = 7 \times \tan 32°$

$\therefore PQ = 4.3740....$

Hence the height of the fountain is 4.37 m to the nearest centimetre.

Accept 437 cm.

1 mark

Mark allocation

- 1 mark for correct answer

Tip

- *Note when the units are in metres and you are asked to round to the nearest centimetre then you can either round the answer to 2 decimal places as your answer is in metres or convert the solution into centimetres hence accept 437 cm.*

Question 3

The LOST? offices *X* are near the base of a mountain. As part of the organisation's orienteering courses they also have a checkpoint *Y* somewhere up the mountain. The relative positions of the office and the checkpoint are shown on the contour map in figure 1.

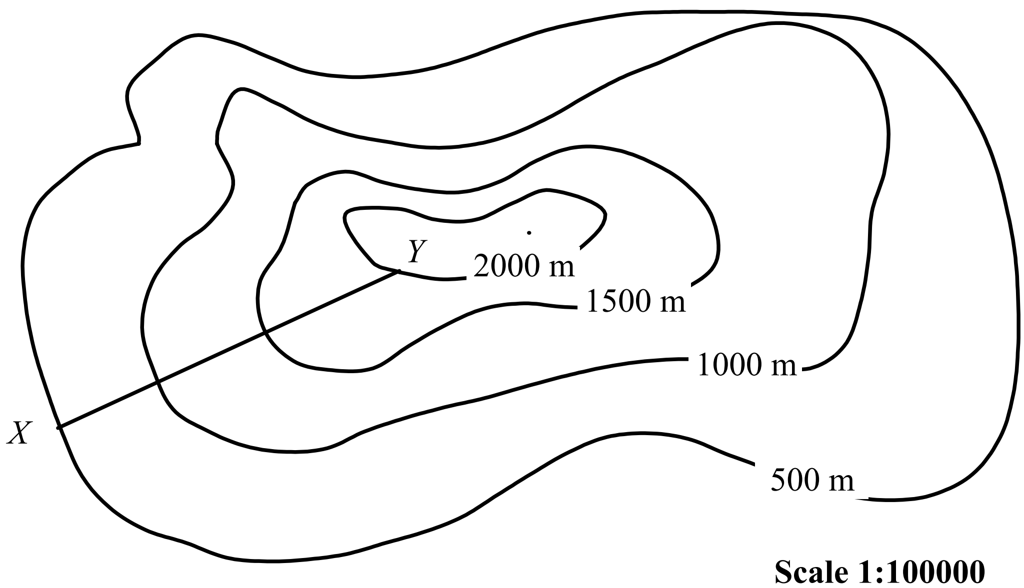


Figure 1

3a. If the map distance *XY* is 5 cm then what is the real life horizontal distance from *X* to *Y* to the nearest kilometre?

Worked solution

$XY = 5 \times 100000$

$= 500000 cm$

This distance when divided by 100 and then by 1000 is equal to 5 km.

1 mark

Mark allocation

- 1 mark for correct answer

3b. Determine the vertical height change between office *X* and checkpoint *Y* in metres.

Worked solution

The difference between contour lines is 2000 – 500 = 1500 m.

1 mark

Mark allocation

- 1 mark for correct answer

3c. Find the average slope between office X and checkpoint Y? Write your answer correct to one decimal place.

Worked solution

$$slope = \frac{1.5}{5} = 0.3$$

1 mark

Mark allocation

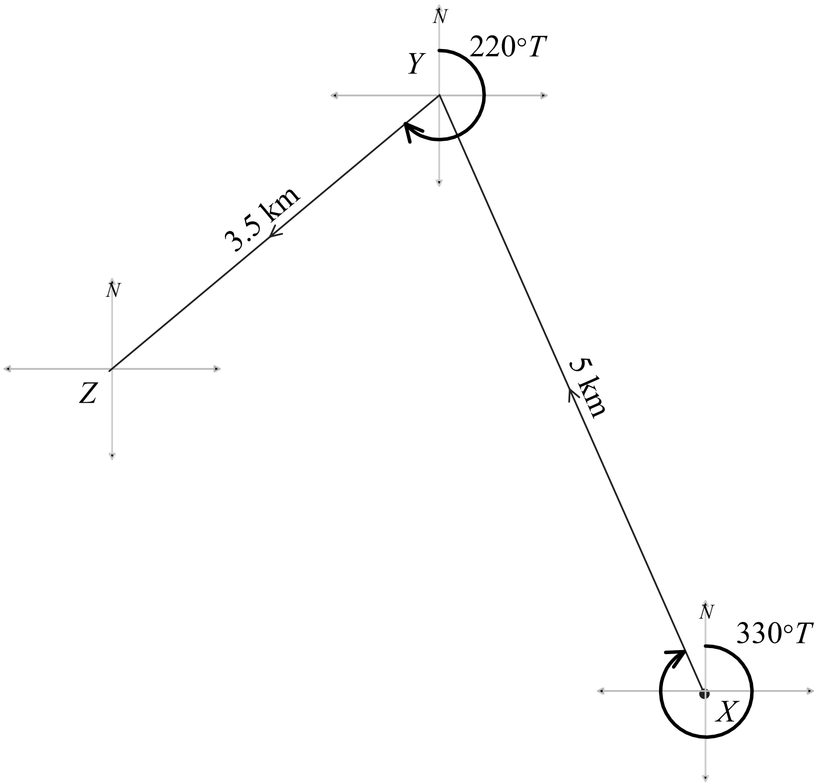
- 1 mark for correct answer

Tip

- *Note we use the two answers from 3a and 3b but the units must be consistent hence in the worked solution we use 1.5 km as the vertical height change as opposed to 1500 m. Ensure units are consistent (the same type) before executing any calculation.*

Question 4

A party goes on an orienteering trip. On the first part of the journey they travel 5 km on a bearing of 330˚T from office X to checkpoint Y. The second leg of the course is a 3.5 km journey on a bearing of 220˚T from Y to a second checkpoint Z. On the third leg they return directly from checkpoint Z to office X.



