2008 Further Mathematics Trial Examination 1 Suggested Solutions



VCE Further Mathematics Trial Examination 1

Suggested Solutions

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Core: Data analysis

Question 1C $\frac{10}{30} \times \frac{100}{1} = 33.3\%$ Question 3 BThis graph has an outlier from 90 to 100, and ithas a tail to the left. This means it is negativelyskewed with an outlier.	Question 2CThe median is 34The lower quartile is 22The upper quartile is 62The interquartile range is $62 - 22 = 40$ Question 4 B $8 + 10 + 2 = 20$
Question 5 E Both variables are numerical∴ scatter plot. One variable is time so the special type of scatter plot is a time series plot. Stem and leaf and box plot are used when one variable is numerical and one is categorical. A bar graph is used when both variables are categorical.	Question 6 D 68% would lie between $600 + 80$ and $600 - 80\therefore not AThe % greater than 520 would have to be greaterthan 50% since the mean is 600 \therefore not B440$ is 2 standard deviations below the mean so 2.5% are less than $440 \therefore$ not C 680 is 1 standard deviation above the mean so about 16% are greater than $680 \therefore$ not E 360 is 3 standard deviations below the mean so about 0.15% are less than $360 \therefore$ D
Question 7 B $Z = \frac{x - \overline{x}}{s}$ $-2 = \frac{60 - \overline{x}}{5}$ $-10 = 60 - \overline{x}$ $\overline{x} = 70$	Question 8 A When the residual plot has no clear pattern, then a linear relationship exists.

Core: Data analysis

Question 9 E	Question 10 A
$r^2 = 0.04$	English mark = $7.25 - 0.18 \times$ Maths mark
$r = \pm 0.2$	$7 = 7.25 - 0.18 \times M$
But r is negative because the gradient of the	-0.25 = -0.18M
regression line is negative $\therefore r = -0.2$	$M = \frac{-0.25}{-0.18} = 1.4$
Question 11 D	Question 12 A
There is only a seasonal pattern here, the graph	Coordinates of median of lower 4 points is
following a similar pattern in the 4 quarters of	(2.5,40)
each year.	Coordinates of median of upper 4 points is
	(10.5,35)
	$m = \frac{35 - 40}{10.5 - 2.5} = -0.625$
Question 13 D	
Using May, June, July and August	
$Mean = \frac{16+15+14+13}{4} = 14.5$	
Using June, July, August and September	
$Mean = \frac{15 + 14 + 13 + 12}{4} = 13.5$	
Centring these two means that would be either side of July, August and September	
4 – mean smoothed value centred at July	
$=\frac{14.5+13.5}{2}=14$	

Module 1 Number patterns

Question 1 D	Question 2 B
Each term in the sequence is found by adding 6	This is a geometric sequence with a common
to the term before it. Hence it is an arithmetic	ratio of 1
sequence with a common difference of 6	$\frac{1}{2}$
	a = 8, n = 12
	$t_n = ar^{n-1}$
	$t_{12} = 8\left(-\frac{1}{2}\right)^{11} = -\frac{1}{256}$
Question 3 C	Question 4 C
$t_7 = 3 \times t_6 + 4$	$t_4 = t_{1+3} = t_{1+2} + t_{1+1} + t_1 = t_3 + t_2 + t_1 = 1 + 1 + 1 = 3$
$8017 = 3 \times t_6 + 4$	$t_5 = t_{2+3} = t_{2+2} + t_{2+1} + t_2 = t_4 + t_3 + t_2 = 3 + 1 + 1 = 5$
$8013 = 3 \times t_6$	
$2671 = t_6$	
$t_6 = 3 \times t_5 + 4$	
$2671 = 3 \times t_5 + 4$	
$2667 = 3 \times t_5$	
$889 = t_5$	
Question 5 C	Question 6 F
4+6+8+	Value at beginning of first year, $2008 = 32,000$
Sum of Arithmetic Sequence	Value at beginning of second year = $32,000(0.85)$
n_{12}	Value at beginning of third year = $32,000(0.85)^2$
$S_n = \frac{1}{2} \left[2a + (n-1)a \right]$	Following the pattern
	Value at hosizating of Effth war = 22,000(0.85) ⁴
$S_{10} = \frac{1}{2} [8 + 9 \times 2] = 5 \times 26 = 130 \text{ m}$	value at beginning of fifth year = $32,000(0.85)^{\circ}$
	= \$16/04.20
	This is closest to \$16704

