Neap:

Trial Examination 2008

VCE Further Mathematics Units 3 & 4

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

Student's Name: ______________________________

Teacher's Name: ______________________________

Structure of Booklet

Section	Number of questions	Number of questions to be answered	Number of marks
A – Core	1	1	15
Section	**Number of modules**	**Number of modules to be answered**	**Number of marks**
B – Modules	6	3	45

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.

Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white-out liquid/tape.

Materials supplied

Question booklet of 28 pages with a detachable sheet of miscellaneous formulas in the centrefold.
Working space is provided throughout the booklet.

Instructions

Detach the formula sheet from the centre of this booklet during reading time.
Please ensure that you write your **name** and your **teacher's name** in the space provided on this page.
All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2008 VCE Further Mathematics Units 3 & 4 Written Examination 2.

Instructions

This examination consists of a core and six modules. Students should answer **all** questions in the core and then select **three** modules and answer **all** questions within the modules selected.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π, surds or fractions.

Page

SECTION A – DATA ANALYSIS – CORE MATERIAL

Question 1

a. The number of visitors for two month periods at the tourist attractions in Touristville are shown in the table below.

	Jan/Feb	Mar/Apr	May/Jun	Jul/Aug	Sep/Oct	Nov/Dec
War memorial	300	250	80	70	120	190
Parliament house	340	260	90	85	140	210
Lookout tower	90	55	40	30	32	56
Museum	12	8	6	7	8	10
Library	28	25	18	17	19	22
Art gallery	104	68	35	32	65	95

i. How many people went to the lookout tower in September/October?

ii. Calculate the mean number of visitors to an attraction in the July/August period (to one decimal place).

iii. Calculate, correct to two decimal places, the Standard Deviation for the number of visitors to the war memorial over the year.

1 + 1 + 1 = 3 marks

b. More detailed analysis of the visitors to the Touristville library over a three year period shows the following data.

	1st quarter	2nd quarter	3rd quarter	4th quarter	Mean for year
2004	45	27	25	32	32.25
2005	52	30	23	35	35
2006	61	35	31	40	41.75

i. Calculate the seasonal index for the first quarter (to two decimal places).

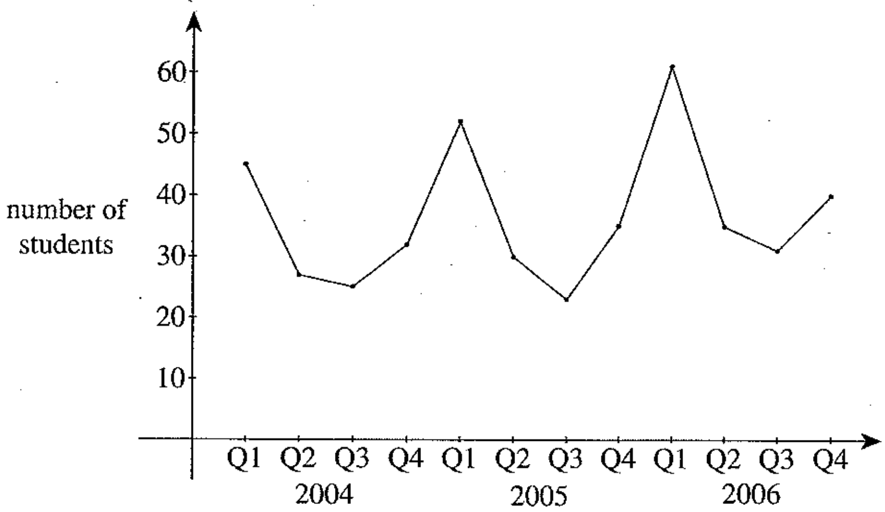
ii. Complete the following table.

	1st quarter	2nd quarter	3rd quarter	4th quarter
Seasonal index		0.84		0.98

iii. Deseasonalise the raw data and complete this table of values.

	1st quarter	2nd quarter	3rd quarter	4th quarter
2004	31.0	32.1	34.2	
2005	35.9	35.7	31.5	
2006	42.1	41.7	42.5	

iv. Add a time series for the deseasonalised data to the time series of the raw data shown.



v. Comment on the effect, if any, of deseasonalising the data.

