Insight Publications

INSIGHT

Trial Exam Paper

2007

FURTHER MATHEMATICS

Written examination 2

STUDENT NAME:

QUESTION AND ANSWER BOOK

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

Structure of book

Core		
Number of questions	*Number of questions to be answered*	*Number of marks*
2	2	15
Module		
Number of modules	*Number of modules to be answered*	*Number of marks*
6	3	45
		Total 60

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference that may be annotated (can be typed, handwritten or a textbook), one approved graphics calculator or approved CAS calculator or CAS software (memory DOES NOT have to be cleared) and, if desired, one scientific calculator.
- Students are NOT permitted to bring blank sheets of paper or white out liquid/tape into the examination.

Materials provided

- The question and answer book of 38 pages, with a separate sheet of miscellaneous formulas.
- Working space is provided throughout the book.

Instructions

- Write your **name** in the box provided.
- You must answer the questions in English.
- Remove the data sheet during reading time.

Students are NOT permitted to bring mobile phones or any other electronic devices into the examination.

Instructions

This examination consists of a core and six modules. Students should answer **all** questions in the core and then select **three** modules and answer **all** question within the modules selected. You do not need to give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π, surds or fractions.

Core

Question 1

The theme park Rollercoaster World earns revenue from attendance sales, merchandise sales and food sales. One aspect of this revenue is the three ice-cream stalls situated within the park.

In order to analyse the performance of the three ice-cream stalls the owner of Rollercoaster World records how many ice-creams each stall sells each day for one week. This data is presented in Table 1 below.

Table 1

	Number of ice-creams sold		
Day of Week	Stall A	Stall B	Stall C
Monday	562	532	362
Tuesday	388	481	305
Wednesday	575	599	416
Thursday	492	63	365
Friday	549	587	425
Saturday	641	683	488
Sunday	617	721	457

a. Complete Table 2 by calculating the mean number of ice-creams sold by Stall A. Write your answer correct to one decimal place.

Table 2

Number of ice-creams sold	Stall A	Stall B	Stall C
mean		523.7	402.6
standard deviation	84.7	219.1	62.6

1 mark

b. Use only the data in Table 2 to state which stall was the most successful in terms of the number of ice-creams sold. Justify your answer.

___

___

1 mark

Stall B appears to have a relatively high standard deviation. The owner believes the main reason for this is because on one particular day a major ride located near to stall B was closed for repairs, and so the stall had far less customers than normal.

c. Identify which day the ride near stall B was closed and prove that the number of ice-creams sold from stall B on this day could be considered to be an outlier.

___

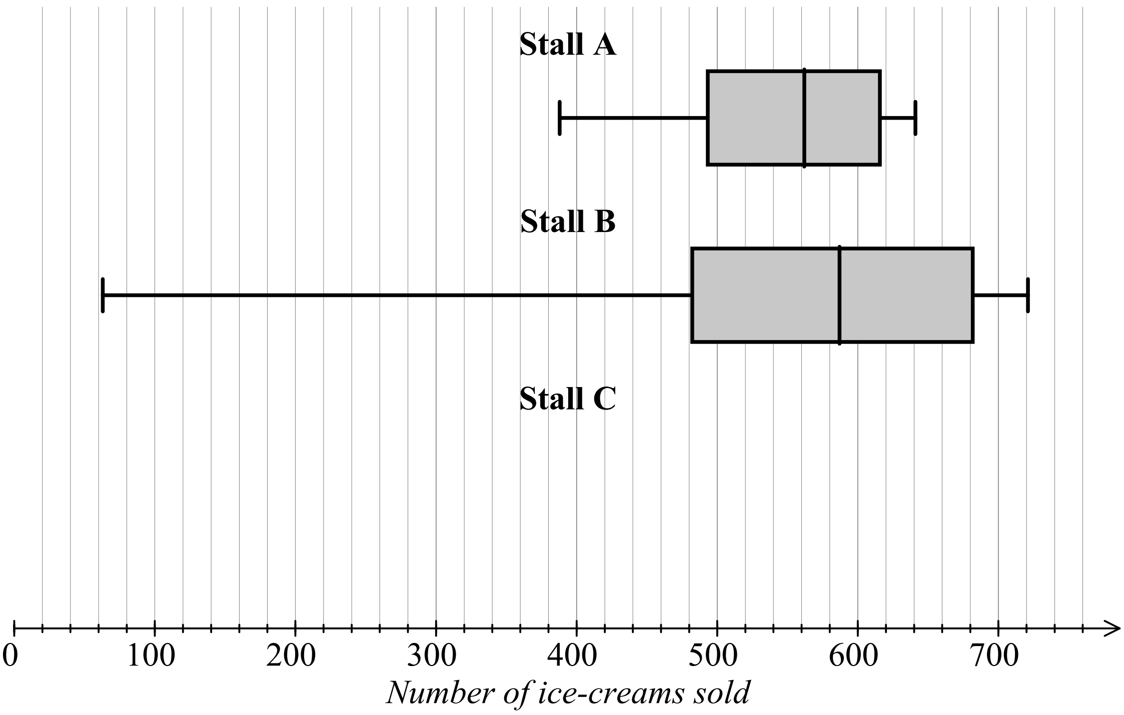
___

___

2 marks

Using the data from Table 1, boxplots are constructed to display the distributions of ice-cream sales for stalls A and B as shown in Boxplot 1.

Boxplot 1



d. Complete Boxplot 1 by constructing and drawing a boxplot that shows the distribution of ice-cream sales for stall C.

1 mark

e. Does Boxplot 1, in particular the median values, support your statement in part (b), which stated which of the three stalls was the most successful in terms of most ice-creams sold? Explain your answer.

1 mark

The owner of Rollercoaster World correctly concludes that the median is more appropriate than the mean as a measure of central tendency for this data

f Explain why the owner's conclusion is correct.

