2007

VCE
Further Mathematics
Trial Examination 2

Suggested Solutions

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Question 1 a.

	Year 7	Year 8	Year 9	Year 10	Year 11	Year 12	Total
Smoke	4	16	21	36	39	50	166
Do not Smoke	71	59	54	39	36	25	284

$\frac{1}{2}$ mark for each correct value

b.

$\frac{93}{150} \times 100 = 62\%$

(1 mark)

c.

Smoking/non smoking is a categorical variable

(1 mark)

Question 2 a.

On graphics calculator enter data for number of cigarettes smoked for 12 children in Stat Edit in L_1. Go to Stat calc 1 Var Stats L_1 Enter and read off the value for the median Med=4.5

$\left(\frac{1}{2} \text{ mark}\right)$

b.

Using the same screen on the calculator read off Q_1, the lower quartile = 2.5

$\left(\frac{1}{2} \text{ mark}\right)$

c.

Using the same screen on the calculator read off Q_3, the upper quartile = 7.5

$\left(\frac{1}{2} \text{ mark}\right)$

d.

Interquartile range = $Q_3 - Q_1 = 7.5 - 2.5 = 5$

$\left(\frac{1}{2} \text{ mark}\right)$

e.

$1.5 \times 5 = 7.5$

$Q_1 - 7.5 < 0$

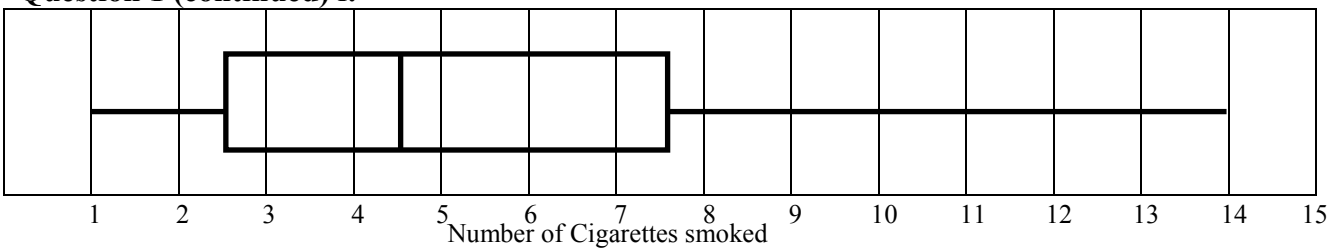
$Q_3 + 7.5 = 15$

No data lies inside the range $0 < x < 15$

$\therefore$ no outliers

(1 mark)

Question 1 (continued) f.



(1 mark)

g. 75%

(1 mark)