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2 + 1 + 1 + 1 + 1 = 6 marks

c. An exciting new attraction at Touristville opened in 2001 and Touristville experienced substantial growth in visitor numbers over the first five years of operation.

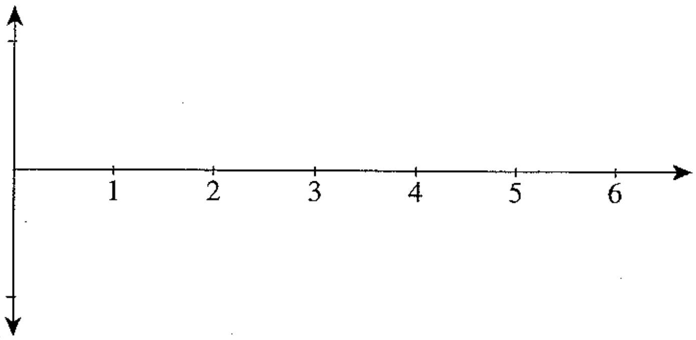
	2001	**2002**	**2003**	**2004**	**2005**	**2006**
Average visitors per day	54	440	1030	2980	6730	12 850

i. Enter the data into L1 and L2 on your calculator (2001 = 1 in L1) to find the value of r, and the regression equation to two decimal places.

r =

average visitors per day =

ii. Sketch the residual plot and comment on what the residual plot shows.



iii. Use your calculator to apply both an x^2 and a $\log(x)$ transformation to the original data.

Which transformation improves the fit of the equation? Justify your answer and write down the new equation, to one decimal place.

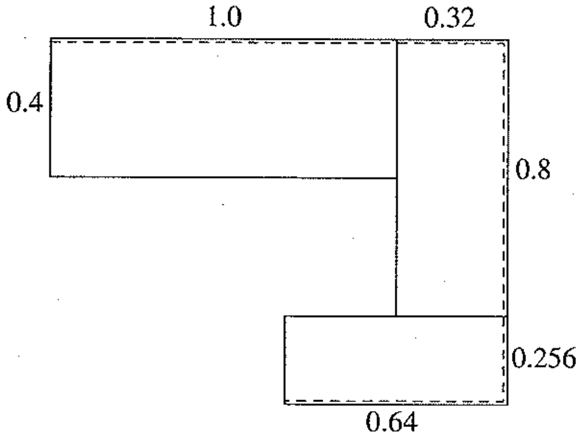
2 + 2 + 2 = 6 marks
Total 15 marks

SECTION B – MODULES

Module 1: Number patterns

Question 1

Steph is making a pattern with some bricks. She selects the first brick with length 1.0 m and width 0.4 m. The second brick is placed at a right angle to the first as shown below. It has length 0.8 m and width 0.32 m. A third brick of length 0.64 m and width 0.256 m is placed at a right angle to the second so that a clockwise spiral is formed.



a. What would be the dimensions of a fourth brick (to the nearest millimetre)?

1 mark

b. Further bricks are added until the last brick which has a length closest to 1 cm.

How many bricks are added to the pattern above?

2 marks

c. An ant runs around the outside of the brick pattern as shown by the dashed line on the diagram.

If the ant was to continue this until it reaches the end of the last brick, how far would it have travelled (to the nearest mm)?

3 marks

d. Steph now considers the pattern if it were possible to continue the pattern forever so that the size of the smaller brick is ignored. She places a rectangle border around the pattern such that the area is minimised, yet it still contains the entire pattern. She now considers the area of the pattern.

i. The sequence D_n gives the area of brick n.

Find the common ratio for this sequence.

ii. What percentage of the bordered region is occupied by bricks (to three decimal places)?

1 + 2 = 3 marks

Question 2

Steph sells her designs. She hires a marketing consultant for this. This involves an initial year's fee of $1800, reducing by $350 every subsequent year.

a. Write down an equation for the fee, P_n, due in the nth year.

1 mark

b. Write down an equation for the total fees, F_n, due over the first n years under this scheme.

2 marks

c. When these marketing fees reduce to a point whereby the calculated amount due is negative, no fee is due in that year or further years.

What is the total amount to be paid in marketing fees?

1 mark

A more complicated arrangement for calculating marketing fees is proposed. According to this scheme, the marketing fees for years 1 and 2 are $1500 and $1200. In the years that follow, the mean (average) of the fees for the preceding two years is calculated and $100 is subtracted. The fee for that year is the result of this calculation.

d. Write down a difference equation for P_{n+1}, the fee in year $n + 1$.