4a. State angle XYZ

Worked solution

Angle XYZ = 220 – 150 = 70˚

1 mark

Mark allocation

- 1 mark for correct answer

4b. What is the direct distance from checkpoint *Z* to office *X*? Write your answer in kilometres correct to three decimal places.

Worked solution

$XZ = \sqrt{XY^2 + YZ^2 - 2 \times XY \times YZ \times \cos Y}$

$= \sqrt{5^2 + 3.5^2 - 2 \times 5 \times 3.5 \times \cos 70°}$

$= \sqrt{25.27929...}$

$= 5.02785...$

Hence $XZ = 5.028$ km

2 marks

Mark allocation

- 1 mark for using appropriate method (cosine rule)
- 1 mark for correct answer

4c. Determine the bearing of *X* from *Z* to the nearest degree.

Worked solution

$\frac{\sin Z}{5} = \frac{\sin 70°}{5.028}$

$\therefore \sin Z = \frac{5 \times \sin 70°}{5.028}$

$\therefore \sin Z = 0.9344...$

$\therefore \angle Z = 69°$

Hence the bearing is 40 + 69 = 109˚T or N109˚E

2 marks

Mark allocation

- 1 mark for appropriate method
- 1 mark for correct answer

Tip

- *The cosine rule could also be used to find the angle at Z as you have all three sides of the triangle.*

Total 15 marks

END OF MODULE 2

Module 3: Graphs and relations

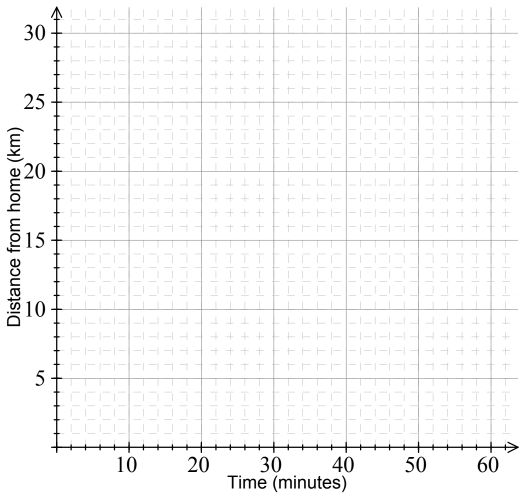
Question 1

When Andy travels from home *H* to work *W* he walks to the train station *T* and then catches a train to the city. Once in the city he does not have to travel any further as his work building is next door to the train station.

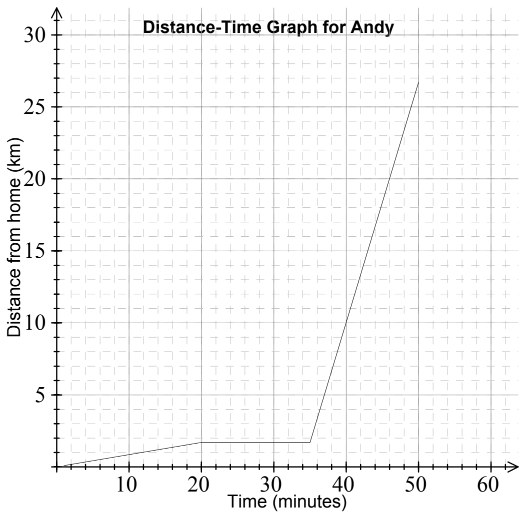
One day it takes Andy 20 minutes to walk the 1.7 km from his house to the train station, he then waits at the station for 15 minutes before catching the train which completes the remaining 25 km of the journey to work in 15 minutes.

1a. Draw and label Graph 1 which is a distance-time graph of Andy's journey to work that day.

Graph 1



Worked solution



2 marks

Mark allocation

- 1 mark for correct time intervals used
- 1 mark for showing cumulative distances up to 26.7 km

1b. Determine the average speed for the train journey (km/h)

Worked solution

15 minutes = 0.25 hours

Speed = distance / time = 25 / 0.25 = 100 km/h

1 mark

Mark allocation

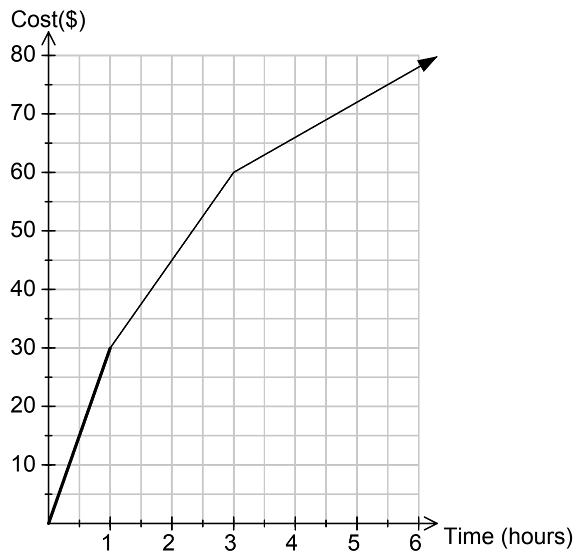
- 1 mark for correct answer only if quoted in km/h

Question 2

Andy works for a mobile phone company. They currently have a special deal for their customers who only use their mobile phones to make phone calls. As customers make use of their phones for longer amounts of time the cost per minute decreases. The cost of the calls decreases after 2 hours of use and 4 hours of use during any particular month.