Question 3

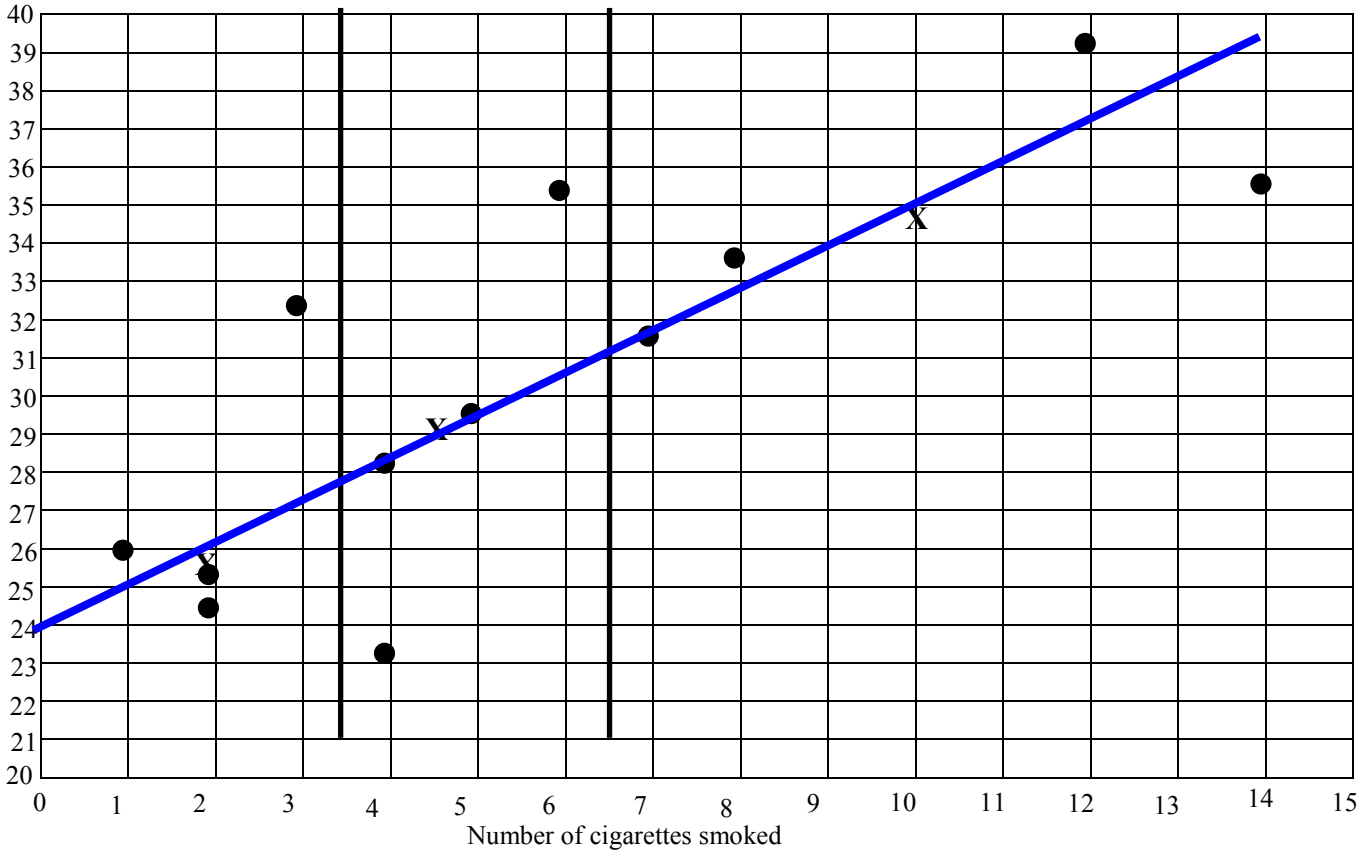
a. On graphics calculator enter data for the time taken to run 200 m for 12 children in Stat Edit in L_2. Go to Stat calc 1 Var Stats L_2 Enter and read off the value for the mean, 30.5 and the standard deviation, 5.0

(1 mark)

b. On graphics calculator go to 2nd Dist normal cdf Enter –1, 2.5) enter. this gives 0.835 = 83.5%

(1 mark)

c.



(1 mark)

Question 3 (continued)

d.

$$\text{gradient} = \frac{y_u - y_l}{x_u - x_l}$$

$x_l = 2$

$$y_l = \frac{25.4 + 26}{2} = 25.7$$

$$x_u = \frac{8 + 12}{2} = 10$$

$$y_u = \frac{33.9 + 35.6}{2} = 34.75$$ (1 mark)

$$\text{gradient} = \frac{34.75 - 25.7}{10 - 2}$$

$= 1.13$ to two decimal places. (1 mark)

e.

Put ruler on line joining points (2, 25.7) and (10, 34.75). Move ruler one third of the way towards (4.5, 29.05).

This is

$$x_m = \frac{4 + 5}{2} = 4.5$$

and

$$y_m = \frac{28.3 + 29.8}{2} = 29.05$$

(1 mark)

Question 1 a.
Customers increase by 30 each year.
2004 = 140 + 30 = 170
2005 = 170 + 30 = 200
(1 mark)

b.
Arithmetic sequence
$t_n = a + (n-1)d$
$t_{11} = 80 + 10 \times 30$
$t_{11} = 80 + 300$
$t_{11} = 380$
or use graphics calculator in sequence mode.
$y =$
enter
$nMin = 1$
$m_n = 80 + (n - 1)30$
$m(nMin) = 80$
Press 2nd table
Go to $n = 11$, to find $m(n) = 380$
(1 mark)

c.
Use sequence mode $y =$
$nMin = 1$
$\mu_n = 80 + (n-1)30$
$\mu(nMin) = 80$
Press 2nd table
Go to $\mu_n = 500$, to find $n = 15$
Year = 2015
(1 mark)

d.
From the table in (c)
Number of customers in 2009 = 320
Number of customers in 2019 = 620
$S_n = \frac{n}{2}(a + l) = \frac{11}{2}(320 + 620) = 5170$
(1 mark)

e.
$N = 800 - 40d$
(1 mark)

Question 2 a.
4%=0.04
3.00 + 0.04 × 3.00 = \$3.12
(1 mark)

b.
This is a geometric sequence with
$r = 1.04$
$a = 3.00$
$C_n = 3.00(1.04)^{n-1}$
(1 mark)

Question 2 c.

