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# Further Mathematics

# 2007

# **Trial Examination I**

Core – Data analysis Module 1 – Number patterns Module 5 – Networks and decision mathematics Module 6 – Matrices

1

# **SECTION A** Instructions

Answer **all** questions A correct answer scores 1, an incorrect answer scores 0. Marks will **not** be deducted for incorrect answers. **No** marks will be given if **more than one** answer is completed for any question.

#### Core – Data analysis

#### **Question 1**

The volumes of soft drink in 100 standard 375-ml cans are measured and recorded to the nearest 0.1 ml. The set of data is

- A. categorical and discrete
- B. numerical and discrete
- C. categorical and continuous
- **D.** numerical and continuous
- E. discrete or continuous

#### The following information relates to Questions 2, 3 and 4

The weight loss in kilograms of 12 people were: 27 13 40 8 24 20 6 15 21 41 23 28

#### **Question 2**

The median weight loss (in kilograms) of the 12 people was

A. 13 B. 20 C. 21 D. 21.2 E. 22

#### Question 3

The percentage of people with weight loss less than 20 kilograms was closest to

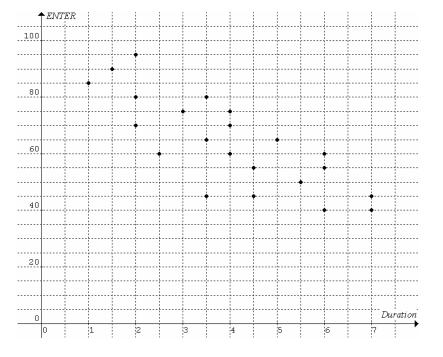
<b>A.</b> 33	<b>B.</b> 33.3	<b>C.</b> 34	<b>D.</b> 41.6	<b>E.</b> 41.7
<b>Question 4</b> The number of	outliers in the set of dat	a was		
<b>A.</b> 0	<b>B.</b> 1	<b>C.</b> 2	<b>D.</b> 3	<b>E.</b> 4

Relationship between a numerical variable and a two-valued categorical variable is best displayed by

- A. a back-to-back stem-and-leaf plot
- B. a segmented bar chart using percentages
- C. a scatterplot
- **D.** a box plot
- E. a frequency histogram

### The following information relates to Questions 6, 7 and 8

The ENTER scores of 22 VCE students are plotted against the average durations at the computer (in hours per night) spent by the students.



#### **Question 6**

The estimated r value is closest to

# Question 7

Which one of the following statements is best inferred by the correlation between ENTER and duration?

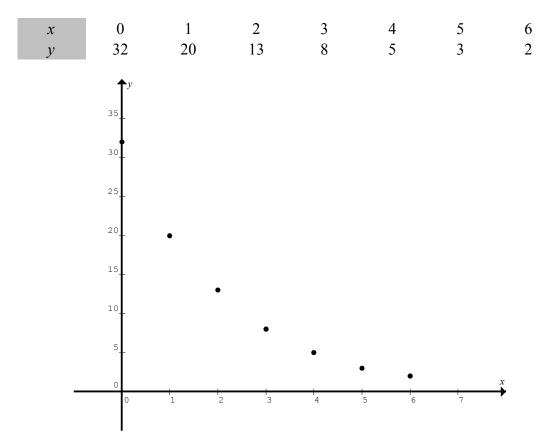
- A. Decrease in ENTER is caused by increase in duration.
- **B.** Increase in duration is caused by decrease in ENTER.
- **C.** Variation in ENTER is caused by variation in duration.
- **D.** Variation in ENTER is partly explained by variation in duration.
- E. Increase in duration corresponds to decrease in ENTER.

