2007

VCE
Further Mathematics
Trial Examination 1

Suggested Solutions

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Question 1 A

$$\frac{61}{73+61+146}\times\frac{100}{1}=\frac{61}{280}\times\frac{100}{1}=21.786\%$$

Question 2 D

Having or not having a water tank or having one on order is categorical data. Scatter plots and histograms are used when both variables are numerical. For a stem and leaf you need two categories, but there are three categories here. You cannot find the median, lower quartile, etc. so cannot draw a box plot. Bar graphs are used for categorical data.

Question 3 C

A long whisker to the left indicates the box plot is negatively skewed.
The interquartile range = 80 – 60 = 20

Question 4 C

99.7% lie within 3 standard deviations of the mean, so 0.3% lie outside this range. Half of 0.3% = 0.15%. this means 0.15% lie above 3 standard deviations above the mean.
$43 + 3 \times 15 = 88$.
So the lowest mark for an A is 88

Question 5 A

The standard deviation remains at 5 g., because the spread is unaltered. The mean weight of each bag is increased by 2g to 452g.

Question 6 B

$r^2 = 0.1296$

$r = \pm\sqrt{0.1296}$

$r = \pm 0.36$

But the gradient of the line is negative, (–0.73) so the correlation coefficient must be negative

$\therefore r = -0.36$

Question 7 B

Draw 2 vertical lines so that there are 3 points to the left of the first vertical line, 4 points in the middle, and 3 points to the right of the last vertical line.

$x_l = 2, y_l = 2$

$x_u = 10, y_u = 18$

$$\text{gradient} = \frac{18-2}{10-2} = \frac{16}{8} = 2$$

Question 8 E

The gradient is approximately –28.
This means that for every dollar increase in fares, the number of passengers decreased by 28.

Question 9 D

For y versus x the graph would look like A or C, not B. However, the graph would contain the point (1,2). Hence, not A, B, or C.
For y versus log x the graph would look like D or E but would contain the point (1,5).
Hence, D

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Question 10 B

Enter data in graphics calculator in L_1 and L_2 using Stat Edit

Press Stat Calc Linear Regression Line L_1, L_2

This gives $r^2 = 0.305$

Question 11 D

To use seasonal indices, there must be a secular trend and the variation must be seasonal.

Question 12 D

Quarterly average = $\frac{240 + 226 + 210 + 120}{4} = 199$

Seasonal Index for 3rd quarter = $\frac{210}{199} = 1.055$

Question 13 A

The median of the first three points is 500 so the point (1985,500) should lie on the graph, hence C, D or E. The median of points 3,4, and 5 is 500, so the third point on the smoothed graph should be (1995,500). Hence E

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Question 1 D

Each term in the sequence is found by adding 6 to the term before it. Hence it is an arithmetic sequence with a common difference of 6

Question 2 B

This is a geometric sequence with a common ratio of $-\frac{1}{2}$

$a = 8, n = 12$

$t_n = ar^{n-1}$

$t_n = 8\left(-\frac{1}{2}\right)^{11} = -\frac{1}{256}$

Question 3 C

$t_7 = 3 \times t_6 + 4$

$8017 = 3 \times t_6 + 4$

$8013 = 3 \times t_6$

$2671 = t_6$

$t_6 = 3 \times t_5 + 4$

$2671 = 3 \times t_5 + 4$

$2667 = 3 \times t_5$

$889 = t_5$

Question 4 C

$t_4 = t_{1+3} = t_{1+2} + t_{1+1} + t_1 = t_3 + t_2 + t_1 = 1 + 1 + 1 = 3$

$t_5 = t_{2+3} = t_{2+2} + t_{2+1} + t_2 = t_4 + t_3 + t_2 = 3 + 1 + 1 = 5$

Question 5 C

$4 + 6 + 8 + \ldots$

Sum of Arithmetic Sequence

$\therefore S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}[8 + 9 \times 2] = 5 \times 26 = 130\,\text{m}$

Question 6 E

Value at beginning of first year = 32,000

Value at beginning of second year = 32,000(0.85)

Value at beginning of third year = $32{,}000(0.85)^2$

Following the pattern,

Value at beginning of fifth year = $32{,}000(0.85)^4$

= \$16704.20

This is closest to \$16704

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Question 7 D

Arithmetic Sequence.