$t_{10} = 3(1.04)^9$

$t_4 = 3(1.04)^3$

$t_{10} - t_4 = 3(1.04)^9 - 3(1.04)^3 = 0.8953$

= 90 cents

or use graphics calculator sequence mode with equation

$m(n) = 3(1.04)\bar{o}\, n$

This gives $m(n) = 4.27$ when $n = 10$

and $m(n) = 3.37$ when $n = 4$

$4.27 - 3.37 = 0.90 = 90$ cents.

(1 mark)

d.

Total cost if she sells one rose per year

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_6 = \frac{3(1.04^{10} - 1)}{(1.04 - 1)} = 36.01832137$$

(1 mark)

total cost for 8000 per year

$= 8000 \times 36.01832137 = \$288,146.57$

(1 mark)

Question 3 a.

$$a = 1 + \frac{20}{100} = 1.2$$

(1 mark)

b.

A minus sign is necessary because overheads decrease the profit.

− 10,000

(1 mark)

c.

Use graphics calculator in sequence mode.

$y =$

enter

$nMin = 1$

$\mu(n) = 1.2\,\mu(n - 1) - 10000$

$\mu(nMin) = 65000$

Press 2nd table

Go to $n = 8$, to find $\mu(n) = 103748$

Profit = \$103,748

(1 mark)

d.

Equation is now $\mu_n = 1.2\mu(n-1) - 30000$

(1 mark)

Use graphics calculator in sequence mode.

$y =$

enter

$nMin = 1$

$\mu(n) = 1.2\,\mu(n - 1) - 30000$

$\mu(nMin) = 103748$

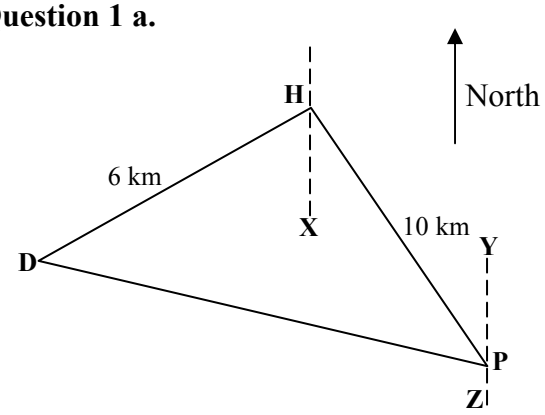
Press 2nd table and look for the first negative value in $\mu(n)$.

8 years later she will make her first loss.

This is the year 2016.

(1 mark)

Question 1 a.



$\angle DHX = 40^0$

$\angle XHP = \angle HPY = 30^0$ (alternate angles)

$\therefore \angle DHP = 40^0 + 30^0 = 70^0$

(1 mark)

b.

Using cosine rule

$DP^2 = 6^2 + 10^2 - 2 \times 6 \times 10 \cos 70^0$

$DP = \sqrt{6^2 + 10^2 - 2 \times 6 \times 10 \cos 70^0}$

$DP = 9.74$ km

(1 mark)

c.

$$\frac{\sin \angle HPD}{6} = \frac{\sin 70^0}{9.74461845}$$

$$\sin \angle HPD = \frac{6 \sin 70^0}{9.74461845}$$

$$\angle HPD = \sin^{-1}\left(\frac{6 \sin 70^0}{9.74461845}\right) = 35^0 \text{ (1 mark)}$$

$\angle DPZ = 360 - (30 + 35) = 295^0 T$ (1 mark)

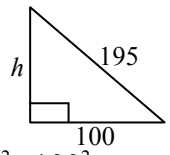
d.

$$\text{Area} = \frac{1}{2} bc \sin A$$

$$\text{Area} = \frac{1}{2} \times 6 \times 10 \sin 70^0 = 28.19 \text{ km}^2$$

(1 mark)

Question 2 a.



$h^2 = 195^2 - 100^2$

$h = \sqrt{195^2 - 100^2}$

$h = 167.4$ m to one decimal place

(1 mark)

b.