1 mark

e. Use this to determine the fees applicable to years 3 and 4.

1 mark

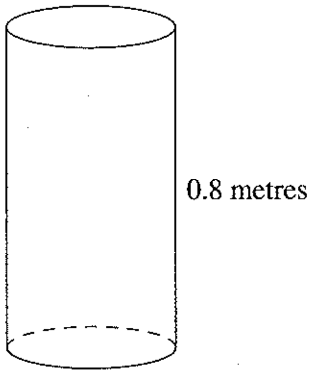
Total 15 marks

END OF MODULE 1

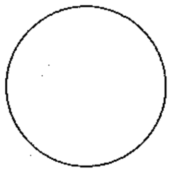
Module 2: Geometry and trigonometry

Question 1

Sam designs light poles to be used along driveways of rural properties. Each pole is the shape of a cylinder and it contains a light which sits on top of the pole. Each light is semi-spherical. The diagram below shows one of the poles.



The diagram below shows a cross-section of the pole.



circumference = 48.2 cm

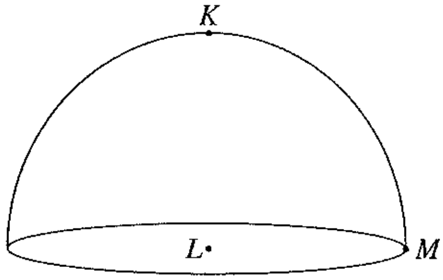
a. Determine the diameter of the pole correct (to two decimal places).

1 mark

b. Determine the area of the cross-section of the pole (to the nearest square centimetre).

1 mark

The diagram below shows the light cover that sits tightly on top of the light pole: in other words, the radius of the light cover is the same as the radius of the light pole. Distance KL is equal to distance LM. Point L represents the centre of the base of the light cover.



c. Write down the size of angle LKM.

1 mark

d. Determine distance KL (to two decimal places).

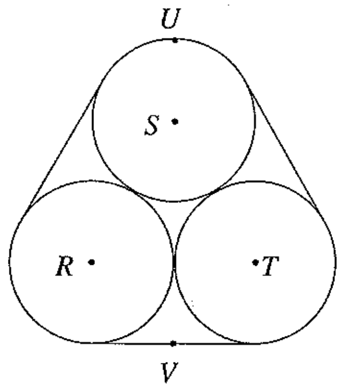
1 mark

e. Danny needs to paint the outside surface of twelve light poles. The light on each pole does not need to be painted, nor does the base of the pole.

Calculate the total surface area that Danny is required to paint (to the nearest 100 square centimetres).

2 marks

To assist in delivering the light poles, each set of three poles is bundled together as shown in the diagram below. The diagram represents the cross-section view of three poles taped together. Points R, S and T represent the centre of each pole.



Carly measures the radius of a single light pole to be 7.7 cm.

f. Using Carly's measurement, calculate

i. the perimeter of triangle RST (to one decimal place).

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ii. distance UV (to one decimal place).

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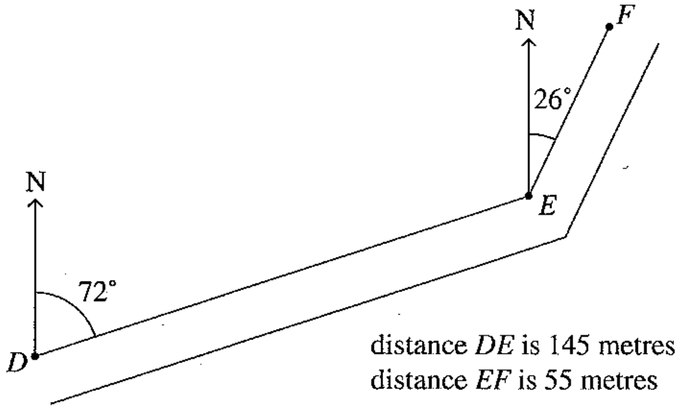
1 + 2 = 3 marks

Question 2

The light poles are to be placed along one edge of two straight sections of a driveway as shown in the diagram below. The diagram is the view from above and it describes the side of the driveway where the light poles will be placed.



a. Calculate the size of the angle DEF.

