2008

VCE
Further Mathematics
Trial Examination 2

Suggested Solutions

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Core

Question 1 a.

For T183 calculator enter data in stat edit in column L_1. Go to stat calc, 1-var stats and enter L_1. This gives mean = 6.1 and standard deviation = 1.6

Mean =6.1, Standard Deviation = 1.6

(1 mark) for each correct value

b.(i)

6 is two standard deviations above the mean. This student is in the top 2.5% of Australian students for artistic ability.

(1 mark)

b.(ii)

$$Z = \frac{x-4}{1} < -2$$

$$x - 4 < -2$$

$$x < 2$$

Since no student in the sample got a score of less than 2 then no student in the sample was in the lowest 2.5% for artistic ability.

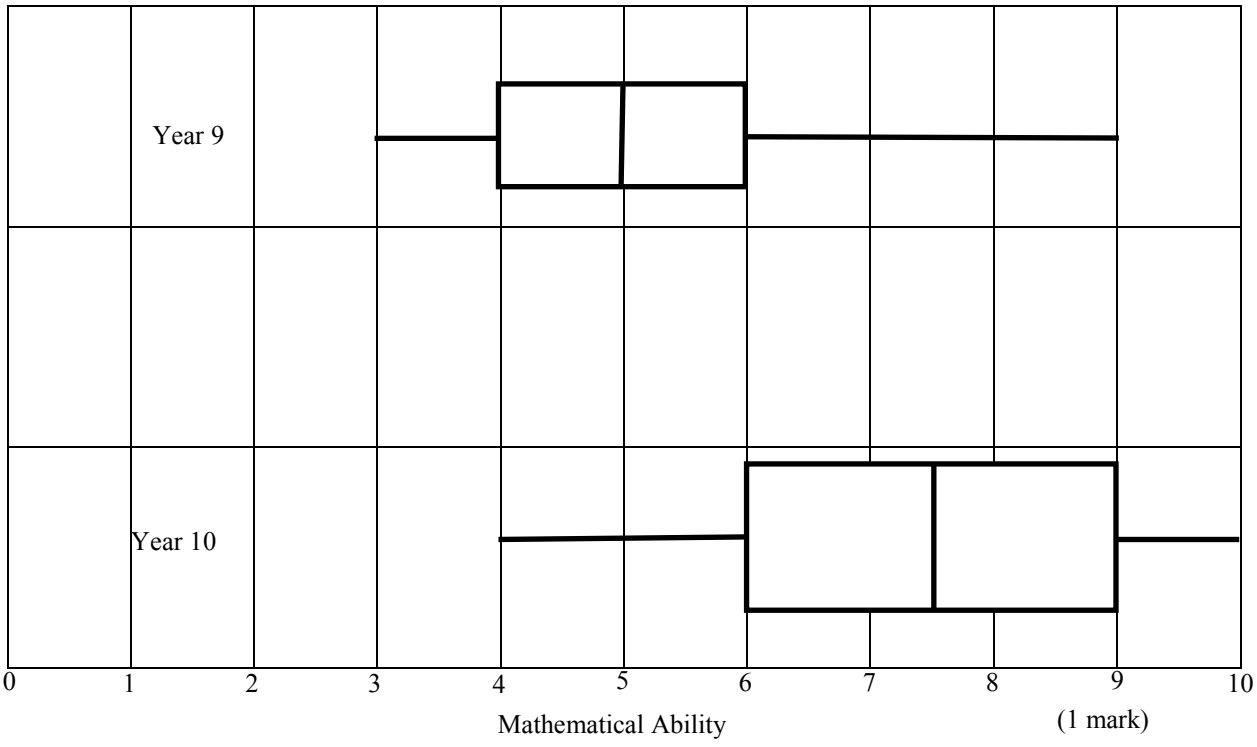
(1 mark)

c.(i)

Dependent Variable: Mathematical Ability
Independent Variable: Year level (1 mark)

c.(ii)

For T183 calculator enter data in stat edit in column L_2. Go to 2nd stat plot and ensure all plots are off. Then, 2nd stat plot enter on, box plot,2nd L_2 .Graph. Use the trace to find Min = 4, LQ = 6, Med = 7.5,UQ = 9, Max = 10