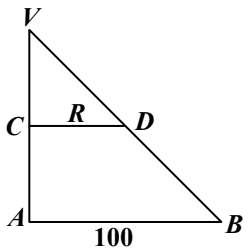
Triangles *VCD* and *VAB* are similar AAA

$$\therefore \frac{R}{100} = \frac{3}{4}$$

$\therefore R = 75$ m.

(1 mark)

c.



(1 mark)

Question 2 (continued) c.

Original volume $= \frac{1}{3}\pi r^2 h$

Original volume $= \frac{1}{3}\pi \times 100^2 \times 167.4$

$= 1753008.701$

Volume removed $= \frac{1}{3}\pi \times 75^2 \times 125.55$

$= 739550.5456$ (1 mark)

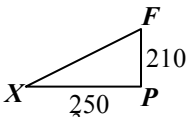
Volume of coal remaining

$= 1753008.701 - 739550.5456$

$= 1013458.2 \text{ m}^3$

(1 mark)

Question 3 a.



$XF^2 = 210^2 + 250^2$

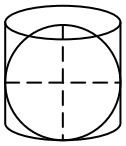
$XF = \sqrt{210^2 + 250^2}$

$= 326.5$ m

(1 mark)

b.

Radius of sphere $= 8$



V of sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi 8^3 = 2144.66 \text{ cm}^3$

(1 mark)

R of cylinder $= 8$

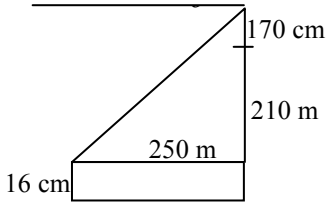
V of cylinder $= \pi r^2 h = \pi \times 8^2 \times 16 = 3216.99$

Space $= 3216.99 - 2144.66 = 1072 \text{ cm}^3$

to the nearest cm^3

(1 mark)

c.



(1 mark)

Question 3 d.

$\tan\theta = \frac{210 + 1.7 - 0.16}{250} = \frac{211.54}{250}$

(1 mark)

$\theta = \tan^{-1}\left(\frac{211.54}{250}\right) = 40^0 14'$

(1 mark)

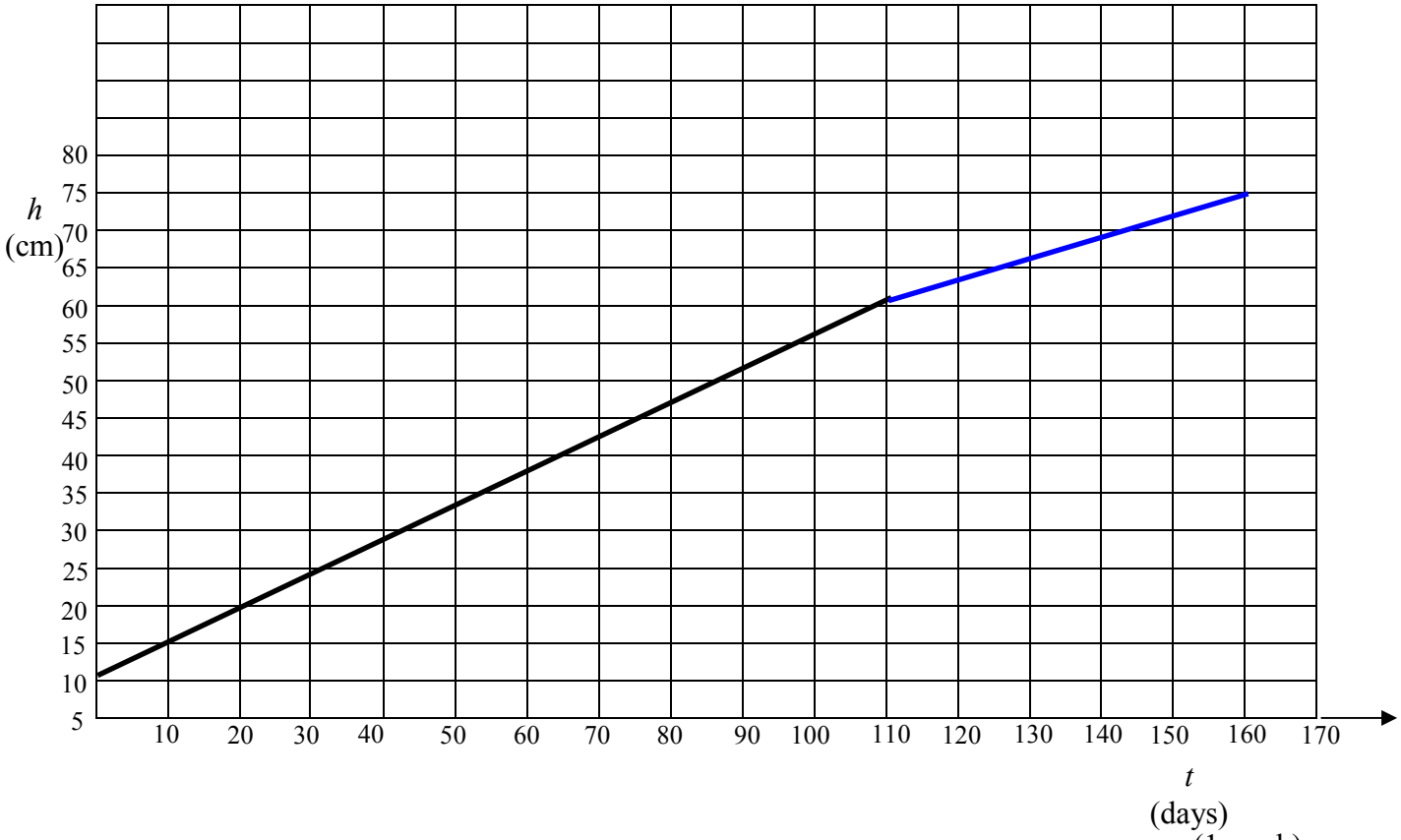
Question 1 a.
30 cm.
(1 mark)