1 mark

b. Calculate the direct distance between point D and point F (to the nearest metre).

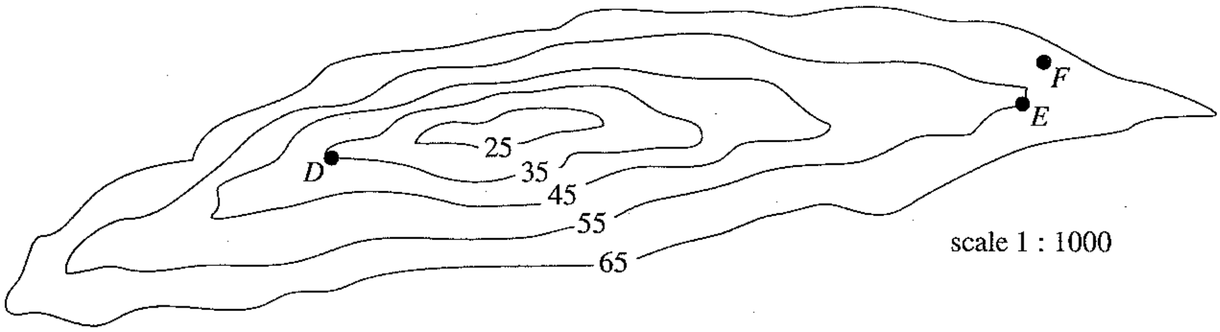
1 mark

c. Calculate the true bearing of point D from point F (to the nearest degree).

2 marks

Question 3

Stan investigates the design of a proposed new job that involves a site of land that is not flat. The contour map below describes the land at the new site but it does not provide specific information about point F. Previous records indicate that the vertical distance between point E and point F is 2.5 metres. The distance between point D and point F is 9.5 cm on the contour map.



Calculate the angle of elevation of point F from point D (to the nearest degree.)

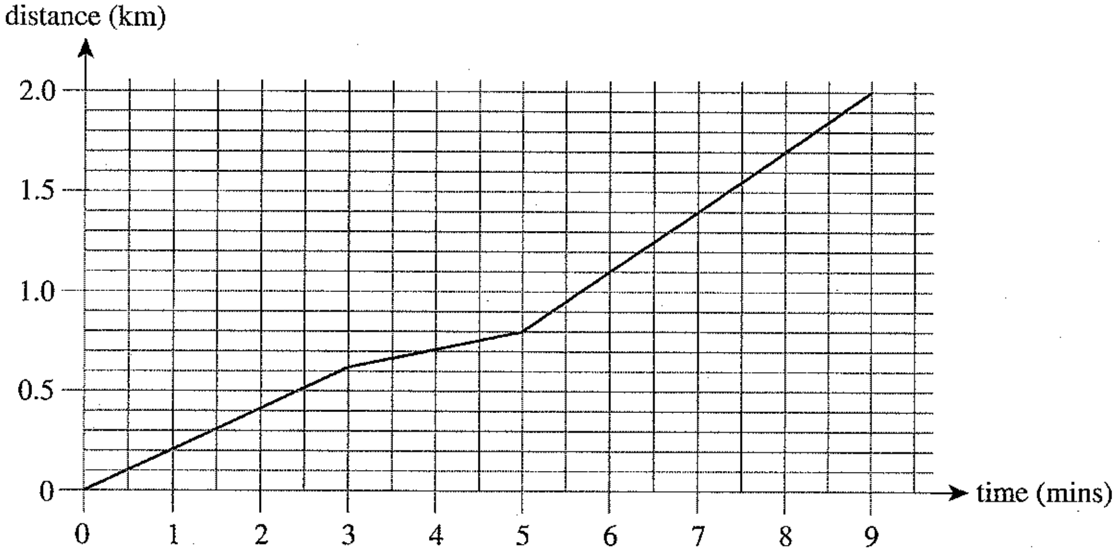
2 marks
Total 15 marks

END OF MODULE 2

Module 3: Graphs and relations

Question 1

Toni runs the town's athletics carnival. One of the main events is the 2000 m race. Due to the terrain of the course this year, Toni believes that the pace of the leading runners will not be constant, but will be slowest up Hill street. There are two other sections of the course – one is downhill and quite fast, the other is horizontal. The graph below shows the progress of one competitor in a training run along the course.



a. What is the time required to complete this race by the competitor in their trial run?