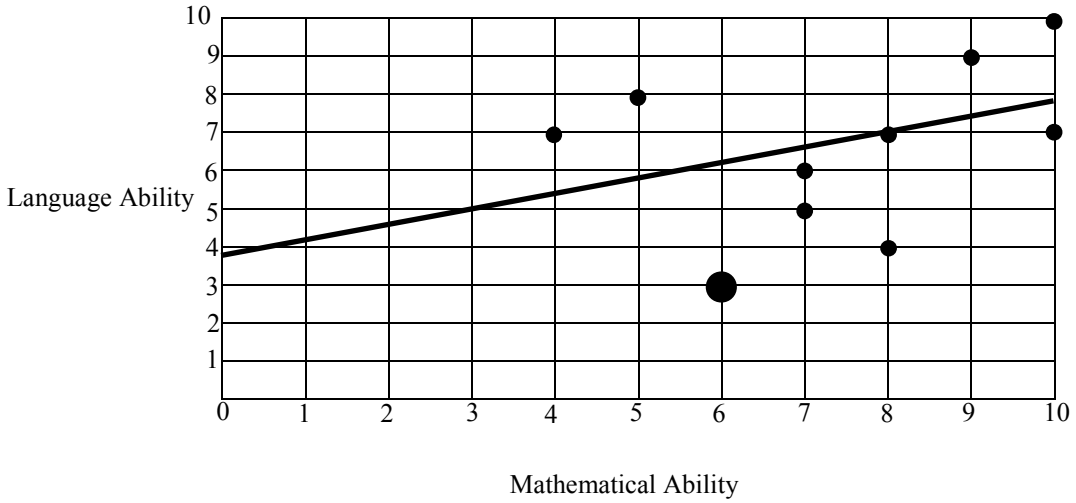
(1 mark)

Core

c.(iii)
Year 10 students on the whole have better mathematical ability than year 9 students. The lowest year 10 mark is equal to the year 9 lower quartile and 75% of year 10 students score a mark greater than the year 9 upper quartile.

(1 mark)

Question 2 a. & c.



(1 mark) for least squares regression line and (1 mark) for required point.

2 b.
For T183 calculator enter year 10 language ability data in stat edit in column L_3. Go to stat calc linear regression (ax+b) L_2, L_3
This gives
$Language\ Ability = 0.4 \times Mathematical$ Ability+3.8

(1 mark)

2 d.
Using the same calculator screen as for 2b.
$r^2 = 0.1198 = 12\%$ to the nearest whole number

(1 mark)

2 e.
12% of the variation in English ability can be explained by the variation in mathematical ability.

(1 mark)

2 f.
$residual = data\ value - predicted$ value
$residual = 3 - 6.2 = -3.2$

(1 mark)

2 g.
$x_L = 5, y_L = 7, x_U = 10, y_U = 9$

$$m = \frac{9-7}{10-5} = 0.4$$

$y = 0.4x + c$

When $x = 5, y = 7$

$7 = 0.4 \times 5 + c$

$7 = 2 + c$

$c = 5$

$Language\ ability = 0.4 \times mathematical\ ability + 5$

(1 mark)

Module 1 Number patterns and applications

Question 1 a.

$7 \times 1 = 7 \text{ cm}^2$

(1 mark)

b.

$\text{Length} = 4 + 6 \times 3 = 22$

$\text{Area} = 22 \times 1 = 22 \text{ cm}^2$

(1 mark)

c.

Areas = 4, 7, 10....

This is an arithmetic sequence

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{20} = \frac{20}{2}[2 \times 4 + 19 \times 3]$

$S_{20} = 650 \text{ cm}^2$

(1 mark)

d.

Use T1-83 graphics calculator in sequence mode.

$y =$

enter

$nMin = 1$

$\mu_n = 4 + (n-1) \times 3$

$\mu(nMin) = 4$

Press 2nd table

Scroll down till the value is 79,

the question asks how many times would it increase

This occurs one less than the value of

n given in the table, so $n = 25$

(1 mark)

Question 2 a.

$A = 9 \times 1.2 = 10.8 \text{ cm}^2$

(1 mark)

b.

Use graphics calculator in sequence mode.

$y =$

enter

$nMin = 1$

$\mu_n = 9 \times (1.2)^{n-1}$

$\mu(nMin) = 9$

Press 2nd table

The 5th press of the button will correspond with

n=6

$\mu_n = 22.4$ to one decimal place

(1 mark)

Module 1 Number patterns and applications

Question 2 c.

Use graphics calculator in sequence mode.

