Further Mathematics

2007

Trial Examination 2

Core – Data analysis
Module 2 – Geometry and trigonometry
Module 3 – Graphs and relations
### Module 4 – Business-related mathematics

Instructions:

Answer all questions in the core and the three modules.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π, surds or fractions.

Core – Data analysis

Question 1 A survey of 100 first marriage couples yields the following frequency distribution of the age at marriage. The column $25 \leq x < 30$ in the histogram is missing.
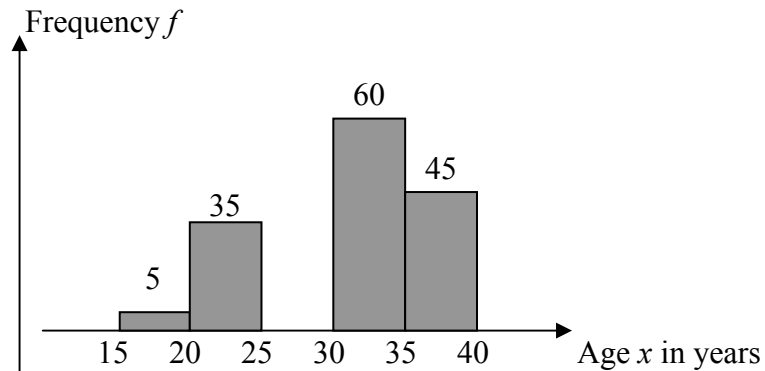
a. Draw the missing column in the diagram above. Write down the frequency just above the column.

1 mark

b. State the interval where the median age at first marriage lies.   1 mark

c. Calculate the average age at first marriage, correct to the nearest whole number.   2 marks

Question 2 A preparation of insulin is studied to determine its effect on reducing the blood-sugar level in 13 mice. The results are shown below.

Dosage D	0.20	0.20	0.25	0.25	0.25	0.30	0.30	0.40	0.40	0.40	0.50	0.50	0.50
Reduction in blood-sugar R	23	30	24	26	40	35	42	49	54	55	56	65	70

a. Determine the least squares regression line for the data set in the form $R = a + bD$ where a and b are constants (2 decimal places), and the correlation coefficient (2 decimal places).

2 marks

b. Residuals are calculated using the least squares regression line and plotted against the dosage D.
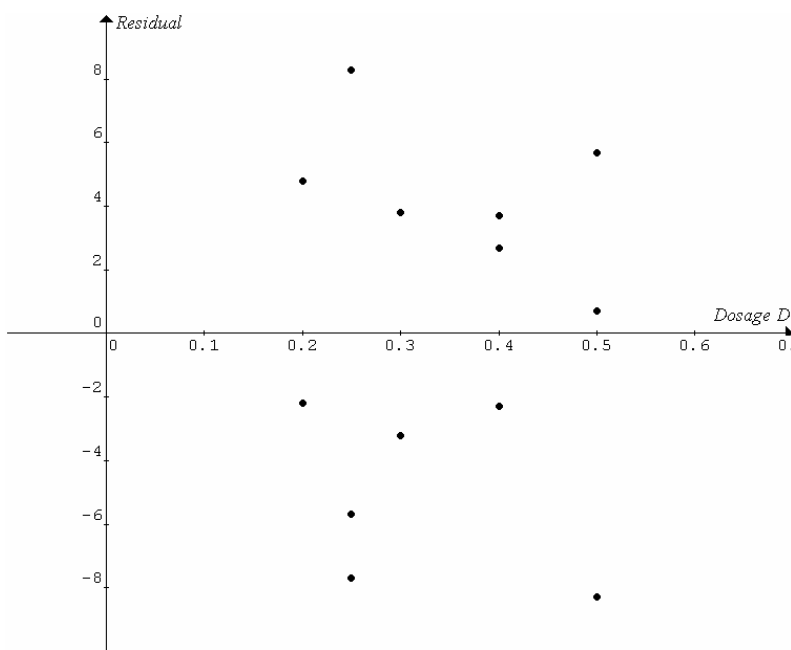
By referring to the above residual plot discuss the adequacy of the least squares regression line as a model for the data set. Would a non-linear relationship be a suitable model?