1 mark

b. How long did the competitor spend running along Hill street, the slowest segment of the race?

1 mark

c. What was the average speed of the competitor along Hill street to the nearest km/h?

1 mark

Question 2

Toni must ensure that the carnival runs at a profit. The hire of equipment and facilities amounts to \$1.7 m, and each event costs \$0.1 m to stage due to labour costs. The carnival receives a \$0.25 m share of betting revenue from each event conducted.

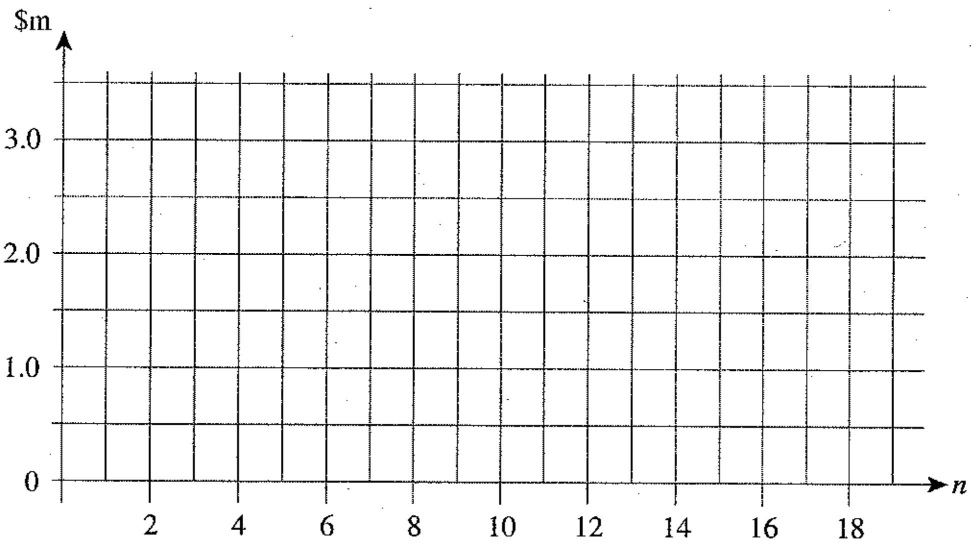
a. Write down an expression for the total cost (C) of n events.

1 mark

b. Write down an expression for the revenue raised (R) when n events are staged.

1 mark

c. Plot graphs for both R and C on the axes below.



1 mark

d. Hence determine the minimum number of events that Toni must conduct in order for the carnival to make a profit.

1 mark

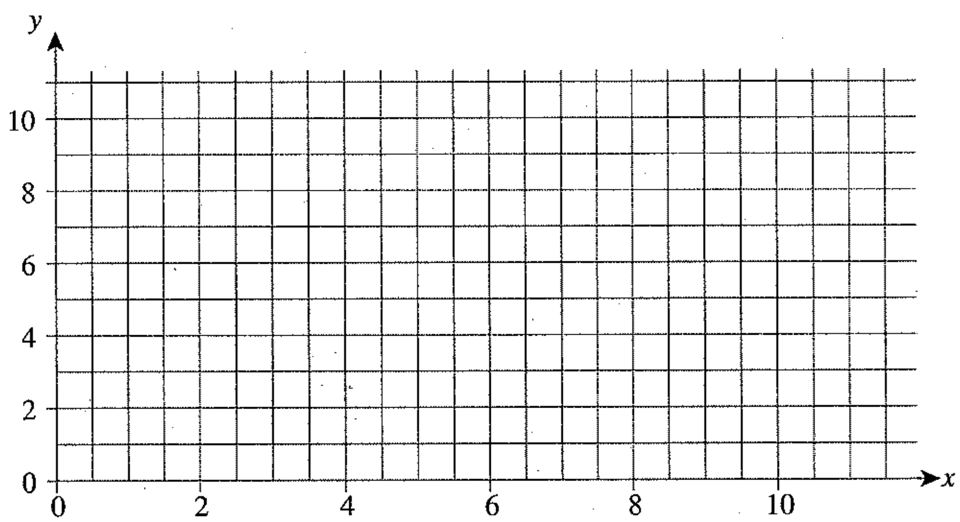
Question 3

Toni must also consider the most efficient use of the facilities. Two locations are available. Location A can host three track events and two field events per hour, while location B can host two track events and four field events per hour. Toni knows that she has at least 24 track events and at least 20 field events that she needs to schedule. It is not possible to book a location for either track or field alone. She also knows that she has a maximum nine hours of daylight in which events can be staged at each location.

a. Write down a set of constraints on x and y, the number of hours that locations A and B are booked respectively.