$y =$

enter

$nMin = 1$

$\mu_n = 9 \times (0.4)^{n-1}$

$\mu(nMin) = 9$

Press 2nd table

When $n = 7, \mu_n = 0.03686$

$n = 7$ corresponds to 6 presses of the button

(1 mark)

Question 3 a.

$t_1 = t_3 - t_2$

$t_3 = t_1 + t_2 = 400 + 700 = 1100\,\text{m}$

(1 mark)

b.

Sequence is 400,700,1100

$\frac{700}{400} \neq \frac{1100}{700}$

$\therefore$ not geometric sequence.

(1 mark)

c.

$t_4 = 1100 + 700 = 1800$

$t_5 = 1800 + 1100 = 2900$

$t_6 = 2900 + 1800 = 4700$ m.

(1 mark)

d.

Use graphics calculator in sequence mode.

$y =$

enter

$nMin = 1$

$\mu(n) = \mu(n-2) + \mu(n-1)$

$\mu(nMin) = \{700, 400\}$

Press 2nd table and look for 357100 in the $\mu(n)$ column

This corresponds to $n = 15$

(1 mark)

e.

Using the same screen as for the previous question

$t_{11} = 52100$

$t_{10} = 32200$

Distance between these stops = $t_{11} - t_{10}$

$= 52100 - 32200 = 19900$ m.

(1 mark)

Module 1 Number patterns and applications

Question 4 a.

$S_3 = 4^3 - 1 = 63$

(1 mark)

b.

$S_1 = 4 - 1 = 3$

$S_2 = 4^2 - 1 = 15$

$S_3 = 4^3 - 1 = 63$

$t_1 = 3$

$t_2 = 15 - 3 = 12$

$t_3 = 63 - 15 = 48$

Distance run on each of first 3 days is 3,12,48

(1 mark)

c.

3, 12, 48... forms a geometric sequence because it has a common ratio of 4

(1 mark)

Module 2 Geometry and trigonometry

Question 1 a.

Isosceles triangles have base angles equal.

Sum of angles of a triangle = 180^0

$\angle ABC = 180 - 2 \times 51.5 = 77^0$

(1 mark)

b.

In triangle $BCX, \angle BCX = 51.5^0, BC = 230$

$\cos 51.5^0 = \dfrac{CX}{230}$

$CX = 230 \cos 51.5^0 = 143.1784$

$AC = 2 \times CX = 2 \times 143.1784 = 286.36$ cm

(1 mark)

c.

In triangle BCX,

$\sin 51.5^0 = \dfrac{BX}{230}$

$BX = 230 \sin 51.5^0 = 180$ cm.

(1 mark)

d.

CF is 2.5 times larger than AC

$CF = 2.5 \times 286.36 = 716$ cm

(1 mark)

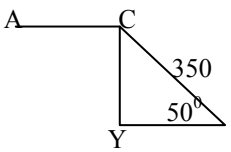
e.

Area of $ACFD = AC \times CF$

$= 286.36 \div 100 \times 715.9 \div 100 = 21$ m^2

(1 mark)

Question 2 a.



$\sin 50^0 = \dfrac{CY}{350}$

$CY = 350 \sin 50^0 = 268.12$

$CY + BX = 268 + 180 = 448$ cm.

(1 mark)

Module 2 Geometry and trigonometry

Question 2 (continued) b.

V = Area of triangular base × height

V = Area of triangular ABC × CF

$$V = \frac{1}{2} \times 286.36 \div 100 \times 180 \div 100 \times 715.9 \div 100$$

$$V = 18 \text{ m}^3$$

(1 mark)

b.

$$SA = \frac{1}{2} \times 4\pi r^2 + \pi r^2$$

$$SA = 3\pi r^2$$

$$SA = 3\pi \times 65 \div 10 \times 6\ 5 \div 10$$

$$SA = 398 \text{ cm}^2$$

(1 mark)

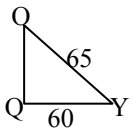
Question 3 a.

$OC =$ radius of sphere $= \frac{1}{2} \times AB$

$$OC = \frac{1}{2} \times 130 = 65 \text{ mm}$$

(1 mark)

c.



Using Pythagoras rule or knowing your triads

$OQ = 25$

$$OQ = 25$$

$$OC = 65$$

$$QC = 65 - 25 = 40 \text{ mm}$$

(1 mark)

Question 4 a.