The best equation of the three median line is

- **A.**  $ENTER = 7.5 \times duration + 94$
- **B.**  $ENTER = 8.6 \times duration + 96$
- **C.**  $ENTER = -7.5 \times duration + 94$
- **D.**  $ENTER = -8.6 \times duration + 96$
- **E.**  $ENTER = 7.5 \times duration + 96$

#### **Question 9**

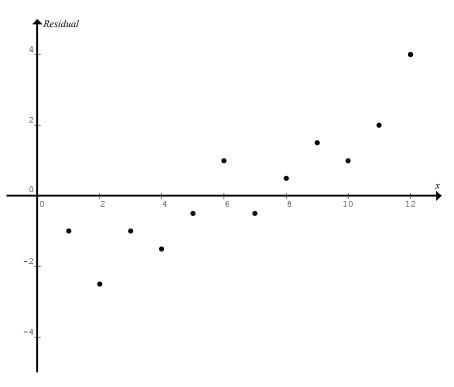
A student uses the following data to construct the scatterplot shown below.



The best method to linearise the scatterplot is to apply

- A. an *x*-squared transformation
- **B.** a *y*-squared transformation
- C. an *x*-reciprocal transformation
- **D.** a  $\log x$  transformation
- **E.** a  $\log y$  transformation

A student determined a least squares regression line y = mx + c and then a residual plot (shown below) from a set of data.



# The residual plot

- A. confirms that a linear relationship exists between residual and independent variable x
- **B.** confirms that a non-linear relationship exists between residual and independent variable x
- C. confirms that a linear relationship exists between dependent variable y and independent variable x
- **D.** confirms that a non-linear relationship exists between dependent variable y and independent variable x
- E. shows that the student determined the regression line or some of the residuals incorrectly

The number of births in a country town in 2006 is shown in the following table.

Month	Number of births	3-moving average	5-moving <i>median</i>
Jan	15		
Feb	20		
Mar	12		
Apr	11		
May	27	X	Y
Jun	23		
Jul	18		
Aug	12		
Sep	14		
Oct	16		
Nov	13		
Dec	21		

The numbers marked as *X* and *Y* are

- Α. X = 20, Y = 18
- X = 20.3, Y = 18B.
- X = 20.3, Y = 18.2C.
- X = 21, Y = 19D.
- X = 16.7, Y = 17E.

# The following information relates to Questions 12 and 13

The quarterly unemployment figures in a country town are shown in the table below.

	First quarter	Second quarter	Third quarter	Fourth quarter		
	1006	956	1210	1352		
<b>Question 12</b> The seasonal index	x for the second qu	uarter is				
<b>A.</b> 0.8	<b>B.</b> - 0.8453	<b>C.</b> 0.84	453 <b>D.</b>	1.1831	<b>E.</b> - 1.183	1
Question 13 The deaseasonalis A. 1131	ed figure for the for <b>B.</b> 1127	ourth quarter is C. 112	3 <b>D</b> .	1119	<b>E.</b> 1115	

# **SECTION B** Instructions

Answer **all** questions A correct answer scores 1, an incorrect answer scores 0. Marks will **not** be deducted for incorrect answers. **No** marks will be given if **more than one** answer is completed for any question.

# Module 1: Number patterns and applications

#### **Question 1**

The rule for the sequence 0, 3, 8, 15, 24, ..... is (or in the form of)

- $\mathbf{A.} \quad t_n = a + (n-1)d$
- **B.**  $t_n = ar^{n-1}$
- **C.**  $t_n = 2n 1$
- **D.**  $t_n = n^2 1$
- **E.**  $t_n = 2^n 1$

#### The following information relates to Questions 2 and 3

The last term of the sequence -121, -117, -113, -109, ..... is 43.