$a = 85$

$d = -7$

$t_n = a + (n-1)d = 85 + (n-1)(-7)$

Use sequence mode on graphics calculator and press $y =$

$n(min) = 1$

$u(n) = 85 + (n-1)(-7)$

$u(n \text{ min}) = 85$

Press second table and scroll down to get -48

This corresponds to $n = 20$

Question 8 A

$$\frac{72}{x} = \frac{x}{8}$$

$x^2 = 576$

$x = \pm\sqrt{576} = \pm 24$

Common ratio $= \dfrac{\pm 24}{8} = \pm 3$

Question 9 D

$$S_\infty = \frac{a}{1-r} = \frac{2}{1-\frac{2}{5}} = 3.3333$$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{2(1-0.4^n)}{0.6}$$

Enter this equation in sequence mode in the graphics calculator

$n(\text{min}) = 1$

$u(n \text{ min}) = 2$

Using the table scroll down

$n = 5 \quad u(n) = 3.2992$

$n = 6 \quad u(n) = 3.3197$

$n = 7 \quad u(n) = 3.3279$

$3.3333 - 3.2992 = 0.0341$ which is too big

$3.3333 - 3.3197 = 0.0136$ which is too big

$3.3333 - 3.3279 = 0.0054$ which is less than 0.01

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Question 1 E

$b^2 = a^2 + c^2 - 2ac\cos 50^0$

$b^2 = 27^2 + 41^2 - 2 \times 27 \times 41 \times \cos 50^0$

$b = \sqrt{27^2 + 41^2 - 2 \times 27 \times 41 \times \cos 50^0}$

$b = 31.4$

AC is closest to 31cm

Question 2 D

$\text{Area} = \frac{1}{2} ac \sin b$

$\text{Area} = \frac{1}{2} \times 4 \times 7 \times \sin 64^0$

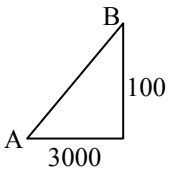
$\text{Area} = 12.58 \text{ cm}^2$

Question 3 E

In this right angled isosceles triangle,
AB = AC, so $\angle BAC = \theta$.
The sum of the angles of a triangle = 180^0
$\therefore \theta = 45^0$

$\sin 45^0 = \cos 45^0 = \frac{1}{-} 0.707$

Question 4 A



$\text{gradient} = \frac{100}{3000} = 0.033$

which is closest to 0.03

Question 5 C

In $\Delta ADB, AD^2 = 20^2 - 16^2$

$AD = \sqrt{20^2 - 16^2} = 12$

In $\Delta BDC, DC^2 = 34^2 - 16^2$

$DC = \sqrt{34^2 - 16^2} = 30$

$AC = AD + DC = 12 + 30 = 42 \text{ cm}$

Question 6 E

Let the length of the original cube = x
Then the original volume = x^3
Length of larger cube = $4x$
Volume of larger cube = $(4x)^3 = 64x^3$
$\therefore$ volume has increased by a factor of 64

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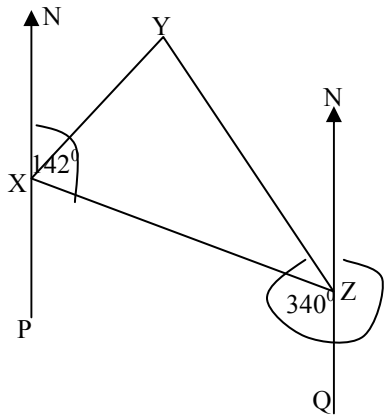
Question 7 A



$\angle NXZ = \angle XZQ$ (alternate angles) $= 142^0$

$\angle NZQ = 180^0$

$\angle XZY = 340 - (180 + 142) = 18^0$

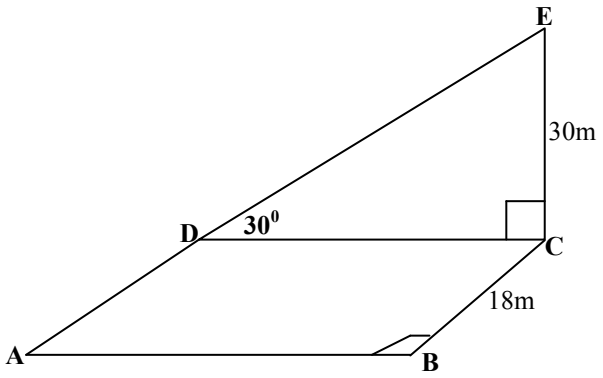
Question 8 C

Δs ABO and CDO are similar (AAA)

$\therefore \dfrac{x}{5.2} = \dfrac{2.5}{10}$

$x = \dfrac{2.5 \times 5.2}{10} = 1.3 \text{ m} = 130 \text{ cm}$

Question 9 A



$\tan 30^0 = \dfrac{30}{DC}$

$DC = \dfrac{30}{\tan 30^0} = 51.9615 = AB$

$AC^2 = AB^2 + 18^2$

$AC^2 = 51.9615^2 + 18^2$

$AC = \sqrt{51.9615^2 + 18^2}$

$AC = 54.99$

$\tan\theta = \dfrac{30}{AC} = \dfrac{30}{54.99}$

$\theta = \tan^{-1}\left(\dfrac{30}{54.99}\right) = 28.6^0$ which is closest to 29^0

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Question 1 C
They both meet where the lines meet. Read the distance from the distance axis. This is 15 km.

