2008

VCE

Further Mathematics

Trial Examination 2

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PURPOSE OF THIS TRIAL EXAMINATION

This Further Mathematics Trial Examination is designed to assess

- understanding and communication of mathematical ideas
- interpretation, analysis and solution of routine problems
- interpretation, analysis and solution of non-routine problems

Assessment is by extended answer questions involving multi-stage solutions of increasing complexity.

STUDENT NUMBER

									Letter
Figures									
Words									

VICTORIAN CERTIFICATE OF EDUCATION 2008

FURTHER MATHEMATICS

Trial Written Examination 2 (Analysis task)

Reading time: 15 minutes

Total writing time: 1 hour 30 minutes

QUESTION AND ANSWER BOOK

Structure of book

Core

Number of questions	Number of questions to be answered
2	2

Modules

Number of modules	Number of modules to be answered
6	3

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator.
Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 26 pages.
- Working space is provided throughout the book.
- There is a detachable sheet of miscellaneous formula supplied.

Instructions

- Detach the formula sheet from the book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

FURTHER MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Further Mathematics Formulas

Core: Data analysis

standardised score: $z = \frac{x - \bar{x}}{s_x}$

least squares line: $y = a + bx$ where $b = r\frac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$

residual value: residual value = actual value – predicted value

seasonal index: seasonal index= $\frac{\text{actual figure}}{\text{deseasonalised figure}}$

Module 1: Number patterns

arithmetic series: $a + (a + d) + ... + (a + (n-1)d) = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a + l)$

geometric series: $a + ar + ar^2 + ... + ar^{n-1} = \frac{a(1-r^n)}{1-r}, r \neq 1$

infinite geometric series: $a + ar + ar^2 + ar^3 + ... = \frac{a}{1-r}, |r| < 1$

Module 2: Geometry and trigonometry

area of a triangle: $\frac{1}{2}bc \sin A$

Heron's formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a + b + c)$

circumference of a circle: $2\pi r$

area of a circle: πr^2

volume of a sphere: $\frac{4}{3}\pi r^3$

surface area of a sphere: $4\pi r^2$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a cylinder: $\pi r^2 h$

volume of a prism: area of base × height

volume of a pyramid: $\frac{1}{3}$ area of base × height

Pythagoras' theorem: $c^2 = a^2 + b^2$

sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$

Module 3: Graphs and relations

Straight line graphs

gradient (slope): $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

equation: $y = mx + c$

Module 4: Business-related mathematics

simple interest: $I = \dfrac{\Pr T}{100}$

compound interest: $A = PR^n$ where $R = 1 + \dfrac{r}{100}$

hire purchase: effective rate of interest $\approx \dfrac{2n}{n+1} \times$ flat rate

annuities: $A = PR^n - \dfrac{Q(R^n - 1)}{R - 1}$, where $R = 1 + \dfrac{r}{100}$

Module 5: Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Module 6: Matrices

determinant of a 2 × 2 matrix: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

inverse of a 2 × 2 matrix: $A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$

END OF FORMULA SHEET

Specific Instructions

This task paper consists a core and six modules. Students should answer **all** questions in the core and then select **three** modules and answer **all** questions within the modules selected.

Core: Data analysis

Question 1

A sample of year 9 and 10 students was tested for mathematical ability, artistic ability and language ability, and the results listed in the table below.

Year 9			Year 10		
Maths	Art	Language	Maths	Art	Language
6	4	8	7	10	5
3	9	5	9	9	9
9	6	5	10	9	7
5	8	7	4	9	7
3	6	6	6	4	3
5	6	6	10	7	10
7	4	9	8	5	4
5	7	10	7	6	6
6	5	6	8	6	7
4	6	5	5	7	8

a. Find the mean and standard deviation for artistic ability for year 9 students. Give your answers to one decimal place.

Mean = ☐

Standard Deviation = ☐

2 marks

b. (i) If all year 9 students in Australia were tested for artistic ability and the distribution was found to be normal with a mean of 4 and a standard deviation of 1, what can be said about a student who gets a score of 6 for artistic ability?

____

1 mark

Core: Data analysis

Question 1 (continued)

b. (ii) Were any of the students from the Year 9 sample in the lowest 2.5% of the Australian population of year 9 students for artistic ability?
Give a reason for your answer.