Graph 2 represents total cost of calls made C($) against the total length of calls made t(minutes).

Graph 2



The same concern as above – I am concerned that the axes are not labelled – Cost($) and Time (minutes).
The equations below give the monthly cost C($) of the phone calls made, given the total time of phone calls made t(hours).

$$C = \begin{cases} at & 0 \le t \le 1 \\ bt + c & 1 < t \le 3 \\ 6t + 42 & t > d \end{cases}$$

2a. Determine the values of *a, b, c* and *d*.

$a =$ ☐ $b =$ ☐ $c =$ ☐ $d =$ ☐

Worked solution

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad y = 15x + c$$

$$m = \frac{30 - 0}{1 - 0} \qquad m = \frac{60 - 30}{3 - 1} \qquad 30 = 15 \times 1 + c$$

$$\qquad \qquad \qquad 30 = 15 + c$$

$$m = \frac{30}{1} = 30 \qquad m = \frac{30}{2} = 15 \qquad c = 15$$

$a = 30$ $b = 15$ $c = 15$ $d = 3$

3 marks

Mark allocation

- 3 marks for all correct values
- 2 marks for 3 correct values
- 1 mark for 2 correct values

2b. For the first hour of calls how much is the call cost in cents/minute?

Worked solution

$30 for 1 hour of call, therefore call cost = 30 / 60 = $0.50 = 50 cents/minute

1 mark

Mark allocation

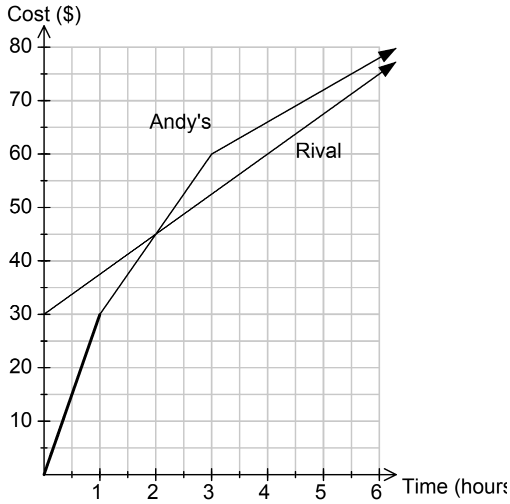
- 1 mark for correct answer

A rival mobile company has only one call cost rate no matter how much you use your phone for. The total cost of calls *C*($) in terms of time usage *t* (hours) is given by the equation,

$$C = 7.5t + 30$$

2c. Draw the line of this cost equation on the same graph as the cost graph for Andy's company (Graph 2)

Worked solution



1 mark

Mark allocation

- 1 mark for correct line

2d. Depending on the total length of phone calls made in a month Andy's company or the rival company would be cheaper. For what total length of calls is the rival company cheaper to use?

Worked solution

Graphically we can see the rival company is cheaper after 2 hours of use, however we can also see that the two lines will cross again beyond 6 hours of use so we must solve the pair of simultaneous equations - $C = 6t + 42$ and $C = 7.5t + 30$ by any means. Their point of intersection is (8, 90) so after 8 hours Andy's company is cheaper. Hence the rival company is cheaper if the phone is used for between 2 to 8 hours in the month.

3 marks

Mark allocation

- 1 mark for stating t > 2
- 1 mark for indicating an attempt to find second value
- 1 mark for stating t < 8

Question 3

The computer server room in Andy's work building has a cooling system to keep the room at a reasonable temperature so that the computer equipment does not malfunction. Unbeknownst to everybody the cooling system breaks down and so the temperature begins to rise above the safe working temperature.

Table 1 shows the temperature T (°C) above the safe working temperature after t hours.

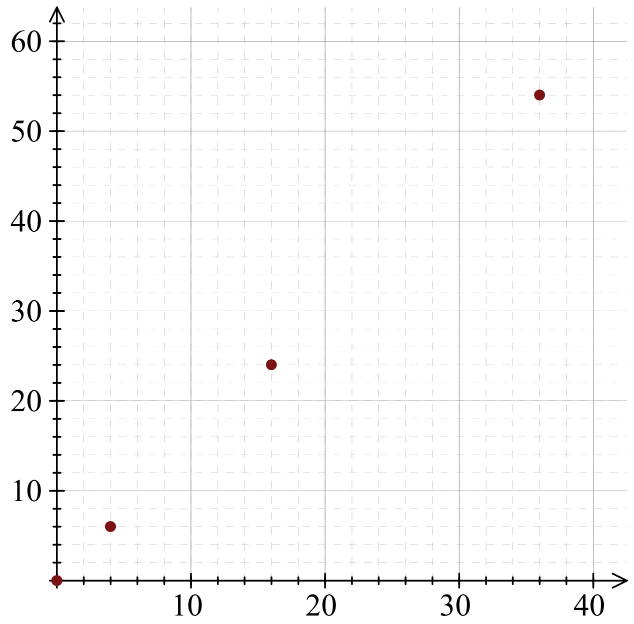
Table 1

t (hours)	0	2	4	6
t^2 (hours^2)				
Temperature T (°C)	0	6	24	54

3a. Complete Table 1 and on the set of axes below, plot the four points (t^2, T) from Table 1.