Question 2 E
Jake travels 25 + 25 = 50 km
Jenny travels 5 + 15 = 20 km.
So Jake travels 30 km further than Jenny

Question 3 E
$28 = a \times 2^3$
$28 = a \times 8$
$a = \frac{28}{8} = 3.5$

Question 4 D
$2 \times 3 - 5 \times 2 + b = 0$
$6 - 10 + b = 0$
$b = 10 - 6 = 4$

Question 5 B

The charge per hour is the gradient of the line
$= \frac{90 - 30}{3 - 0} = \frac{60}{3} = 20$
The fixed cost is the y intercept = 30

Question 6 D

The shaded region is above the line or equal to the line $y = x - 3$
$\therefore y \geq x - 3$
$\therefore$ not C or E
The vertical line is $x = 5$ and the required region is greater than or equal to this line
$\therefore x \geq 5,$ so not B
The horizontal line is $y = 4$ and the required region is less than or equal to this line
so $y \leq 4$
$\therefore$ D

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Question 7 B

$3y = -6x + 2$

$y = -2x + \frac{2}{3}$

gradient of this line is -2

$\therefore$ gradient of parallel line is -2

$y = -2x + c$

when $x = 4$, $y = 10$

$10 = -2 \times 4 + c$

$10 = -8 + c$

$c = 18$

$\therefore y = -2x + 18$

$y + 2x - 18 = 0$

Question 8 E

If you multiplied equation 1 by 2 and equation 2 by 3 you would have $-6b$ in each equation. The b could be eliminated by subtraction.

Question 9 A

Eqn of revenue $R = mx + c$

Points on revenue line are (0,0) and (200,600)

$m = \frac{600 - 0}{200 - 0} = 3$

$y = 3x + c$

c is the y intercept = 0

$R = 3x$

Eqn of cost $C = mx + c$

Points on cost line are (0,400) and (200,600)

$m = \frac{600 - 400}{200 - 0} = \frac{200}{200} = 1$

$c = 400$

$C = x + 400$

When $x = 150$, $C = 150 + 400 = 550$

When $x = 150$, $R = 3 \times 150 = 450$

Therefore, loss = 100

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Question 1 C

$$R = \frac{100I}{PT} = \frac{100 \times 945}{4800 \times 3} = 6.5625$$

This is closest to 6.6%

Question 2 D

Amount paid = $7000 + 24 \times 1330 = \$38,920$

Interest = Amount paid – Cost

Interest = $38920 - 36000 = \$2,920$

Question 3 B

Sale price = 85% of 1300 = 1105

Price paid by customer = 88% of 1105

=\$972.40

Question 4 C

Use TVM solver under finance on a graphics calculator.

$N = 48$

$I = 10$

$Pv = -3000$

$PMT = 0$

$FV =$

$P/Y = 4$

$C/Y = 4$

This gives $FV = 9814.47$

Interest = Final value – Amount invested

Interest = 9814.47 – 3000 = 6814.47

This is closest to \$6814

Question 5 C

On 9 Jan. have 7648 + 2004 = 9652

On 29 Jan have 7632

Amt. taken out of account = 9652 – 7632

= \$2020

Question 6 B

Interest will be paid for the months of Dec., Jan. and Feb.

Dec. min monthly balance = 9243

Jan. min monthly balance = 7632

Feb. min monthly balance = 7532

$$\text{Interest for Dec.} = 9243 \times \frac{1}{100} \times 1 = 92.43$$

$$\text{Interest for Jan.} = 7632 \times \frac{1}{100} \times 1 = 76.32$$

$$\text{Interest for Feb.} = 7532 \times \frac{1}{100} \times 1 = 75.32$$

Total Interest $= 92.43 + 76.32 + 75.32 = \$244.07$

Question 7 E

$(0.92)^n \times 2000 = 800$

Enter $y = (0.92)^n \times 2000$ in graphics calculator

Then 2nd Table

When $x = 11$ $y = 799.27$

So 11 years.