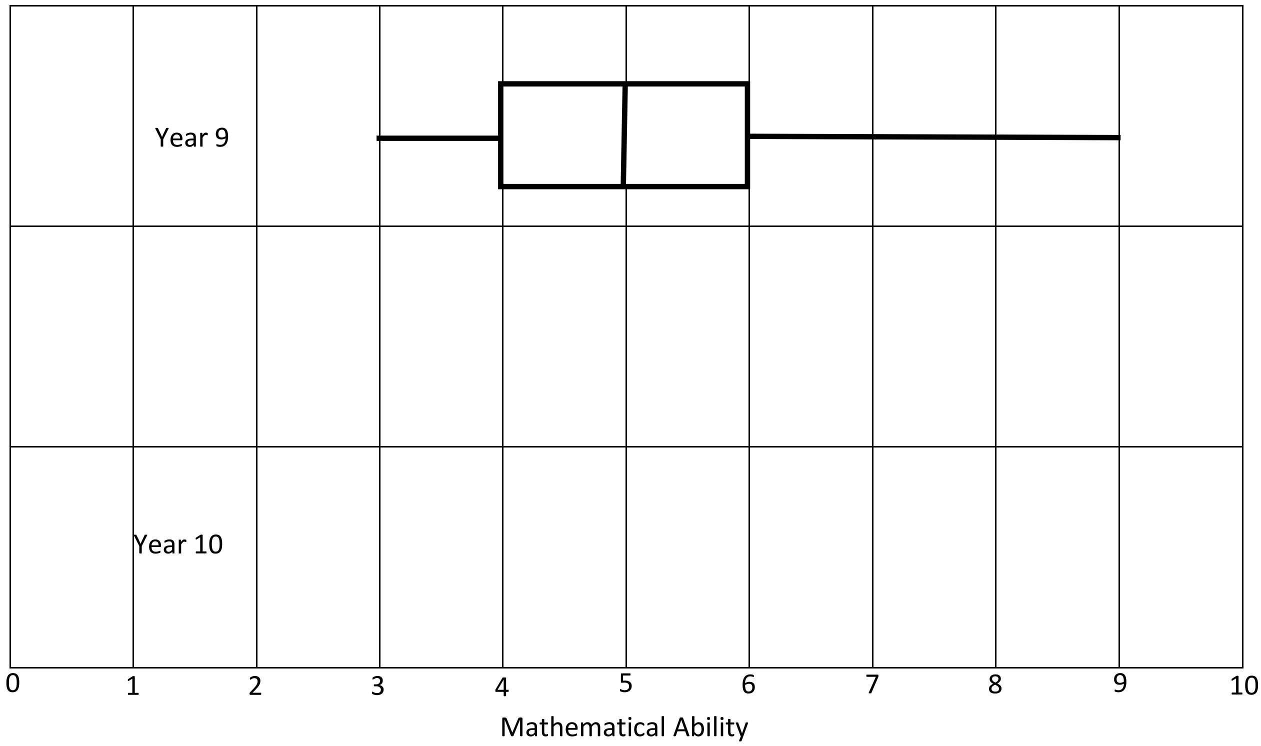
____

____

____

____

1 mark



c. (i) For the box plot above, which is the dependent and which is the independent variable?

Dependent Variable is ________________________________

Independent Variable is ________________________________

1 mark

(ii) On the graph above, draw a box plot for the mathematical ability of the year 10 sample.

1 mark

Core: Data analysis

Question 1 (continued)

c. **(iii)** Use the above graphs to compare the mathematical ability of year 9 and year 10 students.