Worked solution

t (hours)	0	2	4	6
t^2 (hours^2)	**0**	**4**	**16**	**36**
Temperature T (°C)	0	6	24	54



2 marks

Mark allocation

- 1 mark for correct values in table
- 1 mark for correct plot

3b. Assume that this relation is modelled by the equation $T = kt^n$. Justify why $n = 2$.

Worked solution

$n = 2$ because the graph of T against t^2 results in a straight line plot

1 mark

Mark allocation

- 1 mark for relating the value of n to the straight line graph

3c. Given that the relation is now modelled by the equation $T = kt^2$, find the value of *k*.

Worked solution

Substitute (2, 6) into the formula:

$T = kt^2$

$6 = k \times 2^2$

$6 = k \times 4$

$k = \dfrac{6}{4}$

$k = 1.5$

1 mark

Mark allocation

- 1 mark for correct answer

Total 15 marks

END OF MODULE 3

Module 4: Business-related mathematics

Question 1

Megan's quarterly bank account statement is shown in Table 1

Table 1

Date	Transaction	Debit ($)	Credit ($)	Balance ($)
01 April 2007	Brought Forward			850.50
14 April 2007	Wage Payment		3600.00	4450.50
22 April 2007	Credit Card Payment	2436.00		2014.50
02 May 2007	ATM Withdrawal	300.00		1714.50
14 May 2007	Wage Payment		3600.00	5314.50
16 May 2007	ATM Withdrawal	**840.00**		4474.50
25 May 2007	Credit Card Payment	3215.84		1258.66
13 June 2007	Interest		102.35	**1361.01**
14 June 2007	Wage Payment		3600.00	4961.01
24 June 2007	Credit Card Payment	3005.50		1955.51

1a. Complete Table 1 by calculating the ATM withdrawal made on 16 May 2007 and the balance on 13 June 2007.

Worked solution

ATM withdrawal = 5314.50 – 4474.50 = $840.00

Balance = 1258.66 + 102.35 = $1361.01

2 marks

Mark allocation

- 1 mark for each correct value

1b. Interest on this account was based on the minimum monthly balance. If the interest rate was 0.5% per month then how much interest did Megan receive for the month of May?

Worked solution

Interest = 0.5% of 1258.66 = 0.005×1258.66 = $6.29

1 mark

Mark allocation

- 1 mark for correct answer

Question 2

Megan is planning on going on a skiing holiday. To prepare for this she buys some ski equipment and skiwear under a hire purchase agreement with a ski shop. The total value of the goods was $1200 but Megan pays a deposit of $300 and six monthly payments of $164.25

2a. Determine the total amount of interest paid.

Worked solution

Amount paid = 6 × 164.25 + 300 = $1285.50

Interest = 1285.50 – 1200 = $85.50

1 mark

Mark allocation

- 1 mark for $85.50

2b. What is the annual flat rate of interest charged?

Worked solution

$$I = \frac{\Pr T}{100}$$

$$85.50 = \frac{900 \times r \times 0.5}{100}$$

$$\therefore r = \frac{85.50 \times 100}{900 \times 0.5}$$

$$\therefore r = 19\%$$

Flat rate of interest = 19%

2 marks

Mark allocation

- 1 mark for using $T = 0.5$
- 1 mark for correct answer

Question 3

Megan's friend Charlie is also going on the skiing holiday, and so he needs to buy some ski equipment and skiwear. Charlie buys what he requires from a second hand store. The store owner claims the original cost of the goods totalled $1050 three years ago, and that they are willing to sell the goods based on a flat depreciation rate of 12% per annum.

3a. If the sale price is equivalent to the value of the goods after three years of a flat depreciation rate of 12% per annum, how much must Charlie pay?

Worked solution

$$D = \frac{\Pr T}{100}$$

$$D = \frac{1050 \times 12 \times 3}{100}$$

$$D = 378$$

Hence Charlie must pay 1050 – 378 = $672

2 marks

Mark allocation

- 1 mark for correct depreciation value
- 1 mark for correct cost of goods

3b. If the original cost of the goods was still \$1050 and the price after three years was still the answer calculated in part **3a**, but the rate was calculated on a reducing balance basis, what would be the annual depreciation rate? Write your answer correct to one decimal place.

Worked solution:

$672 = 1050 \times x^3$

$0.64 = x^3$

$x = 0.86177...$

$1 - \dfrac{r}{100} = 0.86177...$

$\dfrac{r}{100} = 0.13822....$

$r = 13.822...$

Alternative solution: Use TVM Solver

$N = 3$
$\bullet\ I\% = -13.8226124$
$PV = -1050$
$PMT = 0$
$FV = 672$
$P/Y = 1$
$C/Y = 1$
PMT : END $BEGIN$

Hence the reducing balance depreciation rate is 13.8% per annum.

2 marks

Mark allocation

- 1 mark for method
- 1 mark for correct answer

Tip

- *An alternative method would be to use trial and improvement.*

Question 4

To help pay for the holiday, Charlie borrows \$3000 at 6% interest, per annum, compounding monthly. Charlie will make equal monthly repayments for two years to pay back the loan. The annuities formula used to calculate the monthly repayment is

$$A = PR^n - \frac{Q\left(R^n - 1\right)}{R - 1}$$

4a. What are the values of *A, P* and *n*?