b.
One complete cycle = 4 secs.
(1 mark)

c.
30 cm to the right of the rest position.
(1 mark)

Question 2

a.



(1 mark)

b.
Plant grows 50 cm in 110 days.

$\frac{50}{110} = 0.45$ cm/day
(1 mark)

c.
$h = 0.45t + 10$
(1 mark)

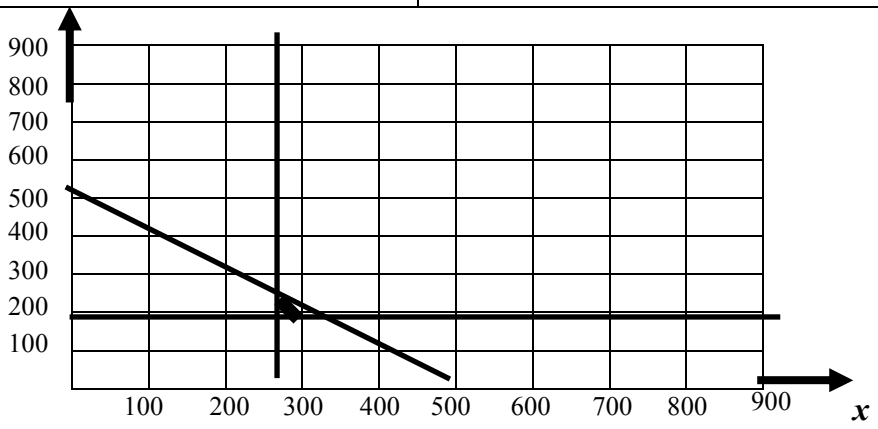
d.
Value of a is h value at end of first period of growth = 60
(1 mark)

e.
$h = 0.1t + 60$
$t = 160$
$h = 0.1 \times 160 + 60 = 76$ cm.
Join (110,60) and (160,76) on graph.
(1 mark)

Question 3 a.
$y \geq 180$
(1 mark)

b.
$x + y \leq 500$
(1 mark)

c.