____

____

____

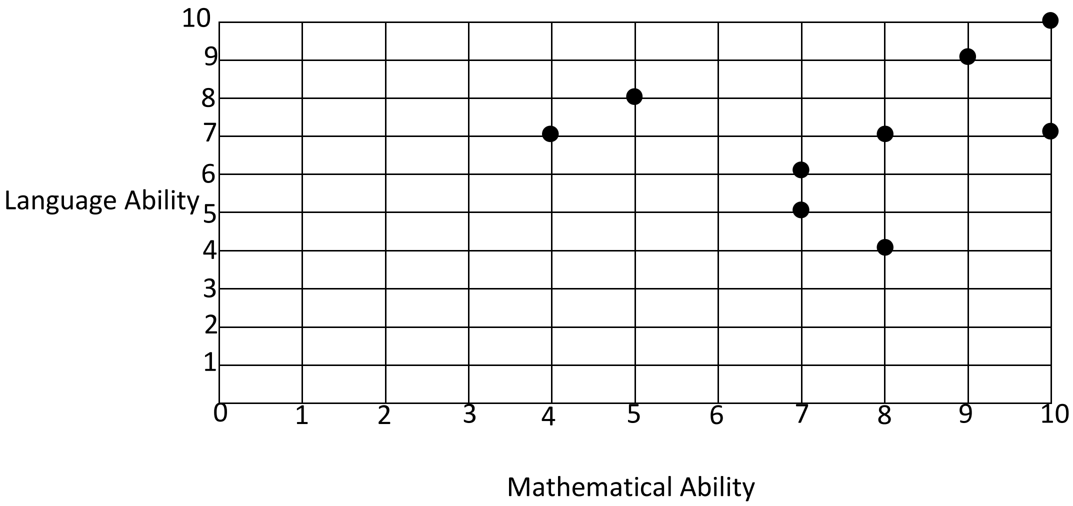
____

1 mark

Question 2

a. On the graph below, complete the scatter graph for year 10 students mathematical and language ability.

1 mark



b. Complete the equation for the least squares regression line for the above scatter graph.

Language ability = ____

1 mark

c. Draw the least squares regression line on the above scatter graph.

1 mark

d. What is the coefficient of determination for this data? Give your answer as a percentage to the nearest whole number.

____

1 mark

Core: Data analysis

Question 2 (continued)

e. What does the coefficient of determination signify in this situation?

____

____

1 mark

f. What is the residual value for a year 10 student who gets a score of six for mathematical ability?

____

1 mark

g. What is the equation of the 3 median regression line for the above scatter plot?

____

____

____

____

____

____

____

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____

____

2 marks

Total = 15 marks

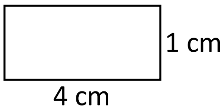
End of Core: Data analysis

Module 1 : Number patterns and applications

If you choose this module, all questions are to be answered.

Question 1

Mai Ling draws a rectangle with a length of 4 cm and a width of 1 cm.



To calculate the area of this rectangle she uses the rule

$$\text{Area} = \text{Length} \times \text{Width}$$
$$\text{Area} = A = 4 \times 1$$
$$A = 4 \text{ cm}^2$$

a. If Mai Ling makes the length 3 cm longer but leaves the width unchanged, what is the area of this new rectangle?

___

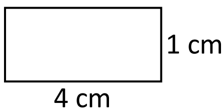
1 mark

b. If Mai Ling makes the length 3 cm longer six times altogether but leaves the width unchanged, what is the area of the rectangle she now makes?

___

1 mark

c. Mai Ling made 20 rectangles beginning with



Each rectangle had a length 3 cm. longer than the previous rectangle while the width remained unchanged. What is the total area covered by all these rectangles?

___

1 mark

d. How many times would Mai Ling have to increase the length of her original rectangle while keeping the width unchanged, in order to have a rectangle with an area of 79 cm^2?

___

1 mark

Module 1: Number patterns and applications

Question 2

Mai Ling's friend, Sue, has a photocopier which has a 120% button. When Sue pushes this button it will increase the area of the picture placed on the machine by 20%. Sue draws a rabbit with an area of $9\,\text{cm}^2$. She then places this picture on the photocopier and presses the 120% button.

a. What is the area of the picture of the rabbit thus generated?

___

1 mark

b. If Sue takes her original picture of the rabbit and keeps pressing the 120% button for a total of five times, what would be the area of the final picture of the rabbit thus produced? Give your answer to one decimal place.

___

1 mark

c. Sue now takes her original rabbit picture and keeps pressing the 40% button until the area is less than $0.04\,\text{cm}^2$. What is the least number of times that she will have to press this button?

___

1 mark

Question 3

Sue makes a collage from her different sized rabbit drawings and takes them by train from the city where she lives to a gallery in the country town of Hicksville. There are several stops on this train line and the distance from the city to each stop in metres can be given by the difference equation

$$t_n = t_{n+2} - t_{n+1} \qquad \text{where } t_1 = 400 \text{ m and } t_2 = 700 \text{ m}$$

a. What is the distance from the city to the third stop?

___

1 mark

b. Is the sequence generated by this difference equation a geometric sequence?
Give a reason for your answer

___

___

1 mark

Module 1: Number patterns and applications

Question 3 (continued)

c. What is the distance from the city to the sixth stop?