1 mark

Question 2

The attendance figures for Rollercoaster World from 2004 to 2006 are given in Table 3.

Table 3

Year	2004				2005				2006			
Season	Aut	Win	Spr	Sum	Aut	Win	Spr	Sum	Aut	Win	Spr	Sum
Attendance (000s)	382	242	351	465	416	309	366	515	429	347	413	562

This data will be used to predict future attendance at the theme park.

a. In this analysis, the **dependent** variable is ☐

1 mark

To start the analysis, the time values have been rescaled as $x = 1$ to $x = 12$ (autumn 2004 = 1, winter 2004 = 2, and so on). This data is displayed in Table 4.

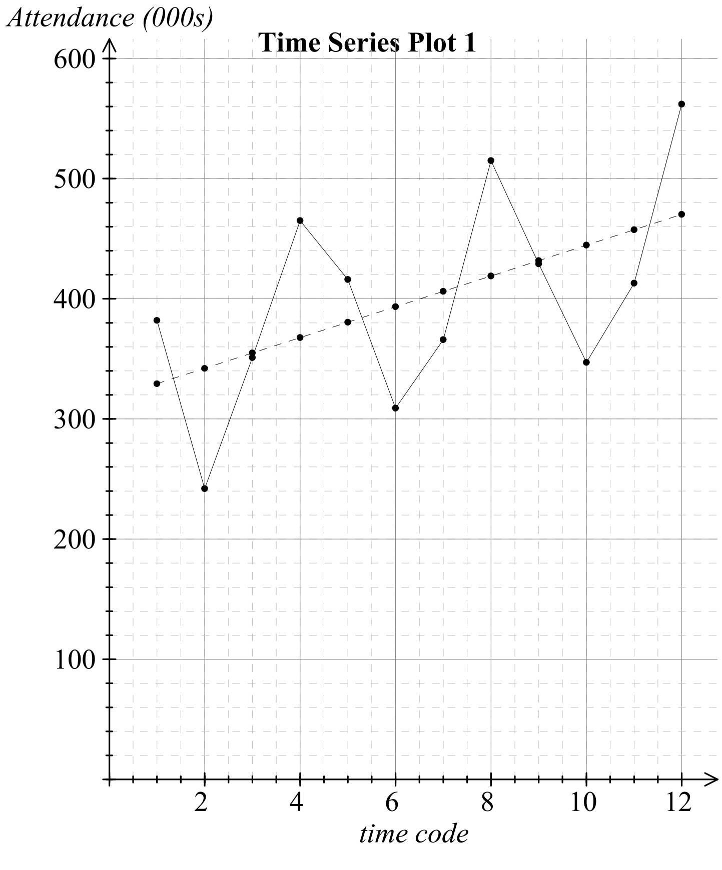
Table 4

Time code	1	2	3	4	5	6	7	8	9	10	11	12
Attendance (000s)	382	242	351	465	416	309	366	515	429	347	413	562

This rescaled data is displayed in Time Series Plot 1 below. Also displayed is the least squares regression line for the rescaled data.

The equation for the least squares regression line is

$$attendance(000s) = 316.5 + 12.8 \times timecode$$



b. Use the information in Time Series Plot 1 to describe the relationship between *attendance* and *time* in terms of **form** and **direction**.

____

____

2 marks

c. Using the regression equation $attendance(000s) = 316.5 + 12.8 \times timecode$ to model attendances at Rollercoaster World, we would say that the average increase in attendances was [ ] people per season.

1 mark

d. Complete Table 5 by calculating the seasonal index for winter.

Table 5

Season	**Index**
Autumn	1.026
Winter	
Spring	0.943
Summer	1.286

1 mark

Using the seasonal indices in Table 5 the attendances from Table 3 are deseasonalised.

e. Complete Table 6 by calculating the deseasonalised attendance value for spring 2006.

Table 6

Year	2004				2005				2006			
Season	Aut	Win	Spr	Sum	Aut	Win	Spr	Sum	Aut	Win	Spr	Sum
Time code	1	2	3	4	5	6	7	8	9	10	11	12
Actual attendance (000s)	382	242	351	465	416	309	366	515	429	347	413	562
Deseasonalised attendance (000s)	372	325	372	362	406	415	388	400	418	466		437

1 mark

The least squares regression line for the deseasonalised data is

$$\textit{deseasonalised} \quad \textit{attendance}(000s) = 340.3 + 9.2 \times \textit{timecode}$$

f. Use the regression equation for the deseasonalised data to predict the actual attendance in summer 2007.

____

____

____

2 marks

Total 15 marks

END OF CORE

Module 1: Number patterns

Question 1

Tree conservation is an important aspect of protecting the world environment. The company Timber Resources Ensuring Environmental Sustainability (or TREES for short) supplies wood for building but replants a tree for every one it cuts down. This not only sustains the tree population but also provides a continuous supply of new trees which can grow and then be cut down.

When initially planted the tree samplings have a diameter of 6 cm, with the diameter increasing by 2.4 cm per year.

a. What would be the diameter today of a tree planted five years ago?

1 mark

b. How many years after planting would a tree have a radius of 18.6 cm?

2 marks

c. A difference equation that generates the terms of this sequence of diameters is

$$t_{n+1} = rt_n + d \qquad \text{where} \qquad t_1 = 6$$

What are the values of r and d?

$r =$ ☐ $d =$ ☐

2 marks

Question 2

The newly planted tree samplings from question 1 begin with a height of 1.23 m.

The height growth for the trees in the first three years after planting is shown in Table 1

Table 1

Year	1	2	3
Height growth (m)	2	1.8	1.62

a. Show that the terms of this height growth sequence are geometric in nature

1 mark

b. What is the percentage decrease in height growth from one year to the next?

1 mark

c. What would the height growth be in the seventh year in metres correct to 2 decimal places?

1 mark

d. If a tree continues to grow according to this pattern, what is the maximum possible height of the tree?