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Question 8 E

Compound interest rate increases more rapidly with time. This means the gradient of the graph increases with time. The only graph that does this is E

Question 9 C

Use TVM solver under finance on a graphics calculator.

$N = 48$

$I = 9.2$

$PV = 20000$

$PMT = -300$

$FV =$

$P/Y = 12$

$C/Y = 12$

This gives $FV = 11,395.90$

Paid back lump sum of 2000 leaves 9395.90 owing

$N =$

$I = 9.2$

$PV = 9395.90$

$PMT = -400$

$FV = 0$

$P/Y = 12$

$C/Y = 12$

This gives N = 25.8 months which is closest to 2 years.

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Question 1 A A complete graph has each vertex linked to every other vertex	**Question 2 E** For an Euler circuit to exist, each vertex must be of even degree.
Question 3 E A to A is 0 B to B is 0 C to C is 1 D to D is 0 Hence, the diagonal is 0,0,1,0	**Question 4 C** A tree is a connected graph with no loops, circuits or multiple edges. A, B, D, and E have loops, multiple edges or circuits.
Question 5 E Degree of A = 3 Degree of B = 4 Degree of C = 5 Degree of D = 3 Degree of E = 5 Degree of F = 4 Need a line joining any two of A, C, D or E, so that two of the vertices only remain odd. From given choices it has to be C to D	**Question 6 D** Cameron is the parent of Donald and Anthea, not of Chris

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Question 7 B	**Question 8 B**
F is a prerequisite for D so not C C is a prerequisite for D so not A or D D is a prerequisite for E so not E	Ben has dominance over Carrie who in turn has dominance over David.

Question 9 B

B,C,D,G lie on the critical path because their float time is zero. Hence, A is not true.
If all the crash times are implemented, only those on the critical path will reduce the overall time of the project. Hence, C is not true. E does not lie on the critical path, so D is not true. The total time for the project is the length of the critical path so E is not true. E does not lie on the critical path, so it does not affect the overall time of the project.

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Question 1 A

det A = (4 × –3) – (–2 × –1) = –12 – 2 = –14

Question 2 D

$$2A = \begin{bmatrix} 2a & 2b \\ 2c & 2d \end{bmatrix}$$

$$2A - B = \begin{bmatrix} 2a-e & 2b-f \\ 2c-g & 2d+h \end{bmatrix}$$

Question 3 B

AB does not exist because the number of columns of A does not equal the number of rows of B.

BA is a 4 × 3 matrix. Order of BA is number of rows of B × number of columns of A

Question 4 C

A has no solutions because the lines are parallel.

B has one, unique, solution. (2,4)

D has a unique solution because

det D = 3 – 2 = 1

E has a unique solution because

det E = –12 – 12 = –24

C has many solutions because the two lines are the same.

Question 5 A

This can be done on the graphics calculator or

$$A - B = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix}$$

$$C^2 = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 4 \\ 6 & 7 \end{bmatrix}$$

$$(A - B)C^2 = \begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix} \times \begin{bmatrix} 7 & 4 \\ 6 & 7 \end{bmatrix} = \begin{bmatrix} 20 & 15 \\ 26 & 22 \end{bmatrix}$$

Question 6 E

From *Y* to *X* is 0.25 so the first row second column is 0.25 so not B or C or D

From *X* to *Z* is 0.05 so not A

Hence, answer E

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Question 7 A

S_0 is the initial number of people choosing milk and dark chocolate.

$$S_0 = \begin{bmatrix} 2180 \\ 1260 \end{bmatrix}$$

Question 8 B

$$T = \begin{bmatrix} 0.72 & 0.35 \\ 0.28 & 0.65 \end{bmatrix}$$

$$T^5 S_0 = \begin{bmatrix} 0.72 & 0.35 \\ 0.28 & 0.65 \end{bmatrix} \text{ ^ } 5 \times \begin{bmatrix} 2180 \\ 1260 \end{bmatrix} = \begin{bmatrix} 1913 \\ 1527 \end{bmatrix},$$

using graphics calculator

Number choosing dark chocolate = 1527

Question 9 D

$$\begin{array}{c} \\ \text{X} \\ \text{Y} \\ \text{Z} \end{array} \begin{array}{c} \begin{array}{ccc} \text{D} & \text{N} & \text{C} \end{array} \\ \begin{bmatrix} 50 & 60 & 200 \\ 20 & 100 & 30 \\ 80 & 40 & 70 \end{bmatrix} \end{array} \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 100+60+600 \\ 40+100+90 \\ 160+40+210 \end{bmatrix} = \begin{bmatrix} 760 \\ 230 \\ 410 \end{bmatrix}$$

End of suggested solutions
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