1 mark

d. If the distance from the city to Hicksville is 357100 m, how many stops are there from the city to Hicksville? (do not count the city as a stop)

1 mark

e. What is the distance between the 10th and 11th stops?

1 mark

Question 4

While she was in Hicksville, Sue went for a run each day. She found that the sum of the distance she ran on n days could be given by the equation $S_n = 4^n - 1$.

a. What is the total distance that Sue runs on the first three days?

1 mark

b. Write down the distance she runs on each of the first three days.

1 mark

c. Does the distance run each day form an Arithmetic sequence or a Geometric sequence or neither? Give a reason for your answer.

1 mark

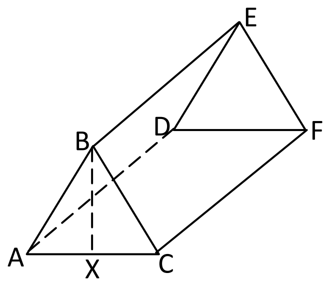
Total = 15 marks

End of Module 1: Number patterns and applications

Module 2: Geometry and trigonometry

If you choose this module, all questions are to be answered.

This diagram refers to Questions 1, 2 and 3.



Question 1

The roof of a garage is in the shape of a triangular prism. The sides *AB* and *BC* are both 230 cm and angle $BAC = 51.5^0$

a. Find angle *ABC*.

1 mark

b. Find the length, *AC*, to two decimal places.

1 mark

Module 2: Geometry and trigonometry

Question 1 (continued)

c. What is the height of the roof, *BX*? Give your answer to the nearest centimetre.

1 mark

d. If *AC* : *CF* = 2:5, then what is the length of *CF*? Give your answer to the nearest centimetre.

1 mark

e. Find the area of the floor of the roof. Give your answer to the nearest square metre.

2 marks

Question 2

The children in the family have a play house in the roof of the garage. To access the play house, a ladder folds down from the floor of the roof to the floor of the garage. The length of this ladder is 350 cm and the ladder makes an angle of 50^0 with the floor of the garage.

a. How high is the top of the garage roof above the floor of the garage?
Give your answer to the nearest centimetre.

2 marks

Module 2: Geometry and trigonometry

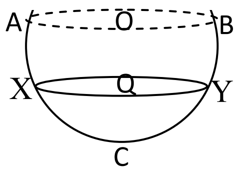
Question 2 (continued)

b. What is the volume of air that the roof can contain?
Give your answer to the nearest cubic metre.

1 mark

Question 3

The playroom contains many toys, one of which is a glass hemisphere which contains a liquid that looks like snow when shaken. *O* is the centre of the top of the hemisphere whose diameter *AB* = 130 mm. *A*, *B* and *C* lie on the surface of the hemisphere. *XY* is the diameter of the surface of the liquid in the hemisphere and equals 120 mm. Q is the centre of the surface of the liquid.



a. Find the length of *OC*.

1 mark

b. Find the total surface area of the hemispherical toy.
Give your answer to the nearest square centimetre.

1 mark

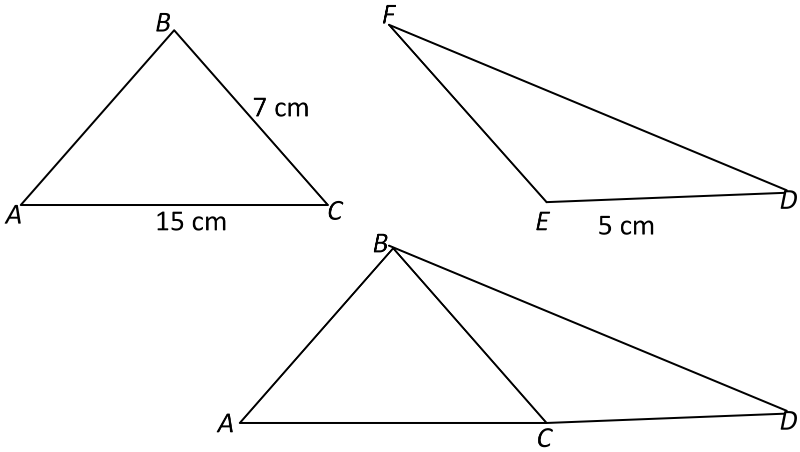
c. Find the depth of the liquid, *CQ*.