Worked solution

$A = \$0$, $P = \$3000$, $n = 24$

2 marks

Mark allocation

- 2 marks if all correct
- 1 mark if only one mistake

4b. Determine the monthly repayment for this loan correct to the nearest cent.

Worked solution

Use TVM Solver

$N = 24$
$I\% = 6$
$PV = 3000$
• $PMT = -132.96183...$
$FV = 0$
$P/Y = 12$
$C/Y = 12$
PMT : END $BEGIN$

Hence the monthly payments will be \$132.96

1 mark

Mark allocation

- 1 mark for correct answer

4c. How much interest is paid over the two years of the loan?

Worked solution

Total loan repayments = 24 x 132.96 = \$3191.04

Interest = 3191.04 – 3000 = \$191.04

1 mark

Mark allocation

- 1 mark for correct answer

4d. By how much has the loan been reduced by after Charlie has made 12 equal monthly repayments?

Worked solution

$$\begin{array}{|l|}\hline N = 12 \\ I\% = 6 \\ PV = 3000 \\ PMT = -132.96 \\ \bullet\ FV = -1544.897062 \\ P/Y = 12 \\ C/Y = 12 \\ PMT: END \quad BEGIN \\ \hline \end{array}$$

Hence amount paid off = 3000 – 1544.90 = $1455.10

1 mark

Mark allocation

- 1 mark for correct answer

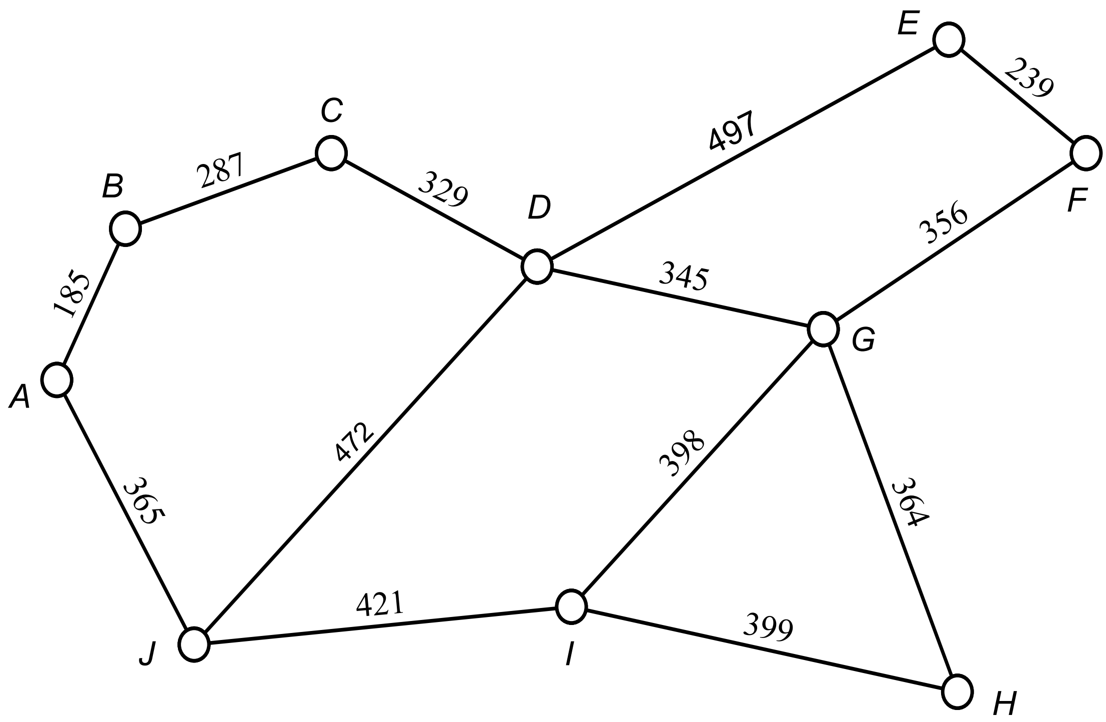
Total 15 marks

END OF MODULE 4

Module 5: Networks and decision mathematics

Question 1

A golf course has a series of paths that link the 18 greens of the course together so that the greenkeeper can maintain and water each green. The network diagram below shows the first 9 greens labelled *A – I* and the clubhouse *J*, with the path distances given in metres.



1a. The degree of vertex *D* is ☐

Worked solution

Degree is 4 as there are 4 edges connected to vertex *D*

1 mark

Mark allocation

- 1 mark for correct answer

1b. The greenkeeper has to get from the clubhouse *J* to the sixth hole *F*. What is the length of the shortest route?

Worked solution

Three main possible routes of *JDEF, JDGF* or *JIGF* of which *JDGF* is the shortest, hence shortest route = 472 + 345 + 356 = 1173 m