The triangles are similar because 3 angles of triangle ABD equal three angles of triangle BCD.

$\angle CBD = \angle EFD$,

so $\angle BAC = \angle CBD$

$\angle BDC$ is common

So the remaining angles of the triangles are equal.

(1 mark)

b.

Because the triangles are similar, then their corresponding sides are in the same ratio.

$$\frac{CD}{BD} = \frac{BD}{AD}$$

$$BD^2 = CD \times AD$$

$$BD^2 = 5 \times 20 = 100$$

$$BD = 10 \text{ cm}$$

(1 mark)

c.

$$\frac{AB}{BC} = \frac{AD}{BD}$$

$$\frac{AB}{7} = \frac{20}{10}$$

$$AB = 14 \text{ cm}$$

(1 mark)

Module 3 Graphs and relations

Question 1 a.
6 km.
(1 mark)

b.
Fastest speed is steepest gradient so A
(1 mark)

c.
$\text{speed} = \dfrac{\text{distance}}{\text{time}} = \dfrac{6}{\frac{1}{4}} = 24 \text{ km/hr}$
(1 mark)

Question 2
a.
$C = 69 + 2x$
(1 mark)

b.
$I = 5x$
(1 mark)

c.
$69 + 2x = 5x$
$69 = 3x$
$x = 23$
(1 mark)

d.
$P = 5x - 69 - 2x$
$P = 3x - 69$
If $x = 30$
$P = 90 - 69 = \$21$
(1 mark)

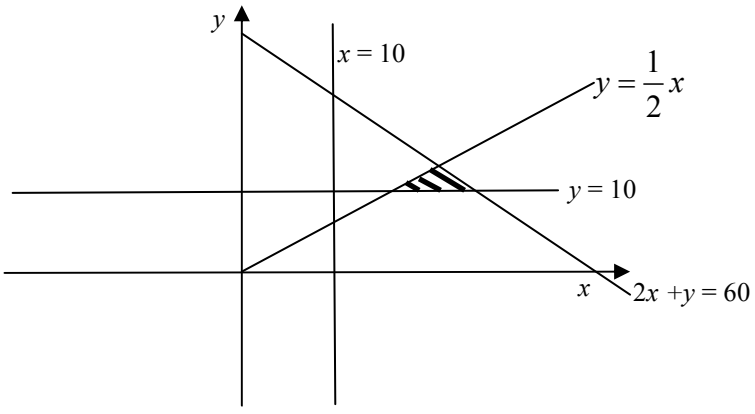
Question 3 a.
$x \geq 10$
$y \geq 10$
(1 mark)

b.
$x \geq 2y$
or $y \leq \frac{1}{2}x$
(1 mark)

c.
$C = 20x + 10y$
(1 mark)

d.
$20x + 10y \leq 600$
$2x + y \leq 60$
(1 mark)

e.



(1 mark)

Module 3 Graphs and relations

Question 3 (continued)

f.

Number of prizes = $x + y$

Points of intersection

$y = 10$ and $y = \frac{1}{2}x$ (20,10) No. prizes = 30

$y = 10$ and $2x + y = 60$ (25,10) No. prizes = 35

$y = \frac{1}{2}x$ and $2x + y = 60$

$2\frac{1}{2}x = 60$

$x = 24$ (24,12) No. prizes = 36

Maximum number of prizes = 36

(1 mark) for realizing number of prizes is $x + y$ and (1 mark) for correct answer)

g.

$C = 20x + 10y$

$C = 20 \times 24 + 10 \times 12 = \$600$

(1 mark)

Module 4 Business-related mathematics

Question 1 a.

Profit $= 302 - 234 = 68$

% Profit $= \frac{68}{234} \times 100 = 29.1\%$

(1 mark)

b.

$85\% \equiv 1200$

$1\% \equiv \frac{1200}{85}$

$100\% \equiv \frac{1200}{85} \times 100 = \$1412$

(1 mark)

c. (i)

Amount paid $= 300 + 70 \times 24 = \$1980$

(1 mark)

c.(ii)

Interest $= 1980 - 1200 = \$780$

(1 mark)

c.(iii)

$R = \frac{100I}{PT} = \frac{100 \times 780}{900 \times 2} = 43.33\%$

(1 mark)

Question 2 a.(i)

$I = \frac{PRT}{100} = \frac{28000 \times 8.5 \times 0.5}{100} = \$1190$

(1 mark)

Module 4 Business-related mathematics

Question 2 (continued)

a.(ii)

Amount repaid $= 620 \times 6 = 3720$ (1 mark)

Amount repaid off principal $= 3720 - 1190$

Amount repaid off principal $= 2530$

Amount owing $= 28000 - 2530 = \$25,470$

(1 mark)

b.(i)

Use graphics calculator

Press Apps Finance Enter TVM Solver Enter

$N = 6$

$I = 8.5$

$PV = 28000$

$PMT = -620$

$FV =$

$P/Y = 12$

$C/Y = 12$

End

Put cursor on FV and press alpha solve.