1 mark

Question 3   The total quarterly rainfall (mm) records in a region over a 3-year period are shown below.

Time t (quarters)	1	2	3	4	5	6	7	8	9	10	11	12
Rainfall r (mm)	98	86	130	112	92	85	125	108	99	87	121	105

In the table the time e.g. $t = 2$ represents the second quarter in the first year; $t = 8$ represents the fourth quarter in the second year etc.

a.   Complete the 3-median smoothing of the data in the following graph.   2 marks
(Note: $t = 7$ and $t = 8$ are missing)
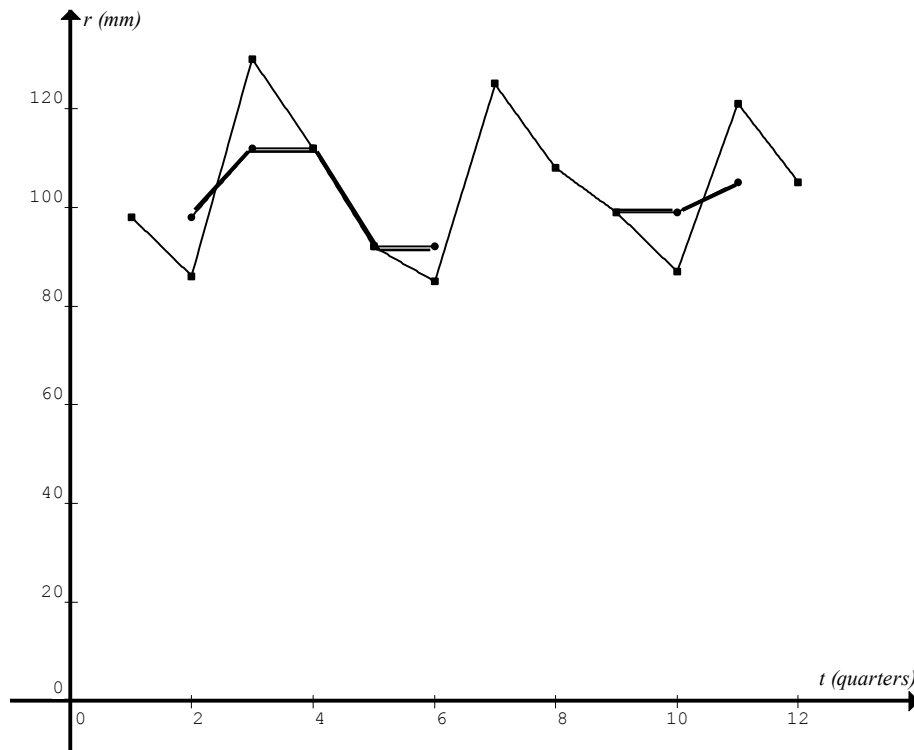
b.   The rainfall data are deseasonalised and shown in the table below.

Time t (quarters)	1	2	3	4	5	6	7	8	9	10	11	12
Deseasonalised rainfall r (mm)	105.8	104.0	107.9	107.5	99.3	102.8	103.7	103.7	106.9	105.2	100.4	100.8

Calculate the seasonal indices (3 decimal places) for the four quarters of any year in the 3-year period.
                                                                                                                        2 marks

c. The least squares regression line for the *deseasonalised* data is $r = -0.3329t + 106.2$ and the correlation coefficient $= -0.4256$. Comment on the rainfall trend in the region over the 3-year period.