1 mark

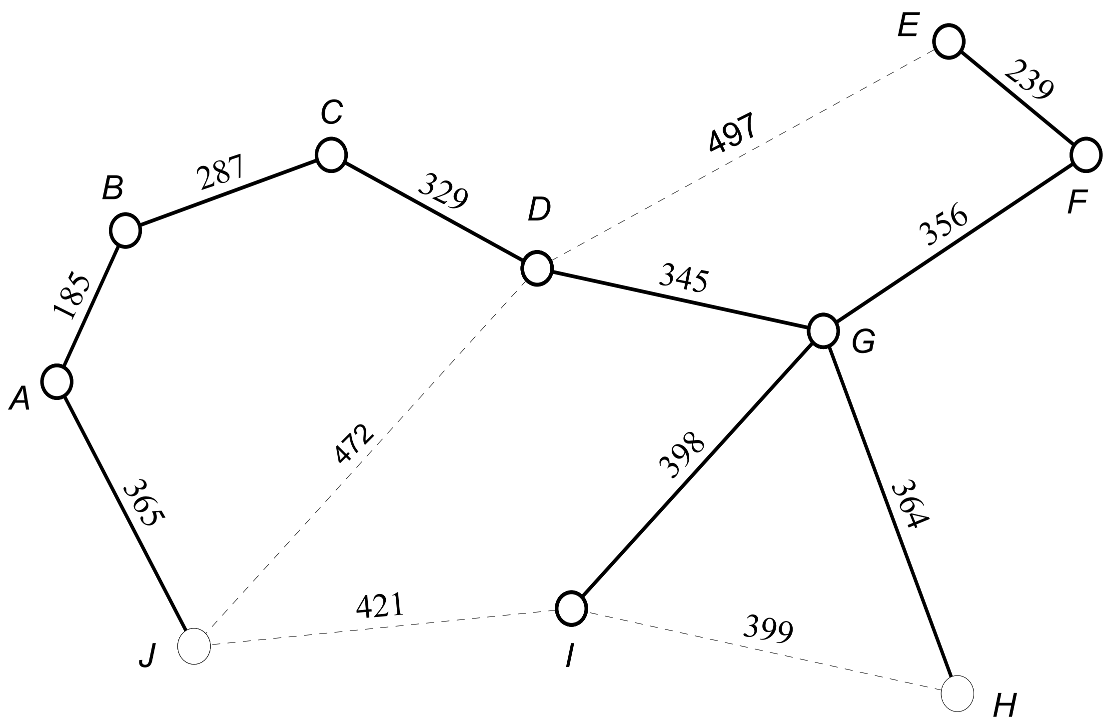
Mark allocation

- 1 mark for correct distance

The golf club plans to lay new water pipes along the length of some of the paths so that all greens have access to water from the clubhouse. The golf club decides to make the water pipe network as short as possible in order to keep the cost down.

1c. On the network diagram below draw the minimal spanning tree which represents the new water pipes.

Worked solution



2 marks

Mark allocation

- 2 marks if tree completely correct
- 1 mark if only one mistake

The greenkeeper is regularly required to inspect the whole golf course and hence must travel around all the paths of the first 9 holes. To save time the greenkeeper only wishes to travel along each path once only.

1d. Write the mathematical term used to describe the route that the greenkeeper plans to take.

Worked solution

Eulerian path

1 mark

Mark allocation

- 1 mark for correct answer

1e. By referring to the **degrees of the nodes** in this network of paths, explain why it is possible to travel every path once only

Worked solution

For an Eulerian path to be possible either every node has to have an even value for its degree or two nodes may have an odd value for their degree. Nodes *J* and *I* are the only vertices with an odd degree hence an Eulerian path is possible.

2 marks

Mark allocation

- 1 mark for stating conditions for an Eulerian path
- 1 mark for relating the conditions to this network

1f. If the greenkeeper starts the Eulerian path at the clubhouse *J*, where will the greenkeeper finish?

Worked solution

Node *I*

1 mark

Mark allocation

- 1 mark for correct answer

Tip

- *If the network has two nodes of odds degree then the Eulerian path must start at one of these nodes and finish at the other.*

1g. The greenkeeper decides that as well as starting at the clubhouse *J* and following the Eulerian path, they must return to the clubhouse. What would be the total length of this journey?

Worked solution

Add together all the path lengths plus include path *IJ* for a second time in order to return to *J*, hence total distance is 5078 m or 5.078 km.

1 mark

Mark allocation

- 1 mark for correct answer

Question 2

The golf club is reconstructing the second 9 holes of the course. The network diagram below shows the activities identified for the reconstruction and the time, in days, it takes to complete each activity.

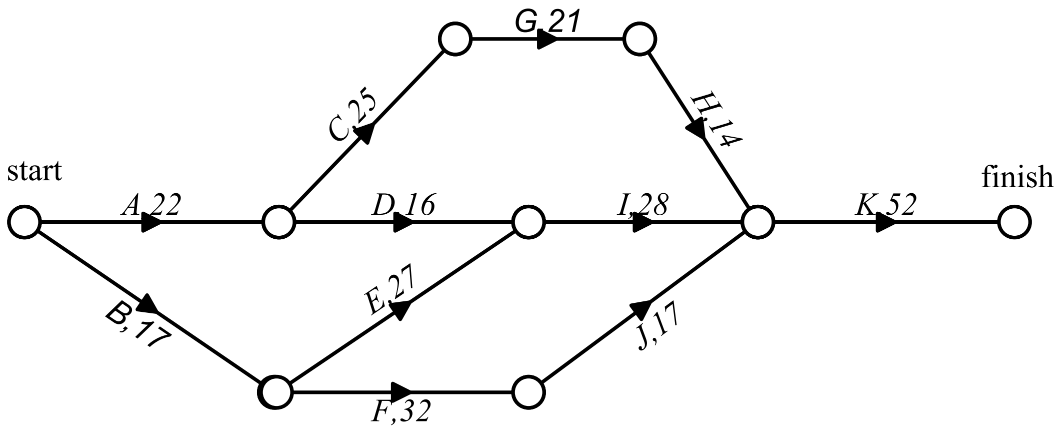


Table 1 shows the same information with immediate predecessor(s) and the earliest start times included.