2 marks

e. The 'TREES' company requires the trees to reach a minimum height of 18 m and a minimum diameter of 50 cm before they can be harvested for timber. What is the minimum number of years after planting that the trees would satisfy these conditions for harvest?

2 marks

Question 3

The ‘TREES’ company decides to change its policy on harvesting and replanting. In its new policy the company commits cutting down only 5% of its tree population during the year and then planting 30 000 new trees just before the end or each year. The initial number of trees planted was 200 000 trees.

a. Write a difference equation that specifies the number of planted trees after *n* years.

1 mark

b. How many trees will there be after three years of this policy?

1 mark

c. Explain why the number of trees will never exceed 600 000 trees.

1 mark

Total 15 marks

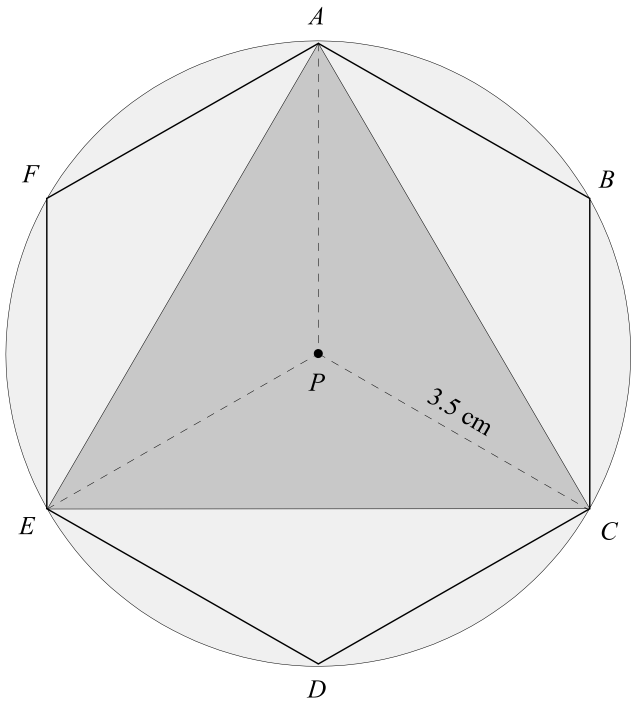
END OF MODULE 1
TURN OVER

Working space

Module 2: Geometry and trigonometry

Question 1

'LOST?' - the orienteering and adventure club, has a logo consisting of an equilateral triangle, inside a regular hexagon, within a circle. All three shapes have the same centre *P* and the distance from *P* to where one of the corners of both the triangle and hexagon meet the edge of the circle at C is 3.5 cm.



a. What is the area of the circle correct to two decimal places?

1 mark

b. What is the size of angle *APC*?

1 mark

c. Determine the area of triangle *ACE* correct to 2 decimal places.

1 mark

Question 2

As well as being a logo on the LOST? uniform, the logo is copied to form a circular landscape design outside the LOST? offices. The landscape feature is an exact enlargement of the design in the diagram with the equivalent distance *PC* being 7 m.

a. What is the linear scale factor of this enlargement?

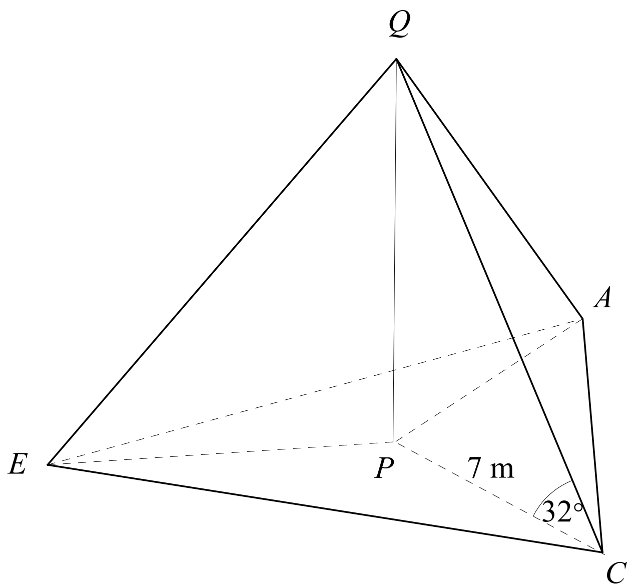
1 mark

In the landscape design the equilateral triangle is a pond with a depth of 50 cm.

b. Determine the volume of water required to fill the pond to the nearest cubic metre.

2 marks

A fountain shoots vertically into the air from the centre of the pond. The angle of elevation of the tip of the fountain Q from a corner of the pond is 32˚



c. Determine the height of the fountain to the nearest centimetre.

1 mark

Question 3

The LOST? offices *X* are near the base of a mountain. As part of the organisation's orienteering courses they also have a checkpoint *Y* somewhere up the mountain. The relative positions of the office and the checkpoint are shown on the contour map in figure 1.