1 mark

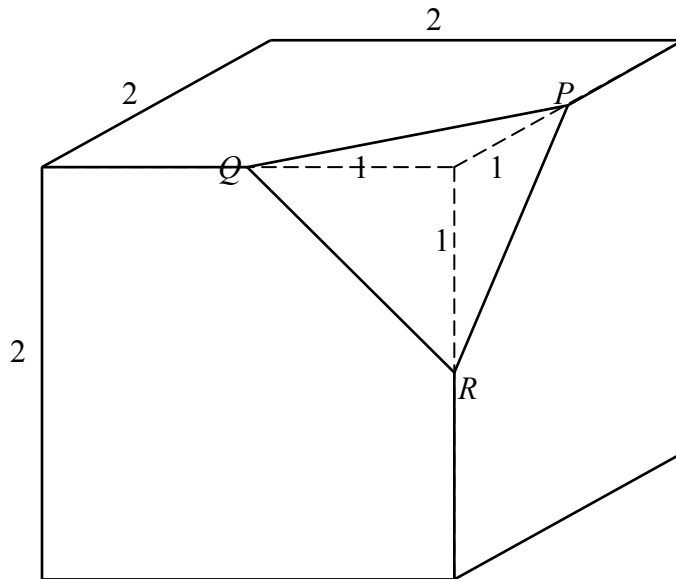**d.** Extrapolate the *seasonal* rainfall for $t = 13$, correct to 1 decimal place.

2 marks

e. Comment on the reliability of the extrapolation in part **d**.

1 mark

Total 15 marks

Module 2: Geometry and trigonometry

Question 1   A corner of a 2-m cube is removed and the resulting shape is shown in the following diagram.


a.   Find the area of ΔPQR in m^2, correct to 1 decimal place.   2 marks

b.   Find the volume in m^3 of the resulting shape if the remaining seven corners are removed the same way, Correct your answer to 1 decimal place.

2 marks

c.   A similar shape has all linear dimensions double that of the one shown in the above diagram (only one corner is removed). Calculate the total surface area of this similar shape in m^2, correct to 1 decimal place.
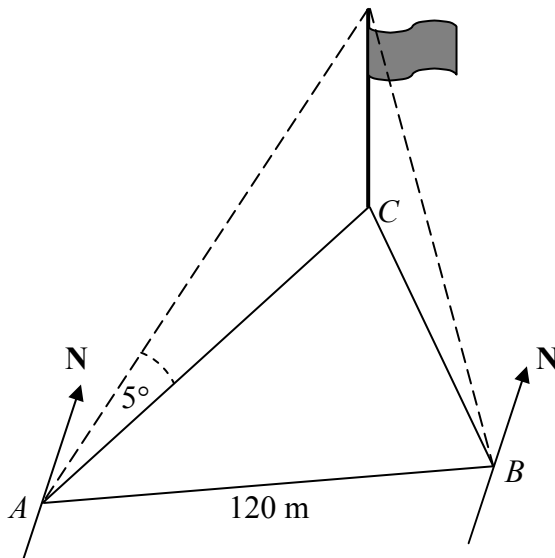
2 marks