FV $= \$25,424.77$

(1 mark)

b.(ii)

Amount paid over 6 months $= 620 \times 6 = 3720$

Amount paid off principal $= PV - FV$

Amount paid off principal $= 28000 - 25424.77$

Amount paid off principal $= 2575.23$ (1 mark)

Amount paid in interest $= 3720 - 2575.23$

Amount paid in interest $= \$1,144.77$

(1 mark)

b.(iii)

Use graphics calculator

Press Apps Finance Enter TVM Solver Enter

$N = 96$

$I = 8.5$

$PV = 28000$

$PMT =$

$FV = 0$

$P/Y = 12$

$C/Y = 12$

End

Put cursor on PMT and press alpha solve.

$PMT = \$402.98$

(1 mark)

Question 3

a. (i)

Depreciation per annum $= \frac{11}{100} \times 60000 = 6600$

Depreciation over 6 years $= 6600 \times 6 = 39600$

BV $= 60000 - 39600 = \$20,400$

(1 mark)

a. ii.

BV $= 60000(1 - 0.09)^6 = \$34,072.16$

(1 mark)

b.

Depreciation $= 60000 - 24000 = 36000$

$2.4 \times$ number of patients $= 36000$ (1 mark)

number of patients $= \frac{36000}{2.4} = 15,000$

Module 5 Networks and decision mathematics

Question 1 a.

From \ To	A	B	C	D
A	0	1	0	1
B	0	0	2	0
C	0	0	0	1
D	0	1	0	1

(1 mark)

b.(i)
2
(1 mark)

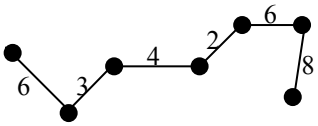
b.(ii)
The number of roads leaving A
(1 mark)

c.
The number of roads coming into D
(1 mark)

Question 2 a.
It is possible to drive along each road once and visit each site if you start at Botanical Gardens and finish at Daffodil display, or vice versa
B-A-B-C-D-E-F-G-D is one possible route.
(1 mark)

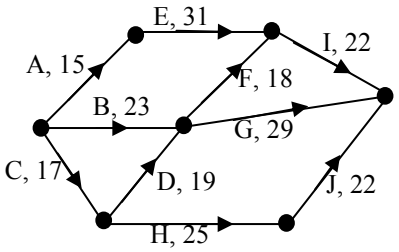
b.
6+9+3+4+2+6+8+7=45 km.
(1 mark)

c.



A-B-C-D-E-F-G
6+3+4+2+6+8=29 km.
(1 mark)

Question 3 a.



(1 mark)

b.
The critical path is the longest path.
C-D-F-I.
(1 mark)

c.
17+19+18+22=76 days
(1 mark)

d.
$17+19-23=13$ days
(1 mark)

e.
0 days, because F is on the critical path.
(1 mark)

Question 3 f.
CDFI are still on the critical path.
Cost to reduce time for C = \$1,200
Cost to reduce time for D = \$900
Cost to reduce time for F = \$800
Cost to reduce time for I = \$4,000
Total cost of reduction = \$6,900 (1 mark)
Number of days saved = 1+1+2+4 = 8
Cost of 8 days = \$40,000
Amount saved = $40,000-6,900=\$33,100$ (1 mark)

g.
There is now a new critical path. A-E-I.
15+31+22 = 68 days.
(1 mark)

Module 6 Matrices

Question 1 a.

$$M = \begin{array}{c} \\ X \\ Y \\ Z \end{array} \begin{array}{c} \begin{array}{ccc} \text{Cotton} & \text{Silk} & \text{Denim} \end{array} \\ \begin{bmatrix} 80 & 10 & 100 \\ 40 & 5 & 150 \\ 60 & 3 & 200 \end{bmatrix} \end{array}$$

(1 mark)

b. $A = \begin{bmatrix} 100+240+80 \\ 150+80+10 \\ 120+360+100 \end{bmatrix} = \begin{bmatrix} 420 \\ 240 \\ 580 \end{bmatrix}$

(1 mark)

c.