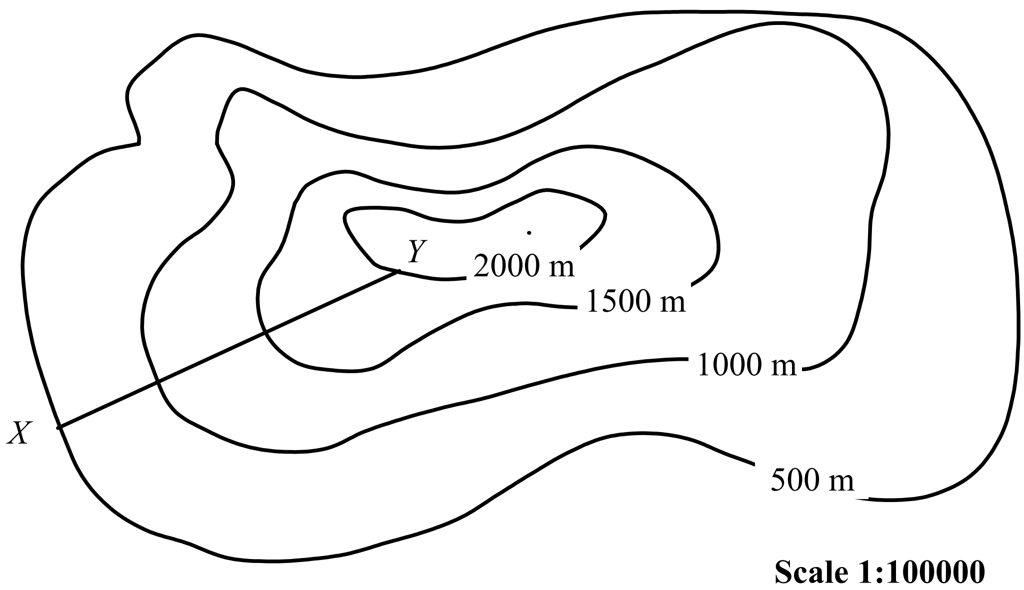


Figure 1

a. If the map distance *XY* is 5 cm then what is the real life horizontal distance from *X* to *Y* to the nearest kilometre?

1 mark

b. Determine the vertical height change between office *X* and checkpoint *Y* in metres.

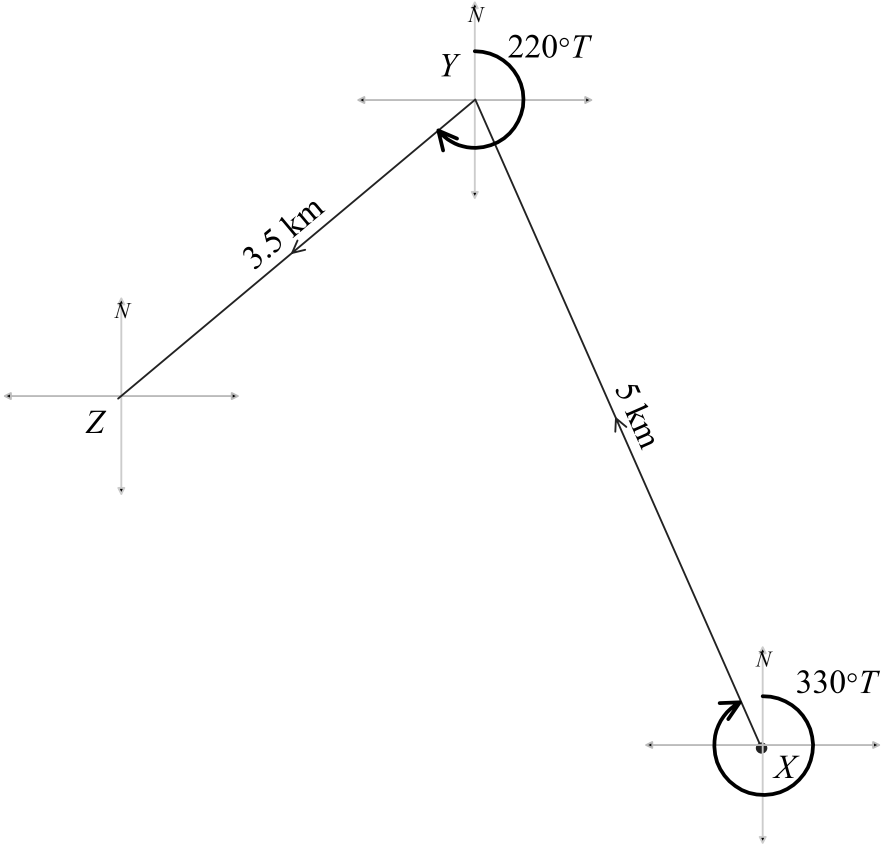
1 mark

c. Find the average slope between office *X* and checkpoint *Y*? Write your answer correct to one decimal place.

1 mark

Question 4

A party goes on an orienteering trip. On the first part of the journey they travel 5 km on a bearing of 330˚T from office *X* to checkpoint *Y*. The second leg of the course is a 3.5 km journey on a bearing of 220˚T from *Y* to a second checkpoint *Z*. On the third leg they return directly from checkpoint *Z* to office *X*.



a. State angle *XYZ*

1 mark

b. What is the direct distance from checkpoint *Z* to office *X*? Write your answer in kilometres correct to three decimal places.

2 marks

c. Determine the bearing of X from Z to the nearest degree.

2 marks

Total 15 marks

END OF MODULE 2
TURN OVER

Module 3: Graphs and relations

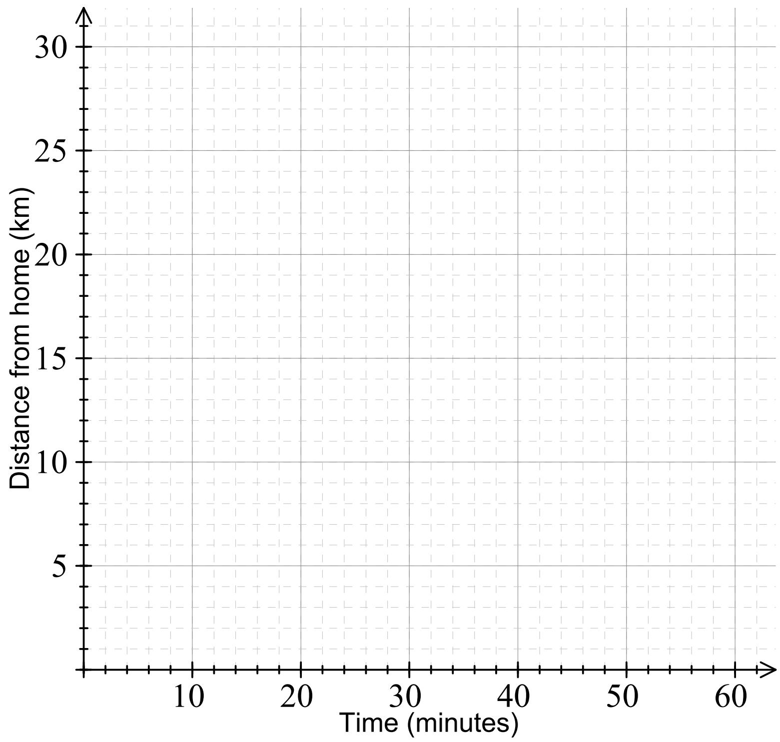
Question 1

When Andy travels from home *H* to work *W* he walks to the train station *T* and then catches a train to the city. Once in the city he does not have to travel any further as his work building is next door to the train station.