Module 1 Number patterns

Question 7 D	Question 8 A
Arithmetic Sequence.	
a = 85 d = -7 $t_n = a + (n-1)d = 85 + (n-1)(-7)$ Use sequence mode on graphics calculator and press $y =$ n(min) = 1 $\mu(n) = 85 + (n-1)(-7)$ $\mu(n \text{ min}) = 85$ Press second table and scroll down to get -48 This corresponds to $n = 20$	$\frac{72}{x} = \frac{x}{8}$ $x^{2} = 576$ $x = \pm\sqrt{576} = \pm 24$ Common ratio = $\frac{\pm 24}{8} = \pm 3$
Question 9 E An arithmetic sequence is linear ∴ not A	
The geometric sequence must have a negative common ratio because the terms go from positive to negative to positive \therefore not B or D	
Because the terms are getting closer to the <i>x</i> axis as <i>n</i> increases, then the common ratio must be a fraction.	

Question 1 D	Question 2 D
$\tan \theta = \frac{8}{2} = 2$	$\angle SOA = 90 - 25 = 65$
$\tan \theta = \frac{1}{4} - 2$	True bearing of $OA = 180 + 65 = 245^{\circ}$ T
$\theta = \tan^{-1}(2) = 63^{\circ}$	
Question 3 B	Question 4 D
$1 \int_{3} x \int_{12} x $	$x^{2} + x^{2} = 144$ $2x^{2} = 144$ $x^{2} = 72$ $x = \sqrt{72} = 8.485$ Perimeter = 8.485 × 2 + 12 Perimeter = 28.97
Question 5 C	Question 6 E
1	$\angle BAC = 180 - (135 + 26) = 19^{\circ}$
Area = $\frac{1}{2}bc\sin A$	$\underline{AC} = \underline{16}$
	$\sin 135^\circ$ $\sin 19^\circ$
Area = $\frac{1}{2} \times 6 \times 6 \times \sin 50^{\circ}$	$4C = \frac{16 \times \sin 135^{\circ}}{\cos 135^{\circ}} = 34.8$
$\Delta rea = 13.8$	$\sin 19^{\circ} = \frac{-54.8}{\sin 19^{\circ}}$
1100 15.0	

Module 2 Geometry and trigonometry



Module 3 Graphs and relations

Question 1 B	Question 2 E
Points are (3,0) and (0,-4)	When $x = 3800$, <i>y</i> is between 4000 and 4500
-4 - 0 - 4 - 4	∴\$4250
$m = \frac{1}{0-3} = \frac{1}{-3} = \frac{1}{3}$	
Question 3 B	Question 4 C
Original means $t = 0$	When Meg stops for lunch her speed would be
P = 0 + 50 = 50	$0 \therefore$ not A or D
	Meg rides home faster than she goes at first,
	\therefore not B
	The distance travelled is not 0 when Meg
Ornerting 5 A	stops for lunch : not E
Question 5 A The equation of the line is $y = r^{3}$	Question o B Using points $(0, 6)$ and $(4, 5)$
The equation of the line is $y = x - 3$	Using points $(0,0)$ and $(4,3)$
Area under or on this line is $y \le x - 3$	$m = \frac{5-6}{2} = -\frac{1}{2}$
This is the same as $y - x \le -3$	4 - 0 4
This is the same as $-y + x \ge +3$	Equation of this part of the function is
This is the same as $x - y \ge 3$	$y = -\frac{1}{4}x + 6 \qquad 0 \le x \le 4$
	Using points (14,0) and (4,5)
	5-0 5 1
	$m = \frac{1}{4 - 14} = -\frac{1}{10} = -\frac{1}{2}$
	Equation of this part of the function is
	$y = -\frac{1}{2}x + c$
	When $x = 14, y = 0$
	0 = -7 + c
	<i>c</i> = 7
	$y = -\frac{1}{2}x + 7$ $4 < x \le 14$