2 marks

b. Sketch the region required on the graph grid below.



3 marks

c. The cost of hiring location A is \$50 per hour while location B costs \$70 per hour.

i. Write down an expression for the total cost of hiring the two locations.

ii. Hence determine the minimum cost of hiring the two locations.

1 + 2 = 3 marks
Total 15 marks

END OF MODULE 3

Module 4: Business-related mathematics

Question 1

Stephanie buys a \$340 new MP3 player on hire-purchase. The conditions are \$40 deposit, 8.1% per annum flat interest and monthly repayments over three years.

a. Calculate the interest charged.

1 mark

b. Calculate the monthly repayments (to the nearest cent).

1 mark

c. How much does Stephanie pay for the MP3 player in total?

1 mark

d. What is the effective interest rate (to two decimal places)?

1 mark

e. Her new MP3 player depreciates at a rate of 35% per annum using the reducing balance method. What is the value of the player after three years (to the nearest cent)?

1 mark

Question 2

In June 2008 Nicholas receives the following bank statement for the previous six months.

Date	Detail	Debit ($)	Credit ($)	Balance ($)
1/1/08	balance forward			3800.00
25/4/08	deposit		750.00	X
13/5/08	withdrawal	2000.00		2550.00
24/5/08	deposit			Y
2/6/08	withdrawal	1000.00		5400.00

a. Calculate the values of X and Y.

$X =$

$Y =$

2 marks

b. The interest rate for Nicholas's account is 5.1% per annum calculated monthly on the minimum monthly balance and added every six months.

i. What is the monthly interest rate?

ii. How much interest is earned in the first three months?

iii. What is the balance of Nicholas's account after adding interest on the 30/6/08?

1 + 1 + 2 = 4 marks

Question 3

Meredith and Peter wish to take out a \$25 000 loan to purchase a new car.

a. The Friendly Bank offers them a personal loan to be paid out over four years at an interest rate of 8.5% per annum compounding monthly.

i. Calculate the monthly repayment.

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ii. Calculate the total to be repaid over the four years.

___

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iii. If the car is sold for \$6500 after four years, how much has the car cost them over four years, disregarding all other costs like petrol, insurance, repairs etc.?

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1 + 1 + 1 = 3 marks

b. The car yard salesman offers them an alternative deal to lease the car rather than buy. They pay a monthly fee of \$200 plus 25 cents per km travelled. In a year Peter and Meredith travel an average of 18 000 km.

Calculate the yearly cost to them.

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1 mark

Total 15 marks

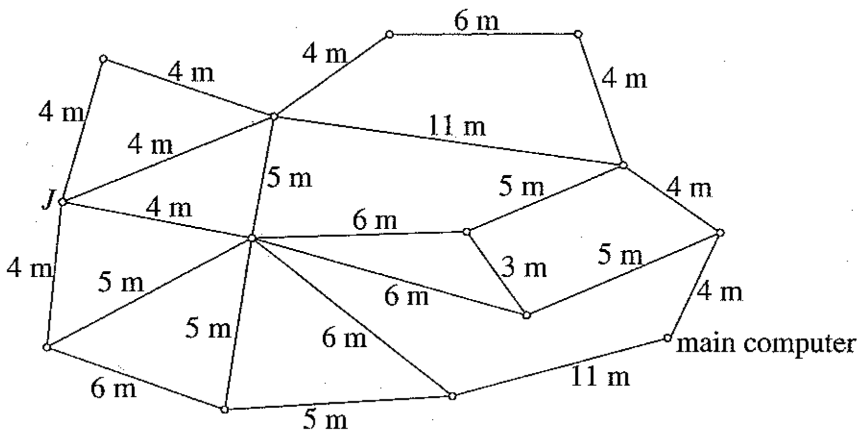
END OF MODULE 4

Module 5: Networks and decision mathematics

Question 1

A computer network exists throughout one floor of a city building. It consists of a main computer and other computers linked by cables.