One day it takes Andy 20 minutes to walk the 1.7 km from his house to the train station, he then waits at the station for 15 minutes before catching the train which completes the remaining 25 km of the journey to work in 15 minutes.

a. Draw and label Graph 1 which is a distance-time graph of Andy's journey to work that day.

Graph 1



2 marks

b. Determine the average speed for the train journey (km/h)

___

___

___

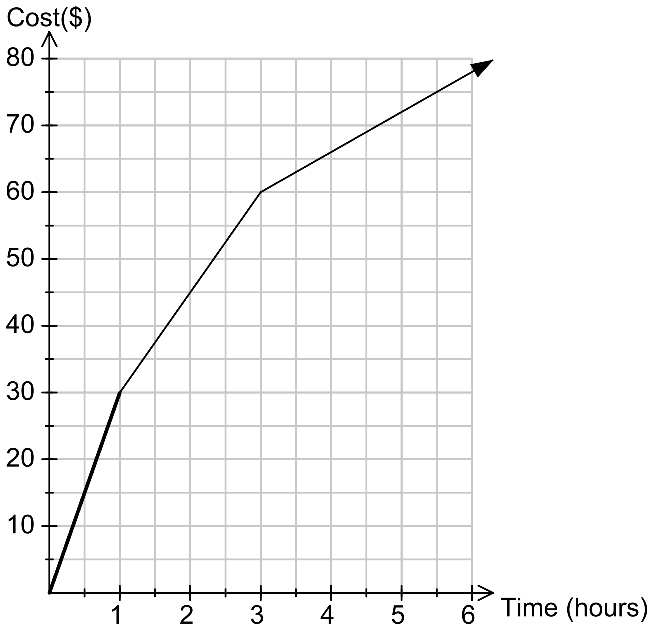
1 mark

Question 2

Andy works for a mobile phone company. They currently have a special deal for their customers who only use their mobile phones to make phone calls. As customers make use of their phones for longer amounts of time the cost per minute decreases. The cost of the calls decreases after 2 hours of use and 4 hours of use during any particular month.

Graph 2 represents total cost of calls made C($) against the total length of calls made t(minutes).

Graph 2



The same concern as above – I am concerned that the axes are not labelled – Cost($) and Time (minutes).
The equations below give the monthly cost C($) of the phone calls made, given the total time of phone calls made t(hours).

$$C = \begin{cases} at & 0 \le t \le 1 \\ bt + c & 1 < t \le 3 \\ 6t + 42 & t > d \end{cases}$$

a. Determine the values of *a, b, c* and *d*.

$a =$ ☐ $b =$ ☐ $c =$ ☐ $d =$ ☐

3 marks

b. For the first hour of calls how much is the call cost in cents/minute?

1 mark

A rival mobile company has only one call cost rate no matter how much you use your phone for. The total cost of calls $C(\$)$ in terms of time usage t (hours) is given by the equation,

$$C = 7.5t + 30$$

c. Draw the line of this cost equation on the same graph as the cost graph for Andy's company (Graph 2)

1 mark

d. Depending on the total length of phone calls made in a month Andy's company or the rival company would be cheaper. For what total length of calls is the rival company cheaper to use?

3 marks

Question 3

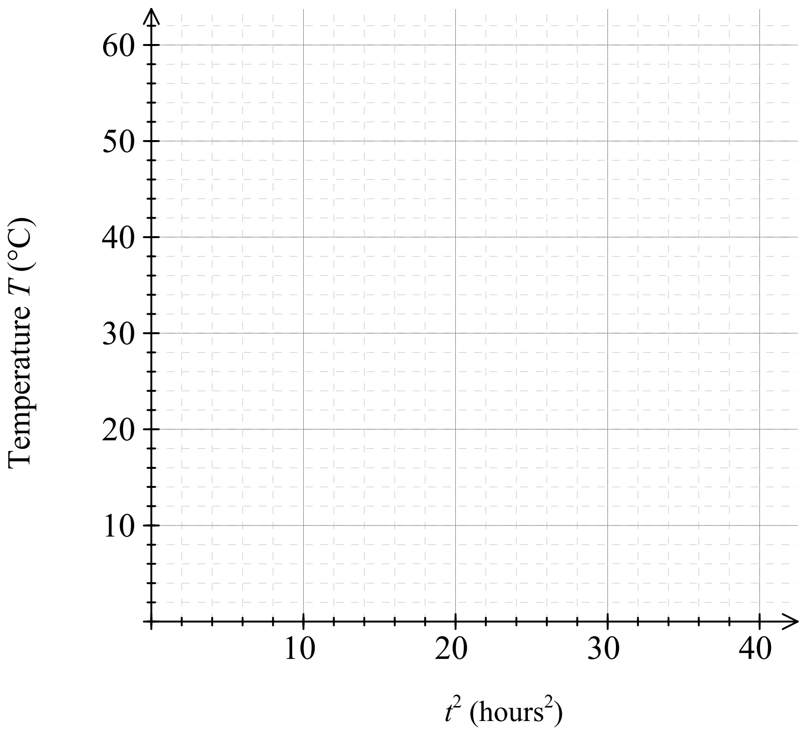
The computer server room in Andy's work building has a cooling system to keep the room at a reasonable temperature so that the computer equipment does not malfunction. Unbeknownst to everybody the cooling system breaks down and so the temperature begins to rise above the safe working temperature.