Module 3 Graphs and relations

Question 7 C	Question 8 D
When $x^2 = 16, y = 4$	In A, one of the lines has a gradient of 2 and the
1 2	other line has a gradient of -2 , \therefore not parallel
$\therefore y = -x^2$	In B, one of the lines has a gradient of 3 and the
	other line has a gradient of -3, not parallel
	In C, both lines are the identical same line, not
	In Δ one of the lines has a gradient of 2 and the
	other line has a gradient of -2 , \therefore not parallel
	In E, one of the lines has a gradient of $-\frac{1}{2}$ and
	the other line has a gradient of $-\frac{1}{2}$, not
Question 9 D	The other line has a gradient of 7 , \cdots not
Move line $Z = 0$ or $3x - y = 0$ keeping	parallel
the movement parallel to the line $Z = 0$.	In D, both lines have the same gradient of 3
Moving to the left passes through P last.	paranei.
Moving to the right passes through S last.	
P would give the minimum and S would	
give the maximum.	

Module 4 Business-related mathematics

Question 1 A	Question 2 E
$0.95^5 = 0.77$	$GST = \frac{118.75}{100} \times \frac{100}{100} = 12.5\%$
50000 - 0.77 is just a little less than 50000	950 1
\therefore not C	
50000 + 0.77 is just a little more than 50000	
\therefore not D	
$50000 \times (1.05)^5$ is larger than 50000	
∴ not B	
$0.95^5 = 0.77$	
$50000 - (1.05)^5$ is just a little less than 49000	
∴ not E	
0 1 1 0	
Question 3 D Total Amount Paid = $66 \times 30+396 = 2376$	Question 4 C
Interest = 2376 - 1200 - \$1176	$A = P\left(1 + \frac{r}{100}\right)$
interest 2570 1200 – \$1170	
	$A = 9400 \left(1 + \frac{7.2}{100}\right)^2 = 10802.33$
	I = 10802.33 - 9400 = \$1402.33
Ouestion 5 E	Question 6 C
Amount repaid = $139 \times 6 = 834$	Let amount invested = x
Interest = $834 - 800 = 34$, PRT
$_{p}$ 100 <i>I</i> 100×34 $_{9.5}$	$I = \frac{1}{100}$
$K = \frac{1}{PT} = \frac{1}{800 \times 0.5} = 8.5$	$740 - x - \frac{x \times 9.6 \times 5}{2}$
Effective interest rate = $\frac{2 \times 6}{2 \times 8} \times 8.5 = 14.6\%$	100
6+1	74000 - 100x = 48x
	74000 = 148x
	$x = \frac{74000}{500} = 500
	148
Question 7 E	
Annual Depreciation = $\frac{13500-920}{10} = 1258$	
%Depreciation = $\frac{1258}{13500} \times 100 = 9.3\%$	

Module 4 Business-related mathematics

Question 8 A	Question 9 B
Use TVM solver	Let original deposit be <i>x</i>
N = 120 I = 9.2 PV = 250000	After 5 years = $x \times (1.08)^5$ After 7 years = $x \times (1.08)^5 \times (1.06)^2 = 12000$ $x = \frac{12000}{5} = \$7268.60$
PMT =	$(1.08)^3 \times (1.06)^2$
FV = -250000	
P / Y = 12	
C / Y = 12	
End	
This gives $PMT = 1916.67$	
Total interest repayments = 120×1916.67	
Total interest repayments $= 230000.40$	
Total amount paid for property	
= 230000.40 + 250000	
=480000.40	
Profit = 485000 - 480000.40 = \$4999.60	