Table 1

Activity	Immediate predecessor(s)	Completion time (days)	Earliest start time EST (days)
A	-	22	0
B	-	17	0
C	*A*	25	22
D	*A*	16	22
E	*B*	27	17
F	*B*	32	17
G	*C*	21	47
H	*G*	14	**68**
I	***D, E***	28	44
J	*F*	17	49
K	*H, I, J*	52	82

2a. Use the network diagram to complete the shaded cells in Table 1.

Worked solution

1st cell is *D, E*

2nd cell is 68

2 marks

Mark allocation

- 1 mark for each cell

2b. Write down the critical path and state the shortest completion time for the project.

Worked solution

Critical path is *A-C-G-H-K,* with the shortest completion time of 134 days (22+25+21+14+52)

2 marks

Mark allocation

- 1 mark for correct critical path
- 1 mark for correct completion time

The completion time of some of the activities of the project can be reduced at a cost. Table 2 shows the new completion times after reduction and the cost of the reductions, per hour.

Table 2

Activity	Usual completion time (days)	Minimum reduced completion time (days)	Cost of reduction per day ($000s)
C	25	15	2
E	27	18	3
G	21	12	1.5

2c. Determine the maximum time, in days, that can be saved by reducing all or some of the completion times.

Worked solution

If you make the maximum reductions then the completion times of possible critical paths are:

Path	Completion time (days)
A-C-G-H-K	115
A-D-I-K	118
B-E-I-K	115
B-F-J-K	118

Hence the new completion time is 118 days so 16 days can be saved.

1 mark

Mark allocation

- 1 mark for correct answer

2d. What is the minimum cost to achieve the maximum time saving?

Worked solution

Paths *A-C-G-H-K* and *B-E-I-K* only need reducing to a total completion time of 118 days, hence the following time reductions should take place:

Activity	Usual completion time (days)	Reduced completion time (days)	Cost of reduction per day ($000s)	Number of reduced days	Total cost of reduction ($000s)
C	25	18	2	7	14
E	27	21	3	6	18
G	21	12	1.5	9	13.5

Hence the total cost of reduction is 14 000 + 18 000 + 13 500 = $45 500

1 mark

Mark allocation

- 1 mark for correct answer

Total 15 marks

END OF MODULE 5

Module 6: Matrices

Question 1

A retailer buys three different products *A, B* and *C* from two separate manufacturers, Pear (P) and Tony (*T*). The cost ($) of each product from each manufacturer if given in *F*, where

$$F = \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 56.00 & 120.00 & 210.00 \\ 50.00 & 100.00 & 250.00 \end{bmatrix} \end{array} \begin{array}{c} \\ P \\ T \end{array}$$

Matrix *G* below shows the transport costs ($) of moving each product from the manufacturers to the retailer.

$$G = \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 4.00 & 4.00 & 4.00 \\ 5.00 & 5.00 & 5.00 \end{bmatrix} \end{array} \begin{array}{c} \\ P \\ T \end{array}$$

1a. Write down the order of matrix *F*.

Worked solution

2 rows and 3 columns hence the order is 2×3

1 mark

Mark allocation

- 1 mark for correct answer

1b. Evaluate the matrix *M*, where $M = F + G$

Worked solution

$$M = \begin{bmatrix} 60.00 & 124.00 & 214.00 \\ 55.00 & 105.00 & 255.00 \end{bmatrix}$$

1 mark

Mark allocation

- 1 mark for correct answer

The retailer sells the three products *A, B* and *C* from manufacturers *P* and *T*. Matrix *H* represents the retail price ($) of these products.

$$H = \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 89.50 & 185.00 & 325.00 \\ 80.00 & 180.00 & 349.95 \end{bmatrix} \end{array} \begin{array}{c} \\ P \\ T \end{array}$$

1c. Evaluate the matrix *N*, where $N = H - M$ (note matrix *M* is your answer from **1b**)

Worked solution

$$N = \begin{bmatrix} 29.50 & 61.00 & 111.00 \\ 25.00 & 75.00 & 94.95 \end{bmatrix}$$

1 mark

Mark allocation

- 1 mark for correct answer

1d. What information do the elements of matrix *N* represent?

Worked solution

The elements represent the profit made on the sale of each product as we have subtracted the cost of buying the products from the sale price.

1 mark

Mark allocation

- 1 mark for the word "profit" or equivalent

Question 2

Matrix *S* represents the price (without GST) of five other products.

$$S = \begin{bmatrix} 75.00 & 125.30 & 287.90 & 14.00 & 22.50 \end{bmatrix}$$

The GST charged on each item is 10% of the price.

2a. Matrix *S* is multiplied by a scalar in order to find the GST to be added for each product. What would this scalar be?

Worked solution

0.1

1 mark

Mark allocation

- 1 mark for correct answer (do not accept 1.1)

Tip

- *10% as a decimal is 0.1 hence this is the correct scalar. Note that if the question asked for a scalar which would calculate the new price including GST then the answer would be 1.1 as this is the original 100% price plus the extra 10%.*

2b. Find the price (including GST) of each of the five products.

Worked solution

$$0.1\begin{bmatrix}75.00 & 125.30 & 287.90 & 14.00 & 22.50\end{bmatrix}$$
$$=\begin{bmatrix}7.50 & 12.53 & 28.79 & 1.40 & 2.25\end{bmatrix}$$

$$\begin{bmatrix}75.00 & 125.30 & 287.90 & 14.00 & 22.50\end{bmatrix}$$
$$+\begin{bmatrix}7.50 & 12.53 & 28.79 & 1.40 & 2.25\end{bmatrix}$$
$$=\begin{bmatrix}82.50 & 137.83 & 316.69 & 15.40 & 24.75\end{bmatrix}$$

1 mark

Mark allocation

- 1 mark for correct five prices, regardless of whether they are in a matrix or not.