Question 2 The contour map of a small island is shown below.
The horizontal distance from O to P is 150 m, and from P to Q is 450 m.
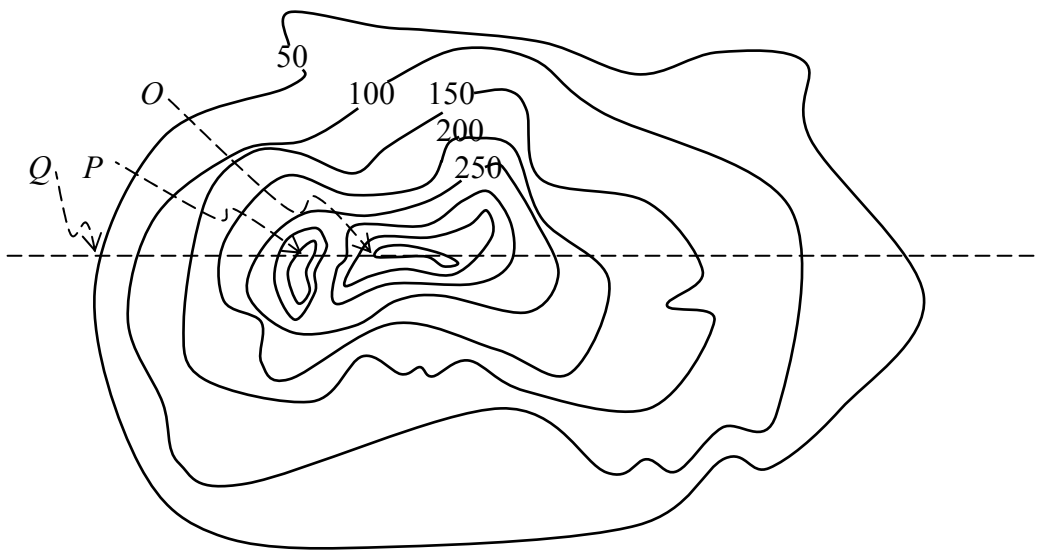
a. Find the average slope between Q and O.   1 mark

b. Can point Q be seen from point O? Justify your answer with calculations.   2 marks

Question 3 A vertical flag pole is positioned at point C. Observations of the flag pole are made from two points A and B on the same horizontal plane as C.

The bearing of C from A is 022°T, the bearing of C from B is 332°T and the bearing of B from A is 082°T. The angle of elevation of the top of the flag pole from A is 5°. The distance of B from A is 120 m.

a.   Calculate $\angle CAB$ and $\angle CBA$.   2 marks

b.   Calculate the distance of C from A, correct to 1 decimal place.   2 marks

c.   Calculate the height of the flag pole, correct to 1 decimal place.   1 mark

d.   Calculate the angle of elevation of the top of the flag pole from B, correct to 1 decimal place.   1 mark

Total 15 marks

Module 3: Graphs and relations

Question 1   The cost (C) and revenue (R) in manufacturing certain consumer goods are related to the number (*n*) of units manufactured. The graph of *C* vs *n* and the graph of *R* vs *n* are plotted on the following set of axes.
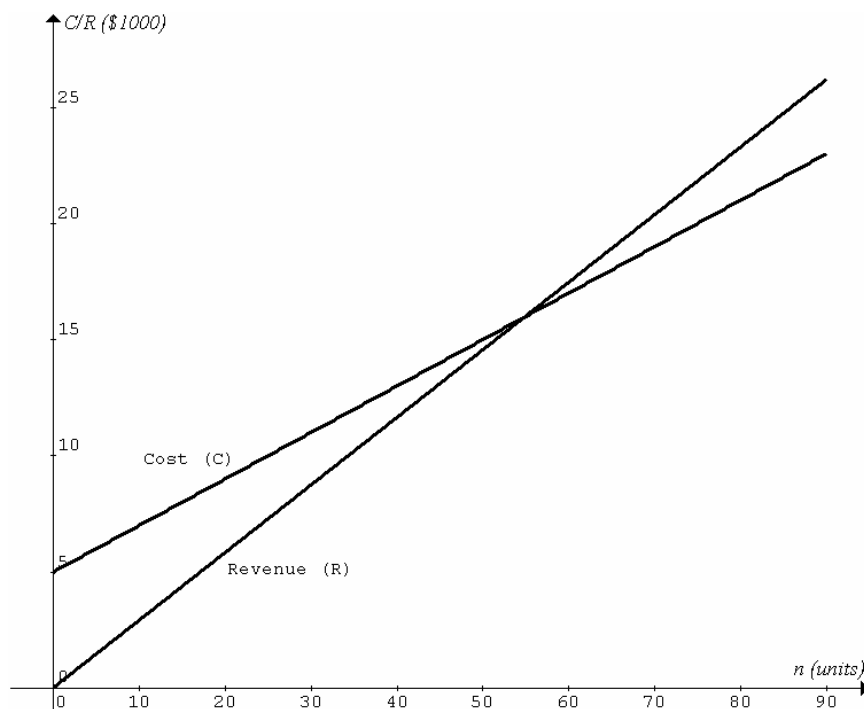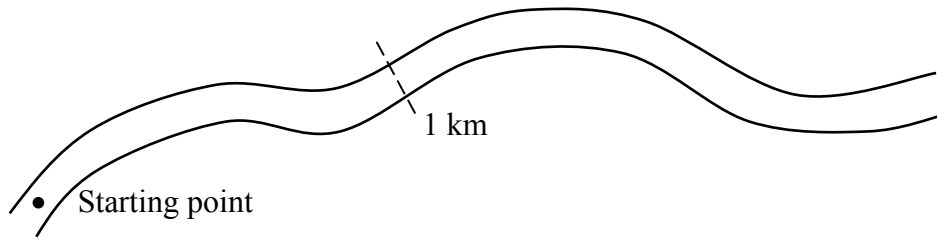