1 mark

Module 2: Geometry and trigonometry

Question 4

Another toy consists of wooden pieces of a jigsaw puzzle. Two of the pieces that fit together are in the shape of triangles. One triangle *ABC* has angle *BAC* equal to angle *EFD* of the other triangle, *EFD*. *ACD* makes a straight line when *C* is placed on top of *E*. *BC* fits exactly on top of *FE*. *ED* = 5 cm, *BC* = 7 cm, and *AC* = 15 cm.



a. Why are triangles *ABD* and *BCD* similar?

1 mark

b. Find the length of *BD*.

1 mark

c. Find the length of *AB*.

1 mark

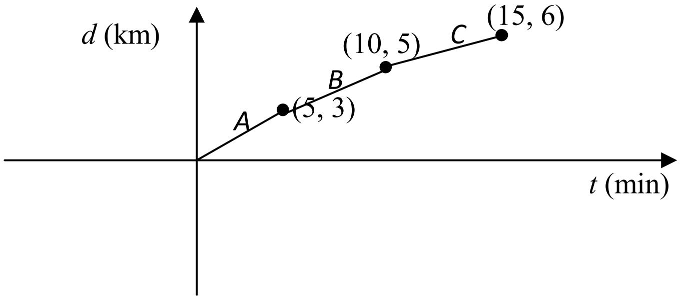
Total = 15 marks

End of Module 2: Geometry and trigonometry

Module 3: Graphs and relations

If you choose this module, all questions are to be answered.

Question 1



The above graph shows Mark's bicycle trip from his home to school

a. How far is the school from his home?

1 mark

b. For which of the three sections of the trip, A, B or C, is he riding the fastest?

1 mark

c. Mark travels home from school along the same road that he travelled to school. If he travels at a constant speed and the home trip takes him 15 minutes, what is his speed in km/hr for the home trip?

1 mark

Question 2

Mark's primary school wishes to raise money for charity. Mark's class makes paintings of their favourite animals to sell at the school art show. The paints cost $69 and each sheet of paper used costs $2.

a. If there are x children in the class and if each child uses one sheet of paper, write an equation in terms of x for the cost, C, of the paintings.

1 mark

Question 2 (continued)

b. Each painting is to be sold for \$5. Write an equation for the total income, I, received for the paintings.

1 mark

c. What is the minimum number of children that must be in the class in order to at least break even?

1 mark

d. If there are 30 children in the class, what profit or loss will be made?

1 mark

Question 3

Mark's teacher runs a mathematics competition for all the year five students. As prizes she buys x books at \$20 each and y calculators at \$10 each.

a. If she must buy at least 10 books and 10 calculators, write two inequations to show these facts.

1 mark

b. The number of books she buys must be at least equal to twice the number of calculators. Use this fact to write an inequation in terms of x and y.

1 mark

Module 3: Graphs and relations

Question 3 (continued)

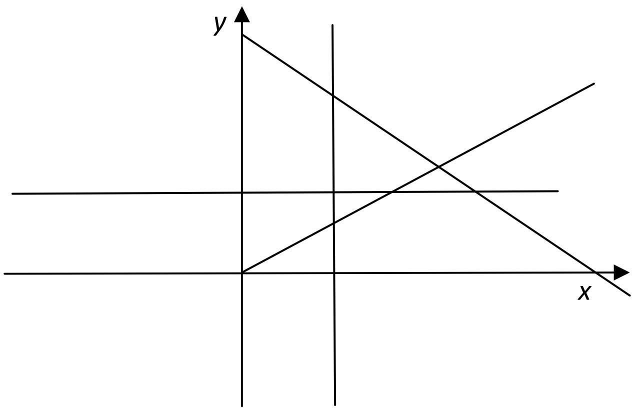
c. Write an equation for the total cost of the books, $\$C$, in terms of x and y.

1 mark

d. If the teacher cannot spend more than $600 on the prizes, write an inequation to show this fact.

1 mark

e.



The graph above shows the lines that satisfy the equalities expressed in the previous sections of this question. On this graph **shade the region** that satisfies all the given constraints.

1 mark

f. What is the maximum number of prizes that can be bought?

2 marks

g. What is the cost of this many prizes?

1 mark

Total = 15 marks

End of Module 3: Graphs and relations

Module 4: Business-related mathematics

If you choose this module, all questions are to be answered.

Question 1

a. Chris buys a bike on E Bay for $234. He then sells it for $302.
What was his percentage profit? Give your answer to one decimal place.