Table 1 shows the temperature T (°C) above the safe working temperature after t hours.

Table 1

t (hours)	0	2	4	6
t^2 (hours^2)				
Temperature T (°C)	0	6	24	54

a. Complete Table 1 and on the set of axes below, plot the four points (t^2, T) from Table 1.



2 marks

b. Assume that this relation is modelled by the equation $T = kt^n$. Justify why $n = 2$.

1 mark

c. Given that the relation is now modelled by the equation $T = kt^2$, find the value of k.

1 mark

Total 15 marks

END OF MODULE 3

Module 4: Business-related mathematics

Question 1

Megan's quarterly bank account statement is shown in Table 1

Table 1

Date	Transaction	Debit ($)	Credit ($)	Balance ($)
01 April 2007	Brought Forward			850.50
14 April 2007	Wage Payment		3600.00	4450.50
22 April 2007	Credit Card Payment	2436.00		2014.50
02 May 2007	ATM Withdrawal	300.00		1714.50
14 May 2007	Wage Payment		3600.00	5314.50
16 May 2007	ATM Withdrawal			4474.50
25 May 2007	Credit Card Payment	3215.84		1258.66
13 June 2007	Interest		102.35	
14 June 2007	Wage Payment		3600.00	4961.01
24 June 2007	Credit Card Payment	3005.50		1955.51

a. Complete Table 1 by calculating the ATM withdrawal made on 16 May 2007 and the balance on 13 June 2007.

2 marks

b. Interest on this account was based on the minimum monthly balance. If the interest rate was 0.5% per month then how much interest did Megan receive for the month of May?

1 mark

Question 2

Megan is planning on going on a skiing holiday. To prepare for this she buys some ski equipment and skiwear under a hire purchase agreement with a ski shop. The total value of the goods was $1200 but Megan pays a deposit of $300 and six monthly payments of $164.25

a. Determine the total amount of interest paid.

1 mark

b. What is the annual flat rate of interest charged?

2 marks

Question 3

Megan's friend Charlie is also going on the skiing holiday, and so he needs to buy some ski equipment and skiwear. Charlie buys what he requires from a second hand store. The store owner claims the original cost of the goods totalled $1050 three years ago, and that they are willing to sell the goods based on a flat depreciation rate of 12% per annum.

a. If the sale price is equivalent to the value of the goods after three years of a flat depreciation rate of 12% per annum, how much must Charlie pay?

2 marks

b. If the original cost of the goods was still $1050 and the price after three years was still the answer calculated in part **3a**, but the rate was calculated on a reducing balance basis, what would be the annual depreciation rate? Write your answer correct to one decimal place.

2 marks

Question 4

To help pay for the holiday, Charlie borrows $3000 at 6% interest, per annum, compounding monthly. Charlie will make equal monthly repayments for two years to pay back the loan. The annuities formula used to calculate the monthly repayment is

$$A = PR^n - \frac{Q(R^n - 1)}{R - 1}$$

a. What are the values of *A, P* and *n*?

$A =$ ☐ $P =$ ☐ $n =$ ☐

2 marks

b. Determine the monthly repayment for this loan correct to the nearest cent.

1 mark

c. How much interest is paid over the two years of the loan?

1 mark

d. By how much has the loan been reduced by after Charlie has made 12 equal monthly repayments?

1 mark

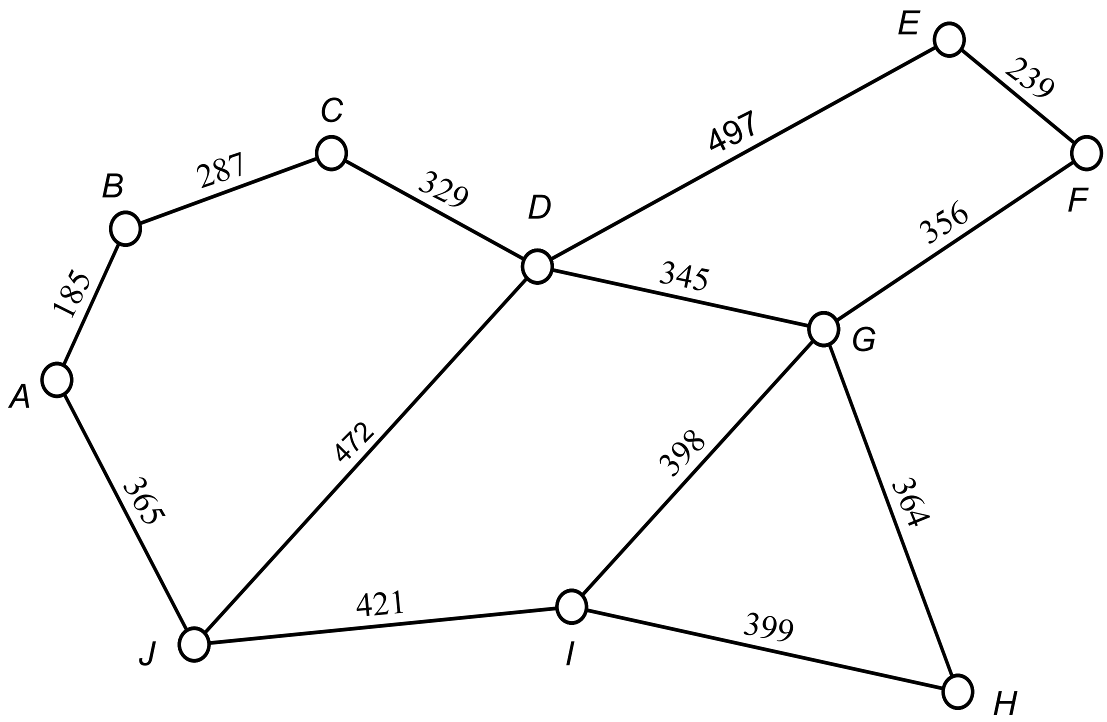
Total 15 marks

END OF MODULE 4

Module 5: Networks and decision mathematics

Question 1

A golf course has a series of paths that link the 18 greens of the course together so that the greenkeeper can maintain and water each green. The network diagram below shows the first 9 greens labelled *A* – *I* and the clubhouse *J*, with the path distances given in metres.