a.   Determine from the graph above the number of units to be sold to break even.   1 mark

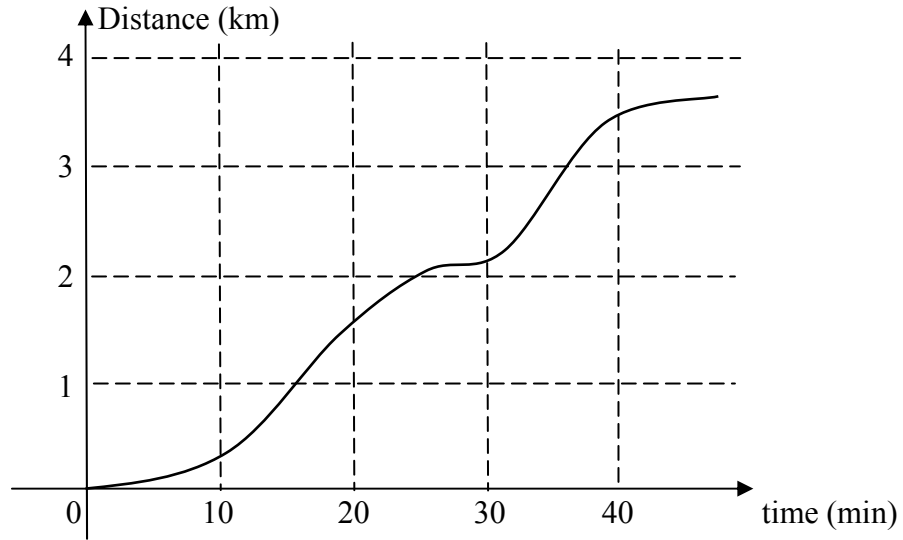**b.**   The equation for revenue is $R = \dfrac{3200}{11}n$. Determine the equation for cost.   1 mark

c.   (**i**) Determine the equation for profit *P*.   1 mark

(**ii**) Hence calculate the loss when 50 units are manufactured and sold.   1 mark

Question 2 Mr. Walker cycles along the path shown below.

The following diagram shows the distance-time graph of the trip.

a. Find the distance (in km) covered in the first 40 minutes.   1 mark

b. Calculate the average speed (in km per hour) in the first 40 minutes, correct to 1 decimal place.

                                                                                                             1 mark

c. Estimate the highest speed in the first 40 minutes, correct to the nearest km per hour.   1 mark

Question 3   Ben Murfey Wine Wholesaler sells two different types of red wine, Type X and Type Y. The profit on the sales of Type X is \$1.80 per litre, and on Type Y is \$1.20 per litre. (Modified VCAA sample question)

The overall profit is P (in **thousands** of dollars) when x **thousand** litres of Type X and y **thousand** litres of Type Y are sold.

a.   Determine a formula for P in terms of x and y.   1 mark

Type X is a blend in which half the wine is Cabernet wine and half is Shiraz wine. Type Y is a blend in which one-sixth is Cabernet wine, one-third is Shiraz wine and the remainder is Mataro wine.