1 mark

b. His sister, Jan, pays $1200 for a bike which is sold in a shop where all goods are discounted by 15%. What was the price of Jan's bike before it was discounted? Give your answer to the nearest dollar.

1 mark

c. Jan does not have $1200 to pay cash for her bike so she agrees to pay a $300 deposit and then $70 per month for two years.

(i) How much does she end up paying for the bike?

(ii) How much interest does she pay?

(iii) What is the simple interest rate that she is charged?
Give your answer to two decimal places.

1 + 1 + 1 = 3 marks

Module 4: Business-related mathematics

Question 2

Linda borrows $28,000 at 8.5% interest to buy a car. She makes repayments of $620 per month for 6 months.

a. If the interest is charged at a flat rate,

(i) What interest does she pay over the six months?

___

___

(ii) How much does she owe at the end of the six months?

___

___

___

___

b. If the interest is a reducible monthly interest,

(i) How much does she owe at the end of six months? Give your answer to the nearest cent.

___

(ii) What interest does she pay over the six months? Give your answer to the nearest cent.

___

___

(iii) If Linda wishes to repay the $28,000 in full by the end of 8 years from the time of borrowing the money, how much should she repay each month of the 8 year period?
Give your answer to the nearest cent.

___

1 + 2 + 1 + 2 + 1 = 7 marks

Module 4: Business-related mathematics

Question 3

The smiling dental company purchases equipment for $60,000.

a. What is the book value of this equipment at the end of six years,

(i) if the flat rate depreciation is 11% per annum?

(ii) if the reducing balance depreciation is 9% per annum. Give your answer to the nearest cent

1 + 1 = 2 marks

b. If the equipment depreciates at an average rate of $2.40 per patient, how many patients can be serviced before the equipment reaches a value of $24,000?

1 mark

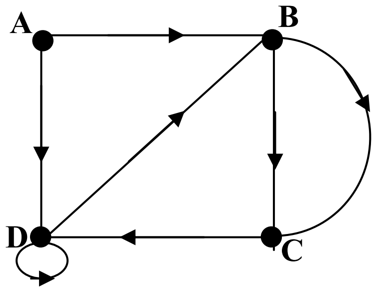
Total = 15 marks

End of Module 4: Business-related mathematics

Module 5: Networks and decision mathematics

If you choose this module, all questions are to be answered.

Question 1



a. Construct an adjacency matrix for the above network.

1 mark

b. (i) What is the sum of the elements in the first row of this matrix?

1 mark

(ii) What does this represent?

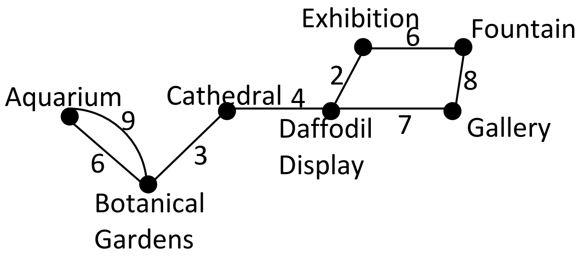
1 mark

c. What does the sum of the elements in the fourth column represent?

1 mark

Module 5: Networks and decision mathematics

Question 2



Jack and Jill wish to visit each of the tourist sites above, and also travel along each of the roads at least once since they are scenic roads.

a. Give a route that can be taken so that each of the above tourist sites can be reached with the minimum amount of driving.

____

____

1 mark

b. What is the distance that Jack and Jill will travel if the distances given on the graph are in kilometres?

____

1 mark

c. If Polly and Peter just wish to travel to each tourist site and are not interested in the scenic roads, what is the shortest distance that they can travel?

____

1 mark

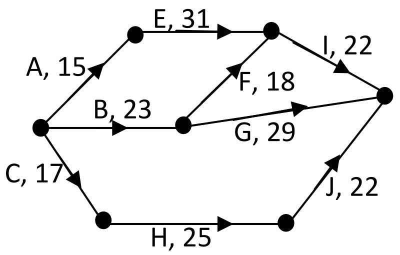
Module 5: Networks and decision mathematics

Question 3

Activities **A, B, C, D, E, F, G, H** and **I** are necessary to establish a garden.

Activity	Immediate Predecessor	Expected Duration (days)	Extra Workers reduce time by (days)	Cost of reduction per day ($)
A	none	15	3	2000
B	none	23	3	1800
C	none	17	1	1200
D	C	19	1	900
E	A	31	3	1300
F	B,D	18	2	400
G	B,D	29	1	800
H	C	25	2	1700
I	E,F	22	4	1000
J	H	22	7	1000

a. Complete the network below to show all the information given in the first three columns of the table.