#### **Question 2**

The number of terms in the sequence is

<b>A.</b> 40 <b>B.</b> 41	<b>C.</b> 42 I	<b>D.</b> 43	Е.	44
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#### Question 3

In the sequence the number of terms that are positive is

	А.	10	В.	11	С.	12	D.	31	E.	32
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#### Question 4

An imaginary ladder to heaven tapers from bottom to top. The length of the bottom rung is 1 metre. For each rung the successive rung is 0.1% shorter. The total length (in metres) of the rungs in this imaginary ladder is

A.	infinite	В.	100000	C.	10000	D.	1000	E.	100
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The fifth term of a sequence generated by the rule  $t_n = 3t_{n-1} + 1$  is -2. The fourth and sixth terms are respectively

**A.** -1 and -5

**B.** 
$$-\frac{1}{3}$$
 and  $-5$ 

**C.** -1 and -7

**D.** 
$$\frac{1}{3}$$
 and 5

**E.** 1 and 5

# Question 6

An example of a first order linear difference equation is

- A.  $u_n + 2u_{n-1} = 0$
- **B.**  $t_n = t_{n+1} t_{n-1}$
- C.  $f_n^2 = -f_{n-1} + 1$
- **D.**  $p_n = 2n + 3$
- **E.**  $2a_{n-1} + 1 = 3a_{n-1} + 2$

### **Question 7**

.....,  $\frac{2}{3}$ ,  $\frac{3}{4}$ , ..... is a geometric sequence. The equation that generates this sequence is

- **A.**  $9t_n = 8t_{n-1}$
- **B.**  $8t_n = 9t_{n-1}$
- **C.**  $t_{n+1} = t_n + \frac{1}{12}$
- **D.**  $12t_n = 12t_{n-1} + 1$
- **E.**  $t_n = t_{n-1} \frac{1}{12}$

8

A person opens an account with a deposit of \$5000 at the beginning of a year. Each year thereafter she deposits \$800. The account earns compound interest of 10% per annum calculated at the end of the year. Let  $A_n$  be the total amount in dollars at the beginning of the nth year after the additional deposit of \$800. The equation that could describe the above information is

- **A.**  $A_{n+1} = 1.1A_n + 800$ ,  $A_1 = 5000$
- **B.**  $A_{n+1} = 0.1A_n + 800$ ,  $A_1 = 5000$
- C.  $A_{n+1} = 1.1A_n + 5000$ ,  $A_1 = 800$
- **D.**  $A_{n+1} = 0.1A_n + 5000$ ,  $A_1 = 800$
- **E.**  $A_{n+1} = 0.1(A_n + 5000), A_1 = 800$

#### **Question 9**

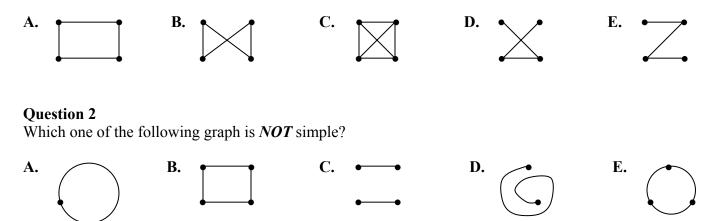
An investor bought a property in the first year, two properties in the second year, and thereafter the number of properties bought in a year was equal to the number of properties bought in the previous two years. The year that the total number of all the properties bought by the investor first reached 200 was the

<b>A.</b>	9th year	B.	10th year	C.	11th year	D.	12th year	Е.	13th year
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### Module 5: Networks and decision mathematics

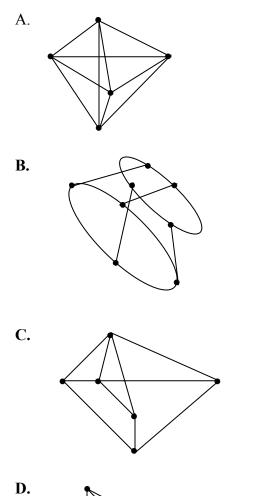
#### **Question 1**

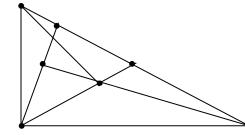
Which one of the following graphs is NOT bipartite?



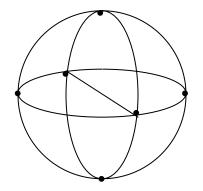
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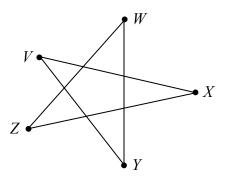
To which one of the following graphs does Euler's formula apply?