($\frac{1}{2}$ mark for horizontal line) , ($\frac{1}{2}$ mark for vertical line) , ($\frac{1}{2}$ mark for line $x + y = 500$)

($\frac{1}{2}$ mark for shading)

d.
$P = 45x + 60y$
(1 mark)

e.
Points of intersection on graph are
(260,180), (260,240) and (320,180)
At (260,180) P = 45 × 260 + 60 × 180 = 22,500
At (260,240) P = 45 × 260 + 60 × 240 = 26,100
At (320,180) P = 45 × 320 + 60 × 180 = 25,200
∴ 260 kg. of day cream and 240 kg. of night cream. (1 mark)

f.
From (e), maximum profit is $26,100
(1 mark)

Question 1 a.
$14,0247
(1 mark)

b.
140247 + 17335 – 12036.55 + 8642
= $154187.45
(1 mark)

c.
Minimum monthly balance for December
= 140247
Interest for December =
$140247 \times \frac{8}{12 \times 100} \times 1 = 934.98$
(1 mark)
Minimum monthly balance for January
=142150.9
Interest for January =
$142150.9 \times \frac{8}{12 \times 100} \times 1 = 947.67$
Minimum monthly balance for February
=142150.9
Interest for February =
$142150.9 \times \frac{8}{12 \times 100} \times 1 = 947.67$
Total interest = 934.98 + 2 × 947.67 = $2829.52
(1 mark)

d.
112% = 160677
$12\% = \frac{160677 \times 12}{112} = \$17215.39$
(1 mark)

Question 2 a.
Increase = 38150 – 35000 = 3150
$\%\text{ increase } = \frac{3150}{35000} \times 100 = 9\%$
(1 mark)

b.
$35000(1.09)^4 = \$49,405$
(1 mark)

c.
Enter formula $y = 35000(1.09) \wedge x$
in graphics calculator. Go to table to where
$x = 21,\ y = 213808$ (1 mark)
So in 21st year, where 2004 is year 0.
$\therefore 2025$ (1 mark)

d.
$35000 + \frac{35000 \times 16 \times 5}{100} = \$63,000$
(1 mark)

Question 2 (continued) e.

Enter $y_2 = 35000 + \dfrac{35000 \times 16 \times x}{100}$

in the graphics calculator. (1 mark)

Keep the y_1 equation from (c) in the calculator.

Go to table.

When $x = 14, y_1 > y_2 \therefore 2018$ (1 mark)

Question 3

a. i.

\$2000

(1 mark)

a. ii.

$1 + \dfrac{9.8}{12 \times 100}$

$= 1.008$

(1 mark)

b.

Use graphics calculator

Press Apps Finance Enter TVM Solver Enter

$N =$

$I = 9.8$

$PV = 200000$

$PMT = -2000$

$FV = 0$

$P / Y = 12$

$C / Y = 12$

Put cursor on FV and press alpha solve

$N = 204.22$ months = 17 years.

(1 mark)

c.

Amount repaid

$= 204.2172486 \times 2000 = 408434.4972$

Interest = 408434.4972 – 200000

=\$208,434.50

(1 mark)

Question 1 a.
A = 2, B = 3, C = 3, D = 3, E = 1
2 + 3 + 3 + 3 + 1 = 12
(1 mark)

b.
D.
(1 mark)
It is the only town that can be reached from A, B, C and E
(1 mark)

c.
A, C and D
(1 mark)

d.

From \ To	A	B	C	D	E
A	0	1	1	2	0
B	1	0	1	2	0
C	1	1	0	2	0
D	0	0	0	0	0
E	0	0	0	1	0

e.g. D can be reached from A in 2 ways, ABD or ABCD
(1 mark) for getting any two rows correct
(1 mark) for getting remaining rows correct.

Question 2 a.	**J**	**K**	**L**	**M**	Step1 – Min. in rows	**J**	**K**	**L**	**M**
R	6	7	5	11	**R**	6-5=1	7-5=2	5-5=0	11-5=6
WS	10	5	3	12	**WS**	10-3=7	5-3=2	3-3=0	12-3=9
CP	11	3	4	13	**CP**	11-3=8	3-3=0	4-3=1	13-3 =10
WP	8	4	6	10	**WP**	8-4=4	4-4=0	6-4=2	10-4=6

(1 mark)

Step 2 – Minimum in columns

	J	**K**	**L**	**M**		**J**	**K**	**L**	**M**
R	1-1=0	2-0=2	0-0=0	6-6=0	**R**	0	2	0	0
WS	7-1=6	2-0=2	0-0=0	9-6=3	**WS**	6	2	0	3
CP	8-1=7	0-0=0	1=1=0	10-6=4	**CP**	7	0	0	4
WP	4-1=3	0-0=0	2-0=2	6-6=0	**WP**	3	0	2	0

(1 mark)

Question 2 a. (continued)
Allocating jobs.
R ⟶ J
CP ⟶ K
WS ⟶ L
WP ⟶ M
(1 mark)

b.
Minimum time = 6 + 3 + 3 + 10 = 22 hours
(1 mark)

Question 3 a.