a. The degree of vertex *D* is ☐

1 mark

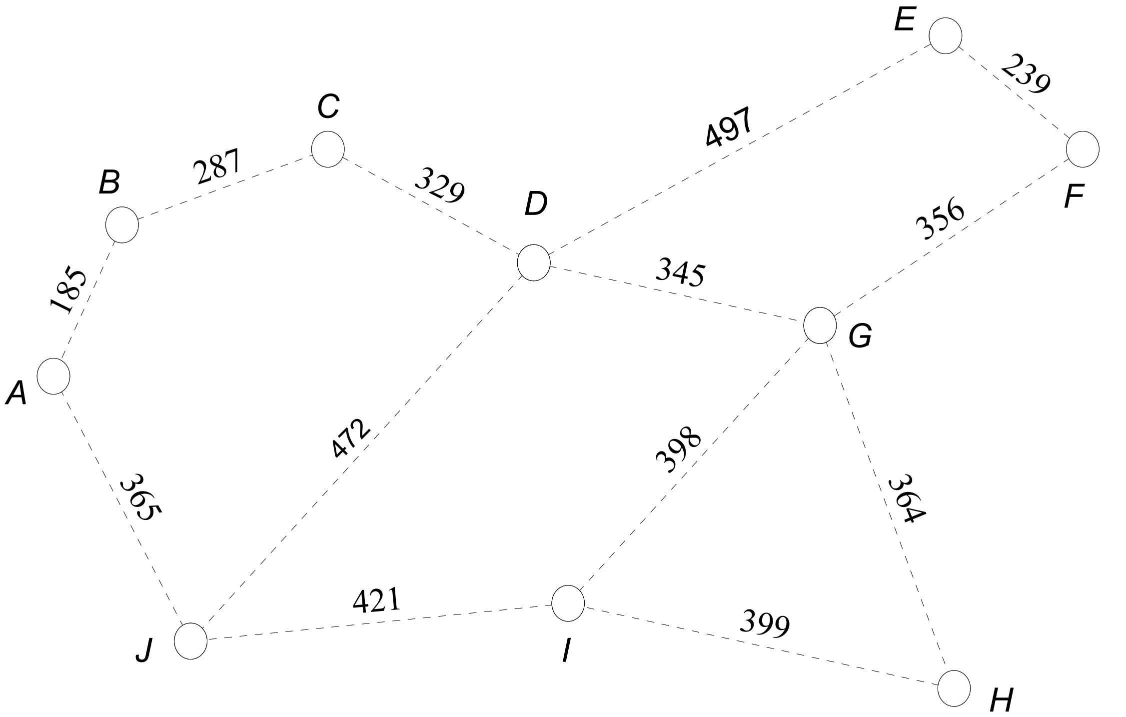
b. The greenkeeper has to get from the clubhouse *J* to the sixth hole *F*. What is the length of the shortest route?

____

1 mark

The golf club plans to lay new water pipes along the length of some of the paths so that all greens have access to water from the clubhouse. The golf club decides to make the water pipe network as short as possible in order to keep the cost down.

c. On the network diagram below draw the minimal spanning tree which represents the new water pipes.



2 marks

The greenkeeper is regularly required to inspect the whole golf course and hence must travel around all the paths of the first 9 holes. To save time the greenkeeper only wishes to travel along each path once only.

d. Write the mathematical term used to describe the route that the greenkeeper plans to take.

___

1 mark

e. By referring to the **degrees of the nodes** in this network of paths, explain why it is possible to travel every path once only

___

___

___

2 marks

f. If the greenkeeper starts the Eulerian path at the clubhouse *J*, where will the greenkeeper finish?

___

1 mark

g. The greenkeeper decides that as well as starting at the clubhouse *J* and following the Eulerian path, they must return to the clubhouse. What would be the total length of this journey?

___

1 mark

Question 2

The golf club is reconstructing the second 9 holes of the course. The network diagram below shows the activities identified for the reconstruction and the time, in days, it takes to complete each activity.

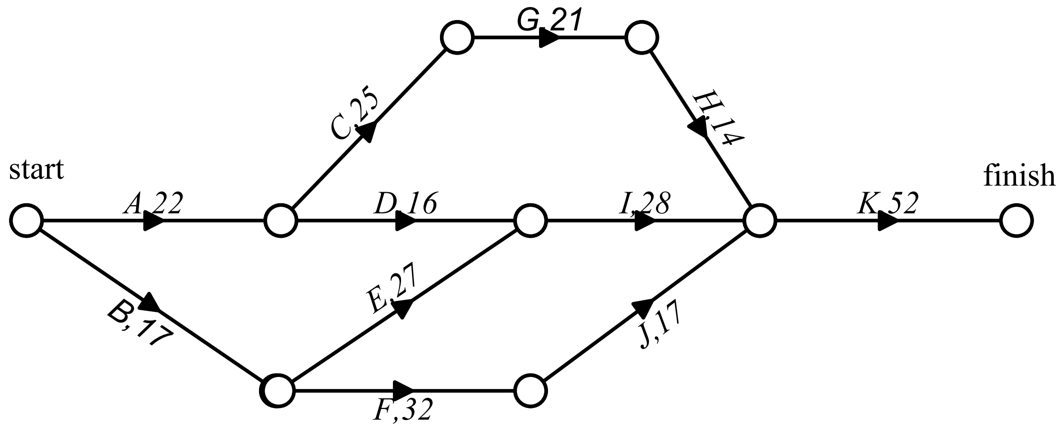


Table 1 shows the same information with immediate predecessor(s) and the earliest start times included.