E.

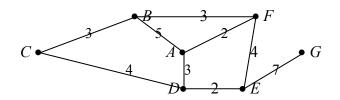




WYVXZW in the graph above is

- A. an Euler circuit but not a Hamiltonian circuit
- **B.** a Hamiltonian circuit but not an Euler circuit
- **C.** an Euler circuit but not an Euler path
- D. a Hamiltonian circuit but not a Hamiltonian path
- E. both a Hamiltonian path and an Euler path

# **Question 5**



A minimum spanning tree for the weighted graph above is

- A. AFBCDEF
- **B.** ADCBFED
- C. GEFADCB
- **D.** *GEDAFBC*
- E. BCDAFEG

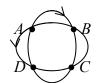
	A	В	С	D
A	$\left[ 0 \right]$	1	0	1]
A B	0	1 0 0	1	0
С	1		0	1
D	1	1	0	0

The digraph that corresponds to the adjacency matrix above is

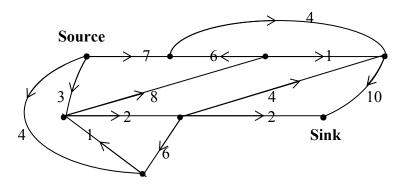
A. B. C. D.



E.



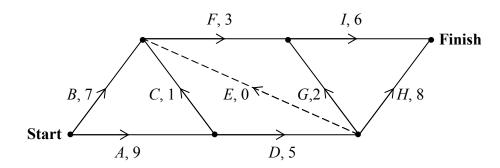
# **Question** 7



# The maximum flow for the network shown above is

14 B. 11 **C.** 8 **D.** 7 **E.** 1 A.

A project diagram with durations in hours next to the activities A, B, C, ...., I is shown below.



The shortest time in hours that the project can be completed is

A. 41 B. 23 C. 22 D. 19 E. 16

#### **Question 9**

Four grocery items *A*, *B*, *C* and *D* are for sale by four stores *W*, *X*, *Y* and *Z*. The prices (\$) of the items from each store are shown in the following table.

	A	В	С	D
W	1.2	0.95	1.5	0.9
X	1.15	0.85	1.55	0.9
Y	1.35	0.8	1.5	0.85
Ζ	1.15	0.9	1.45	0.95

A shopper selects a different item from each store. The minimum total cost (\$) for the four items is

**A.** 4.45 **B.** 4.40 **C.** 4.35 **D.** 4.30 **E.** 4.25

#### **Module 6: Matrices**

#### **Question 1**

Matrix  $X = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$  and there are 6 elements in the matrix product XY, matrix Y is possibly

A. 
$$\begin{bmatrix} 3 \\ -1 \end{bmatrix}$$
 B.  $\begin{bmatrix} -1 & 1 \end{bmatrix}$ 
 C.  $\begin{bmatrix} 0 & 1 \\ -1 & 0 \\ 0 & 3 \end{bmatrix}$ 
 D.  $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ 
 E.  $\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 3 \end{bmatrix}$ 

If 
$$2\begin{bmatrix} x^2 - 1 & -1 \\ ax & 2 \end{bmatrix} = \begin{bmatrix} 0 & c \\ b & x + 5 \end{bmatrix}$$
 and  $a, b, c$  and  $x$  are real numbers, the value of  $x$  is

**C.** 0 **A.** -2 **B.** −1 **D.** 1 **E.** 2

# **Question 3**

-z + 2y = -12z + x = 1

The set of simultaneous equations

$$-x+y=3$$

can be generated by the matrix equation

A. 
$$\begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 \\ 2 & 0 & 1 \\ -1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 3 \end{bmatrix}$$
  
B.  $\begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} 0 & 2 & -1 \\ 1 & 0 & 2 \\ 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 3 \end{bmatrix}$   
C.  $\begin{bmatrix} -1 & 2 & 0 \\ 2 & 1 & 0 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$   
D.  $\begin{bmatrix} 0 & 2 & -1 \\ 1 & 0 & 2 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$   
E.  $\begin{bmatrix} 0 & 1 & 1 \\ 2 & 0 & -1 \\ -1 & 2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$ 

**Question 4** 

If  $X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$  and  $Y = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ , then  $(X - Y)^4$  equals 8(X-Y)A. 8(X+Y)B.