1 mark

b. What is the critical path for this project?

____

1 mark

c. What is the shortest time possible to complete this project?

____

1 mark

d. What is the maximum number of days late that activity B can start and still not delay the finishing time?

____

1 mark

Module 5: Networks and decision mathematics

Question 3 (continued)

e. What is the maximum number of days late that activity F can start and still not delay the finishing time?

1 mark

f. If each day the total project takes costs the company $5000, use the information in the table given above to find the maximum amount of money that can be saved if extra workers are brought in?

2 marks

g. If activity C could be completed in just three days, how long would the whole project take if no other activity had its time reduced?

1 mark

Total = 15 marks

End of Module 5: Networks and decision mathematics

Module 6: Matrices

If you choose this module, all questions are to be answered.

Question 1

Jack Reid has 3 factories where he manufactures women's clothes, men's clothes and children's clothes. The types and quantity of material he uses for these garments at each of the factories is listed in the table below.

	Factories	Cotton (metres)	Silk (metres)	Denim (metres)
Women's Clothes	X	100	150	120
	Y	250	164	80
	Z	160	260	75
Men's Clothes	X	240	80	360
	Y	120	60	280
	Z	175	70	200
Children's Clothes	X	80	10	100
	Y	40	5	150
	Z	60	3	200

a. Complete the matrix, M, for the materials required for children's clothes at each of the three factories.

Cotton Silk Denim

$$M = \begin{matrix} X \\ Y \\ Z \end{matrix}\begin{bmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \end{bmatrix}$$

1 mark

b. Find matrix, A, which shows the total requirements of each of the three materials at factory X.

$$A = \begin{matrix} \text{Cotton} \\ \text{Silk} \\ \text{Denim} \end{matrix}\begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix}$$

1 mark

Module 6: Matrices

Question 1 (continued)

c. What is the order of matrix A?

---

1 mark

d. Complete matrix B for the total materials required for each of the three factories.

$$B = \begin{array}{c} \text{Cotton} \\ \text{Silk} \\ \text{Denim} \end{array} \overset{\begin{array}{ccc} X & Y & Z \end{array}}{\begin{bmatrix} \quad & \quad & \quad \\ & & \\ & & \end{bmatrix}}$$

1 mark

e. If the cost per metre of cotton, silk and denim is \$35, \$70 and \$20 respectively, write down the cost matrix, C, for one metre of each of the three types of material.

$$C = \begin{array}{c} \text{Cotton} \\ \text{Silk} \\ \text{Denim} \end{array} \begin{bmatrix} \quad \\ \\ \\ \end{bmatrix}$$

1 mark

f. If Q is the matrix that gives the total cost of materials at each of the three factories, and if $Q = P \times C$, write down matrix P.

1 mark

g. Find matrix Q.

1 mark

h. If all the clothes made at factory Y are sold for \$156000, what profit is made at factory Y?

---

1 mark

Module 6: Matrices

Question 2

Rex owns a company where his employees use either Orange or Lemon computers. The Orange computers have a 70% probability of working tomorrow if they work today and a 40% probability of needing repair tomorrow if the needed repair today.

a. Write down the transition matrix, T, for the Orange computers.

$$T = \text{Tomorrow} \overset{\text{Today}}{\begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}}$$

1 mark

b. Of the Orange computers, 60 are working today and 20 are being repaired.
Construct the initial state matrix, S_0

$$S_0 = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$$

1 mark

c. Construct the matrix S_1 that shows the number of Orange computers working and needing repairs after one week.

$$S_1 = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$$

1 mark

d. In the long term, how many Orange computers can Rex expect to be working?

____

____

1 mark

Module 6: Matrices

Question 2 (continued)

For the Lemon computers, Rex notes that they have an 80% probability of working tomorrow if they work today and a 50% probability of needing repair tomorrow if the needed repair today.

e. If initially, 70 of the Lemon computers are working and 10 are not working, how many will be working after 2 weeks?

1 mark

f. If Rex wishes to buy more computers for his firm, which brand, Oranges or Lemons should he buy? Give a reason for your answer.

2 marks

Total = 15 marks

End of Module 6: Matrices

END OF QUESTION AND ANSWER BOOK
2008 Further Mathematics Trial Examination 2

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