In a typical year, a total of 15 thousand litres of Cabernet wine, 20 thousand litres of Shiraz wine and 22.5 thousand litres of Mataro wine are available for blending into Type X and Type Y wines.

x thousand litres of Type X wine and y thousand litres of Type Y wine are produced in a typical year.

b.   Write down a set of 5 constraints for x and y as linear inequalities.   2 marks

The constraints are sketched on the same set of axes as shown in the following diagram.
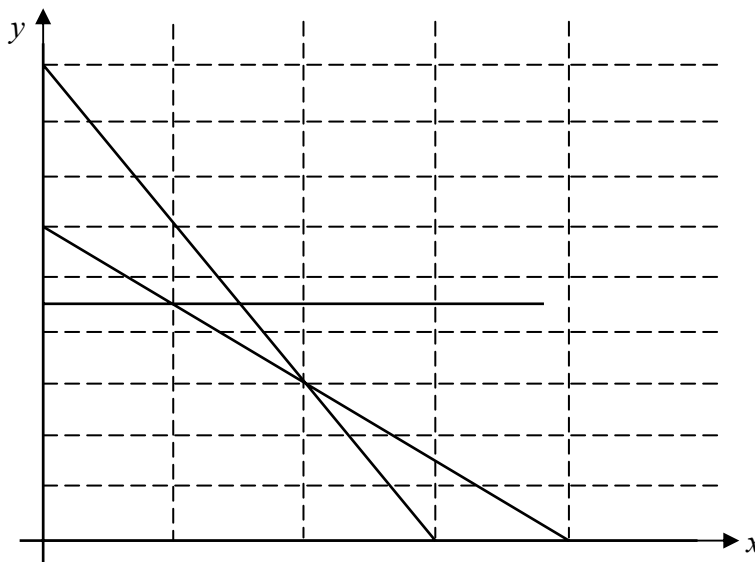
c.   In the above diagram label the vertices of the feasible region with coordinates.   2 marks

d. Find the greatest number of litres of Type X wine and the least number of litres of Type Y wine that should be produced for a maximum profit.

2 marks

e. Determine the maximum possible profit. 1 mark

Total 15 marks

Module 4: Business-related mathematics

Question 1 Inflation rates in a certain country for 2005 and 2006 were 5% and 10% respectively.

a. Use these rates to estimate the total cost of 20 selected grocery items at the start of 2005 if the total cost of the same items was $100.00 at the start of 2007.

2 marks

b. Calculate the average annual inflation rate over the two years (2005 and 2006). Correct your answer to two decimal places.

2 marks

Question 2   A bank savings passbook showed the following entries for August.

Date	Deposit	Withdrawal	Balance
1/8			$2325.80
3/8		$201.50	
12/8	$570.00		
17/8		$89.75	
23/8		$364.20	
29/8	$230.00		

a.   Calculate the minimum monthly balance for August.   2 marks

b.   The bank pays 3.5% p.a. monthly on the minimum monthly balance. Calculate the interest for August.

2 marks

c.   Calculate the balance on 1st September after the interest is entered into the savings account.   1 mark

Question 3 A bank offers a reducing balance (adjusted monthly) home loan of $180000 over 20 years at 8%.

a. Find the minimum amount of monthly repayment required. 2 marks

b. Calculate the total amount of interest paid over the 20 years. 1 mark

A borrower negotiated a reduced rate of 7.35% and agreed to repay the $180000 reducing balance (adjusted monthly) home loan with monthly repayment of $1500.

c. Calculate the term of the loan in years and months. 1 mark

d. Calculate the amount of the repayment in the last month of the term. 2 marks

Total 15 marks

End of exam 2