- C. 4(X-Y)
- 4(X+Y)D.
- $-4Y^{2}$ E.

Given  $P = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ , which one of the following statements is **true**?

- **A.**  $P^{-1}$  is a singular matrix.
- **B.**  $P^{-1}$  does not exist.
- C. The inverse of  $P^{-1}$  is P.
- **D.** There is a matrix X such that XP = PX = I.

$$\mathbf{E}. \quad P^{-1} = \frac{I}{P}.$$

### Question 6

Matrix [0.454 0.0284] is used to convert weights in pounds and ounces to kilograms.

1 pound = 16 ounces.

If matrix  $\begin{bmatrix} 10 & 18 \\ 12 & 5 \end{bmatrix}$  gives the weights of two parcels in pounds and ounces, the matrix that gives the weights of

the parcels in kilograms is

A. 
$$\begin{bmatrix} 4.88 \\ 8.31 \end{bmatrix}$$
B.  $\begin{bmatrix} 4.88 & 8.31 \end{bmatrix}$ C.  $\begin{bmatrix} 5.05 \\ 5.59 \end{bmatrix}$ D.  $\begin{bmatrix} 5.05 & 5.59 \end{bmatrix}$ E.  $\begin{bmatrix} 5.59 & 5.05 \end{bmatrix}$ 

## **Question 7**

Which one of the following matrices is a possible transitional matrix?

A.
$$\begin{bmatrix} 1.2 & 0.8 \\ -0.2 & 0.2 \end{bmatrix}$$
B. $\begin{bmatrix} 0.6 & 0.4 \\ 0.2 & 0.5 \\ 0.2 & 0.1 \end{bmatrix}$ C. $\begin{bmatrix} 0.3 & 0.2 & 0.2 \\ 0.4 & 0.4 & 0.5 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ D. $\begin{bmatrix} 0.3 & 0.2 & 0.5 \\ 0.4 & 0.2 & 0.4 \\ 0.3 & 0.3 & 0.4 \end{bmatrix}$ E. $\begin{bmatrix} 0.6 & 0.5 & 0.1 \\ 0.4 & 0.5 & 0.9 \end{bmatrix}$ 

Two restaurants, A and B, are opened in a country town. In the first week 400 people dine at A and 600 people dine at B. In any week, 60% of people who dine at A return the next week and the remaining 40% switch to B, and 50% of people who dine at B return the next week and the remaining 50% switch to A.

In the first week the column matrix that describes the number of diners at the two restaurants is  $\begin{bmatrix} 400\\600 \end{bmatrix}$ .

#### **Question 8**

In the second week the column matrix that describes the number of diners at the two restaurants is

A.
$$\begin{bmatrix} 540 \\ 460 \end{bmatrix}$$
B. $\begin{bmatrix} 460 \\ 540 \end{bmatrix}$ C. $\begin{bmatrix} 560 \\ 440 \end{bmatrix}$ D. $\begin{bmatrix} 440 \\ 560 \end{bmatrix}$ E. $\begin{bmatrix} 520 \\ 480 \end{bmatrix}$ 

#### **Question 9**

A year later the column matrix (corrected to nearest whole numbers) that describes the number of diners at the two restaurants is

A. 
$$\begin{bmatrix} 555\\445 \end{bmatrix}$$
B.  $\begin{bmatrix} 556\\444 \end{bmatrix}$ C.  $\begin{bmatrix} 565\\435 \end{bmatrix}$ D.  $\begin{bmatrix} 566\\434 \end{bmatrix}$ E.  $\begin{bmatrix} 566\\435 \end{bmatrix}$ 

# End of Exam 1