Activity	Immediate Predecessor
A	-----
B	*A*
C	*A*
D	*B*
E	*C*
F	*E*
G	*A ,B*
H	-----
I	*D,F,G,H*

(1 mark)

b.
The critical path is the longest path.
ACDI
(1 mark)

c.
5 + 8 + 12 + 8 = 33 hours.
(1 mark)

d.
This will alter the critical path to ABEFI
Time taken = 5 + 6 + 4 + 14 + 8 = 37 hours.
This means it will take four hours longer to finish the project.
(1 mark)

e.
No effect, as F is not on the critical path.
(1 mark)

Question 1 a.

$$M = \begin{matrix} \\ X \\ Y \end{matrix}\begin{matrix} \begin{matrix} R & W & C \end{matrix} \\ \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \end{matrix}$$

(1 mark)

b.

2 rows and 3 columns so 2×3

(1 mark)

c.

$$N = \begin{bmatrix} P \\ Q \\ R \end{bmatrix}$$

(1 mark)

d.

$$MN = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}\begin{bmatrix} P \\ Q \\ R \end{bmatrix} = \begin{bmatrix} aP + bQ + cR \\ dP + eQ + fR \end{bmatrix}$$

(1 mark)

e.

$dP + eQ + fR$ represents the selling price of wines for company Y

(1 mark)

Question 2 a.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -\frac{7}{31} & \frac{2}{31} & \frac{8}{31} \\ \frac{8}{31} & \frac{11}{31} & -\frac{18}{31} \\ \frac{9}{31} & \frac{-7}{31} & \frac{3}{31} \end{bmatrix}\begin{bmatrix} 36.1 \\ 30.65 \\ 32.45 \end{bmatrix} = \begin{bmatrix} 2.20 \\ 30.65 \\ 32.45 \end{bmatrix}$$

Use graphics calculator to get inverse matrix of matrix $\begin{bmatrix} 3 & 2 & 4 \\ 6 & 3 & 2 \\ 5 & 1 & 3 \end{bmatrix}$ which is $\begin{bmatrix} -\frac{7}{31} & \frac{2}{31} & \frac{8}{31} \\ \frac{8}{31} & \frac{11}{31} & -\frac{18}{31} \\ \frac{9}{31} & \frac{-7}{31} & \frac{3}{31} \end{bmatrix}$

Then use graphics calculator to do the multiplication to get the answer.

1 mark for inverse matrix
1 mark for giving answers as fractions
1 mark for correct solution

(3 marks)

b.

The cost of a tin of dog food is \$2.20

(1 mark)

Question 3 a.

0.05 = 5 %

(1 mark)

b.

$$T = \text{Current Purchase} \begin{matrix} \\ A \\ B \\ C \end{matrix} \begin{matrix} \text{Next Purchase} \\ A \quad B \quad C \end{matrix} \begin{bmatrix} 0.9 & 0.1 & 0.2 \\ 0.05 & 0.8 & 0.1 \\ 0.05 & 0.1 & 0.7 \end{bmatrix}$$

(1 mark)

c.

$$\begin{bmatrix} 0.3 \\ 0.6 \\ 0.1 \end{bmatrix}$$

(1 mark)

d. i.

$$\begin{bmatrix} 0.9 & 0.1 & 0.2 \\ 0.05 & 0.8 & 0.1 \\ 0.05 & 0.1 & 0.7 \end{bmatrix} \begin{bmatrix} 0.3 \\ 0.6 \\ 0.1 \end{bmatrix} = \begin{bmatrix} 0.35 \\ 0.505 \\ 0.145 \end{bmatrix}$$

(1 mark)

d. ii.

The percentage of customers expected at Charisma pharmacy next week is 14.5%.

(1 mark)

e.

Find $T \wedge 40\ S_0$

and $T \wedge 50\ S_0$

These both give $K = \begin{bmatrix} 0.588 \\ 0.235 \\ 0.176 \end{bmatrix}$

To enter $T \wedge 40\ K_0$ into calculator press

2nd matrix A $\wedge$ 40 2nd matrix B enter

So expect 17.6% of customers in the Charisma pharmacy in the long term.

(1 mark)

End of suggested solutions 2007 Further Mathematics VCE Trial Examination 2

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