3×1

(1 mark)

d.

100+240+80 = 420
150+80+10 = 240
120+360+100 = 580
250+120+40 = 410
164+60+5 = 229
80+280+150 = 510
160+175+60 = 395
260+70+3 = 333
75+200+200=475

$$A = \begin{array}{c} \\ \text{Cotton} \\ \text{Silk} \\ \text{Denim} \end{array} \begin{array}{c} \begin{array}{ccc} X & Y & \text{Z} \end{array} \\ \begin{bmatrix} 420 & 410 & 395 \\ 240 & 229 & 333 \\ 580 & 510 & 475 \end{bmatrix} \end{array}$$

(1 mark)

e.

$$C = \begin{array}{c} \\ \text{Cotton} \\ \text{Silk} \\ \text{Denim} \end{array} \begin{array}{c} \\ \begin{bmatrix} 35 \\ 70 \\ 20 \end{bmatrix} \end{array}$$

(1 mark)

f.

$$P = \begin{array}{c} \\ X \\ Y \\ Z \end{array} \begin{array}{c} \begin{array}{ccc} \text{Cotton} & \text{Silk} & \text{Denim} \end{array} \\ \begin{bmatrix} 420 & 240 & 580 \\ 410 & 229 & 510 \\ 395 & 333 & 475 \end{bmatrix} \end{array}$$

(1 mark)

g.

Use graphics calculator to get

$$Q = \begin{bmatrix} 420 & 240 & 580 \\ 410 & 229 & 510 \\ 395 & 333 & 475 \end{bmatrix} \times \begin{bmatrix} 35 \\ 70 \\ 20 \end{bmatrix} = \begin{array}{c} \\ X \\ Y \\ Z \end{array} \begin{array}{c} \\ \begin{bmatrix} 43100 \\ 40580 \\ 46635 \end{bmatrix} \end{array}$$

(1 mark)

h.

$156000 - 40580 = \$115,420$

(1 mark)

Module 6 Matrices

Question 2 a.

$$\begin{array}{c} \\ \text{Tomorrow} \end{array} \begin{array}{c} \begin{array}{cc} \text{Today} & \\ \text{W} & \text{NR} \end{array} \\ \begin{array}{c} \text{W} \\ \text{NR} \end{array} \begin{bmatrix} 0.7 & 0.6 \\ 0.3 & 0.4 \end{bmatrix} \end{array}$$

(1 mark)

b.

$$S_0 = \begin{bmatrix} 60 \\ 20 \end{bmatrix}$$

(1 mark)

c.

$$S_1 = \begin{bmatrix} 0.7 & 0.6 \\ 0.3 & 0.4 \end{bmatrix} \begin{bmatrix} 60 \\ 20 \end{bmatrix} = \begin{bmatrix} 54 \\ 26 \end{bmatrix}$$

54 are working and 26 need repair.

(1 mark)

d.

$$S_{30} = T^{30} S_0$$

$$S_{30} = \begin{bmatrix} 0.7 & 0.6 \\ 0.3 & 0.4 \end{bmatrix}^{30} \begin{bmatrix} 60 \\ 20 \end{bmatrix} = \begin{bmatrix} 53 \\ 27 \end{bmatrix}$$

In the long term Rex can expect 53 of the Orange computers to be working.

(1 mark)

e.

$$S_2 = \begin{bmatrix} 0.8 & 0.5 \\ 0.2 & 0.5 \end{bmatrix}^2 \begin{bmatrix} 70 \\ 10 \end{bmatrix} = \begin{bmatrix} 58 \\ 22 \end{bmatrix}$$

(1 mark)

f.

$$S_{30} = T^{30} S_0$$

$$S_{30} = \begin{bmatrix} 0.8 & 0.5 \\ 0.2 & 0.5 \end{bmatrix}^{30} \begin{bmatrix} 70 \\ 10 \end{bmatrix} = \begin{bmatrix} 57 \\ 23 \end{bmatrix}$$

He should buy more of the Lemon brand because in the long term 57 out of 80 would be expected to be working whereas only 53 of the Orange computers would be expected to be working.

(1 mark)

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