Table 1

Activity	Immediate predecessor(s)	Completion time (days)	Earliest start time EST (days)
A	-	22	0
B	-	17	0
C	*A*	25	22
D	*A*	16	22
E	*B*	27	17
F	*B*	32	17
G	*C*	21	47
H	*G*	14	
I		28	44
J	*F*	17	49
K	*H, I, J*	52	82

a. Use the network diagram to complete the shaded cells in Table 1.

2 marks

b. Write down the critical path and state the shortest completion time for the project.

2 marks

The completion time of some of the activities of the project can be reduced at a cost. Table 2 shows the new completion times after reduction and the cost of the reductions, per hour.

Table 2

Activity	Usual completion time (days)	Minimum reduced completion time (days)	Cost of reduction per day ($000s)
C	25	15	2
E	27	18	3
G	21	12	1.5

c. Determine the maximum time, in days, that can be saved by reducing all or some of the completion times.

1 mark

d. What is the minimum cost to achieve the maximum time saving?

1 mark

Total 15 marks

END OF MODULE 5
TURN OVER

Working space

Module 6: Matrices

Question 1

A retailer buys three different products *A, B* and *C* from two separate manufacturers, Pear (P) and Tony (*T*). The cost ($) of each product from each manufacturer if given in *F*, where

$$F = \begin{array}{c} \\ \end{array}\begin{array}{c}\begin{matrix} \quad A & \quad\quad B & \quad\quad C \end{matrix}\\ \begin{bmatrix} 56.00 & 120.00 & 210.00 \\ 50.00 & 100.00 & 250.00 \end{bmatrix}\end{array}\begin{matrix} \\ P \\ T \end{matrix}$$

Matrix *G* below shows the transport costs ($) of moving each product from the manufacturers to the retailer.

$$G = \begin{array}{c}\begin{matrix} \quad A & \quad B & \quad C \end{matrix}\\ \begin{bmatrix} 4.00 & 4.00 & 4.00 \\ 5.00 & 5.00 & 5.00 \end{bmatrix}\end{array}\begin{matrix} \\ P \\ T \end{matrix}$$

a. Write down the order of matrix *F*.

___

1 mark

b. Evaluate the matrix *M*, where $M = F + G$

1 mark

The retailer sells the three products *A, B* and *C* from manufacturers *P* and *T*. Matrix *H* represents the retail price ($) of these products.

$$\begin{array}{c} \\ H = \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 89.50 & 185.00 & 325.00 \\ 80.00 & 180.00 & 349.95 \end{bmatrix} \end{array} \begin{array}{c} \\ P \\ T \end{array}$$

c. Evaluate the matrix *N*, where $N = H - M$ (note matrix *M* is your answer from **1b**)

1 mark

d. What information do the elements of matrix *N* represent?

___

___

1 mark

Question 2

Matrix S represents the price (without GST) of five other products.

$$S = \begin{bmatrix} 75.00 & 125.30 & 287.90 & 14.00 & 22.50 \end{bmatrix}$$

The GST charged on each item is 10% of the price.

a. Matrix S is multiplied by a scalar in order to find the GST to be added for each product. What would this scalar be?

___

1 mark

b. Find the price (including GST) of each of the five products.

___

___

___

1 mark

Question 3

Researchers for the manufacturer Pear (*P*) claim that in order to maximise profits the three products *A, B* and *C* should be produced in certain quantities *a, b* and c respectively at their factory. The researchers also claim these three quantities can be determined by solving the equations

$$a - b + c = 650$$
$$a + b + 2z = 4300$$
$$a - c = 350$$

a. Write the system of equations in matrix form using the template below.

$$\begin{bmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \end{bmatrix}\begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix}$$

1 mark

b. Write down an inverse matrix that can be used to solve the equations

1 mark

c. Solve the equations and hence state the numbers of *A, B,* and *C* that should be produced in order to maximise profit.

1 mark

Question 4

Products *A, B* and *C* are software packages which protect computers from viruses. The software has to be bought every year in order to ensure it is up to date. In one year the manufacturer Tony (*T*) sells 305 000 units of *A*, 250 000 unit of *B* and 160 000 units of *C*.

a. Write this information in terms of a column matrix, K_0, below.

$$K_0 = \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} \begin{matrix} A \\ B \\ C \end{matrix}$$

1 mark

Tony's researchers predict that the following changes will take place in regards to the sales of their three products:

84% of customers who bought product *A* last year will again buy product *A* next year
8% of customers who bought product *A* last year will instead buy product *B* next year
8% of customers who bought product *A* last year will instead buy product *C* next year

75% of customers who bought product *B* last year will again buy product *B* next year
15% of customers who bought product *B* last year will instead buy product *A* next year
10% of customers who bought product *B* last year will instead buy product *C* next year

88% of customers who bought product *C* last year will again buy product *C* next year
7% of customers who bought product *C* last year will instead buy product *A* next year
5% of customers who bought product *C* last year will instead buy product *B* next year

b. Enter this information into the transition matrix *R* below (express percentages as proportions, e.g. write 88% as 0.88)

this year

$$\begin{matrix} A & B & C \end{matrix}$$

$$R = \begin{bmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \end{bmatrix} \begin{matrix} A \\ B \\ C \end{matrix} \quad \textbf{next year}$$

2 marks

Module 6: Matrices – Question 4 – continued
TURN OVER

c. Use R and K_0 to **write** and **evaluate** a matrix product that determines how many units of each product *A, B* and *C* are sold during the next year.

2 marks

d. If these trends continue each year the numbers of each product sold each year will stabilise. How many of each product will be sold when this steady state is reached?

1 mark

Total 15 marks

END OF QUESTION AND ANSWER BOOK