Question 3

Researchers for the manufacturer Pear (*P*) claim that in order to maximise profits the three products *A, B* and *C* should be produced in certain quantities *a, b* and c respectively at their factory. The researchers also claim these three quantities can be determined by solving the equations

$$a-b+c=650$$
$$a+b+2z=4300$$
$$a-c=350$$

3a. Write the system of equations in matrix form using the template below.

Worked solution

$$\begin{bmatrix}1 & -1 & 1\\ 1 & 1 & 2\\ 1 & 0 & -1\end{bmatrix}\begin{bmatrix}a\\ b\\ c\end{bmatrix}=\begin{bmatrix}650\\ 4300\\ 350\end{bmatrix}$$

1 mark

Mark allocation

- 1 mark for correct answer

3b. Write down an inverse matrix that can be used to solve the equations

Worked solution

$$\begin{bmatrix}0.2 & 0.2 & 0.6\\ -0.6 & 0.4 & 0.2\\ 0.2 & 0.2 & -0.4\end{bmatrix}$$

1 mark

Mark allocation

- 1 mark for correct matrix

3c. Solve the equations and hence state the numbers of *A, B,* and *C* that should be produced in order to maximise profit.

Worked solution

$$\begin{bmatrix} 0.2 & 0.2 & 0.6 \\ -0.6 & 0.4 & 0.2 \\ 0.2 & 0.2 & -0.4 \end{bmatrix}\begin{bmatrix} 650 \\ 4300 \\ 350 \end{bmatrix} = \begin{bmatrix} 1200 \\ 1400 \\ 850 \end{bmatrix}$$

Hence Pear should manufacture 1200 units of product *A*, 1400 units of product *B* and 850 units of product *C*.

1 mark

Mark allocation

- 1 mark for correct solution

Question 4

Products *A, B* and *C* are software packages which protect computers from viruses. The software has to be bought every year in order to ensure it is up to date. In one year the manufacturer Tony (*T*) sells 305 000 units of *A*, 250 000 unit of *B* and 160 000 units of *C*.

4a. Write this information in terms of a column matrix, K_0, below.

Worked solution

$$K_0 = \begin{array}{c} \begin{bmatrix} 305000 \\ 250000 \\ 160000 \end{bmatrix} \end{array}\begin{array}{c} A \\ B \\ C \end{array}$$

1 mark

Mark allocation

- 1 mark for correct matrix

Tony's researchers predict that the following changes will take place in regards to the sales of their three products:

84% of customers who bought product *A* last year will again buy product *A* next year
8% of customers who bought product *A* last year will instead buy product *B* next year
8% of customers who bought product *A* last year will instead buy product *C* next year

75% of customers who bought product *B* last year will again buy product *B* next year
15% of customers who bought product *B* last year will instead buy product *A* next year
10% of customers who bought product *B* last year will instead buy product *C* next year

88% of customers who bought product *C* last year will again buy product *C* next year
7% of customers who bought product *C* last year will instead buy product *A* next year
5% of customers who bought product *C* last year will instead buy product *B* next year

4b. Enter this information into the transition matrix R below (express percentages as proportions, e.g. write 88% as 0.88)

Worked solution

$$\begin{array}{c c l}
 & \textbf{this year} & \\
 & \begin{array}{ccc} A & B & C \end{array} & \\
R = & \begin{bmatrix} 0.84 & 0.15 & 0.07 \\ 0.08 & 0.75 & 0.05 \\ 0.08 & 0.10 & 0.88 \end{bmatrix} & \begin{array}{l} A \\ B \\ C \end{array} \quad \textbf{next year}
\end{array}$$

2 marks

Mark allocation

- 2 marks if answer correct
- 1 mark if one mistake only or values correct but columns entered as rows and vice versa

4c. Use R and K_0 to **write** and **evaluate** a matrix product that determines how many units of each product A, B and C are sold during the next year.

Worked solution

$$\begin{bmatrix} 0.84 & 0.15 & 0.07 \\ 0.08 & 0.75 & 0.05 \\ 0.08 & 0.10 & 0.88 \end{bmatrix}\begin{bmatrix} 305000 \\ 250000 \\ 160000 \end{bmatrix} = \begin{bmatrix} 304900 \\ 219900 \\ 190200 \end{bmatrix}$$

2 marks

Mark allocation

- 1 mark for correct method
- 1 mark for correct matrix answer

4d. If these trends continue each year the numbers of each product sold each year will stabilise. How many of each product will be sold when this steady state is reached?

Worked solution

Need to take the initial state matrix K_0 and keep multiplying it by the transition matrix R until the solution reaches a steady state (does not change)

$$R^{99} \times K_0 = \begin{bmatrix} 268393.3934 \\ 146006.006 \\ 300600.6006 \end{bmatrix} \qquad R^{100} \times K_0 = \begin{bmatrix} 268393.3934 \\ 146006.006 \\ 300600.6006 \end{bmatrix}$$

Hence $K = \begin{bmatrix} 268393 \\ 146006 \\ 300601 \end{bmatrix}$

Hence the sales for products A, B and C are 268 393, 146 006 and 300 601 units respectively.

1 mark

Mark allocation

- 1 mark for correct values

END OF WORKED SOLUTIONS

Total 15 marks