Module 5 Networks and decision mathematics

0 1 1 0	
Question 1 C	Question 2 D
This is the only graph where you cannot get to	Number of edges = $\frac{n \times (n-1)}{n-1} - \frac{20 \times 19}{n-190} - 190$
the bottom left point from any other point.	$\begin{array}{c} 1 \text{ value of of edges} \\ 2 \\ 2 \\ 2 \end{array}$
Ouestion 3 D	Ouestion 4 A
Sum of vertices = $6 \times 3 + 6 = 24$	P and R have more than 2 odd vertices \therefore no
24	Euler path and no Euler circuit
Number of edges = $\frac{24}{24}$ = 12	T has all even vertices : it has an Fuler circuit
2	O and S have 2 add vertices : they have an
	Q and S have 2 oud vertices they have an Euler noth and no Euler circuit
	Euler pain and no Euler circuit.
Question 5 B	Question 6 D
Only E goes to E so the number in the number in	
the 5 th row and 5 th column must be 1. A to A is	$A \bullet = A \bullet B \bullet$
0 so the number in the first row and first	5 5 5 8
oolumn is 0	F C
	3
	3+5+5+5+8=26
Ouestion 7 C	Ouestion 8 A
11 + 2 + 5 = 18	
	10 2 2 14
	Input Output
	9 • • 3
	5
	Maximum flow = minimum cut
	= 11 + 2 + 3 = 16
Question 9 E	
The critical path is ADFI : not A	
C can be delayed for 5 hours without delaying the finishing time \therefore not B	
C can be delayed for 5 hours without delaying the finishing time so delaying it by 6 hours will only	
delay the project by 1 hour \therefore not C	
E can be delayed for 1 hour without delaying the t	finishing time so delaying it by 6 hours will only
delay the project by 5 hours : not D	inising time so delaying it by o nours will only

Statement E is true.

Module 6 Matrices

Question 1 A	Question 2 C
Use calculator or	If the determinant $= 0$ then there is no unique
	solution.
$2 \ 2 \ 6 \ -6 \ -18$	For A, determinant = $1212 = 24$
$\begin{vmatrix} -3 \\ -4 \\ 1 \end{vmatrix} - \begin{vmatrix} -12 \\ -3 \end{vmatrix}$	For B, determinant = $-12 - 12 = -24$
	For D, determinant = $-3210 = 22$
	For E, determinant = $3 - 12 = 15$
	For C, determinant $=24-24=0$
Question 3 D	Question 4 D
$\begin{bmatrix} -2 & 0 \end{bmatrix} \begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} -2 & a & -b \end{bmatrix}$	From
$\begin{vmatrix} X = \begin{vmatrix} 2 & 0 \\ 4 & 2 \end{vmatrix} - \begin{vmatrix} a & 0 \\ b & a \end{vmatrix} = \begin{vmatrix} 2 & a & 0 \\ 4 + b & 2 & a \end{vmatrix}$	A B C
$\begin{bmatrix} 4 & -5 \end{bmatrix} \begin{bmatrix} -b & a \end{bmatrix} \begin{bmatrix} 4+b & -5-a \end{bmatrix}$	$4\begin{bmatrix}0.5 & 0.25 & 0.1\end{bmatrix}$
	A = 0.5 = 0.25 = 0.1
	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
	$C [0.3 \ 0.15 \ 0.55]$
Question 5 E	Question 6 A
Steady State = $T^{30}S_{o}$	det A
	$=3p \times s - 2q \times 4r$
$ = 0.4 \ 0.8 ^{50} 200 $	=3ps+8qr
$\begin{bmatrix} 0.6 & 0.2 \end{bmatrix} \begin{bmatrix} 300 \end{bmatrix}$	
	$\frac{1}{\det A} = \frac{1}{3ns+8ar}$
Use calculator to get	
	$A^{-1} = \frac{1}{2} \begin{bmatrix} s & 2q \\ s & s \end{bmatrix}$
	$3ps + 8qr \lfloor -4r 3p \rfloor$
Question 7 C	Question 8 E
$2x + 2y + 2z = 1 \equiv x + y + z = 0.5$	Columns must add to $1 \therefore$ not A, B or C
1x + 0y + 1z = -1	0.1 must be in the umbrella column and row
0x - 5y + 4z = 5	0.1 must be in the uniorena column and low.
Matrix is	
$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \end{bmatrix} \begin{bmatrix} 0.5 \end{bmatrix}$	
$\begin{vmatrix} 1 & 0 & 1 \end{vmatrix} y = \begin{vmatrix} -1 \end{vmatrix}$	
$\begin{vmatrix} 1 & -5 & 4 \\ z & z \\ z & 5 \end{vmatrix}$	
Question 9 E	

B and *C* must be of the same order which would have to be 4×2

A multiplied by a 4×2 matrix would have to have 4 columns, so it must be a 3×4 matrix.

End of suggested solutions 2008 Further Mathematics VCE Trial Examination 1

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