The diagram below shows the distances along the cables (in metres) that connect the computers to each other.



An upgrade is required at the computer labelled *J*.

a. **i.** On the diagram above, clearly draw the shortest path between the main computer and computer *J*.

ii. Determine the length (in metres) of the shortest path between the main computer and computer *J*.

____

1 + 1 = 2 marks

b. Kylie, the computer network manager, begins testing computers and some cables. Starting at the main computer she intends to visit each computer in the network only once and in a continuous order. In other words, she will test a computer then test the cable leading from that computer to the next, then test that computer and so on. She does not intend to test all the cables. She does not need to return to test the main computer.

i. What is the mathematical term used to describe this journey?

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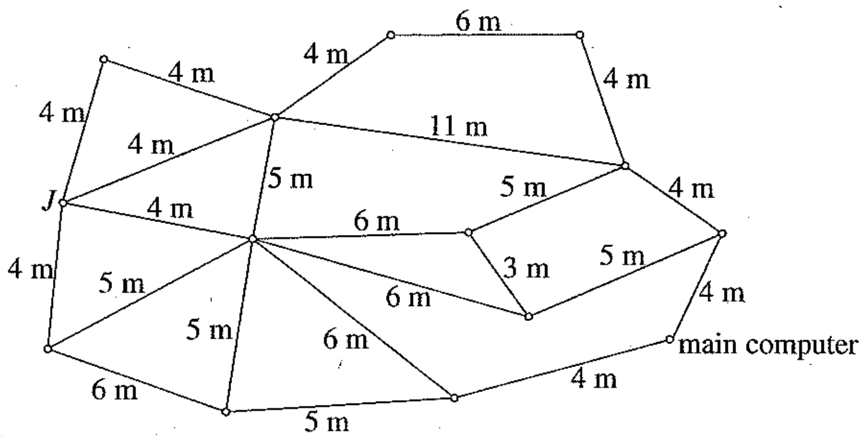
ii. On the following diagram, draw a complete journey that Kylie could take.

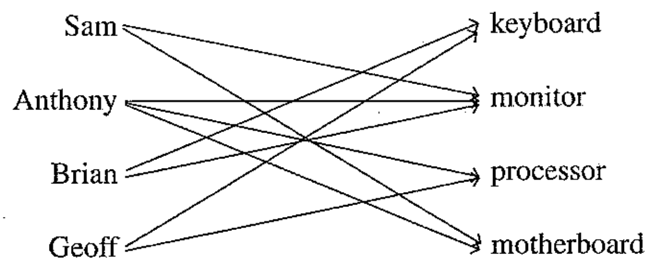


1 + 1 = 2 marks

Question 2

Kylie employs four assistants – Sam, Anthony, Brian and Geoff – to help with a major network upgrade. Four tasks are required on each computer. The tasks involve the keyboard, the monitor, the processor and the motherboard.

The bipartite graph below shows the expertise of each assistant.



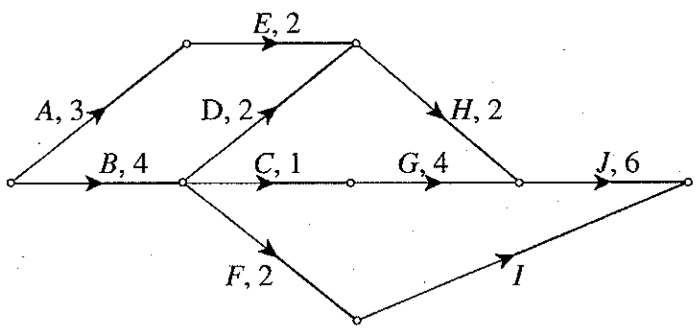
Kylie allocates the keyboard task to Geoff.

Which task **must** Sam perform?

1 mark

Question 3

The four computer assistants are required to rebuild the computer network. This will involve ten activities. The activities, their immediate predecessors and completion times (in hours) are shown in the network graph below.



The duration of activity I is not yet known, but it is known that

- activity I takes no longer than four hours,
- there is only one critical path, and
- activity B and activity J are on the critical path.

a. Write down the critical path for this project.

1 mark

b. Write down the duration of the critical path.

1 mark

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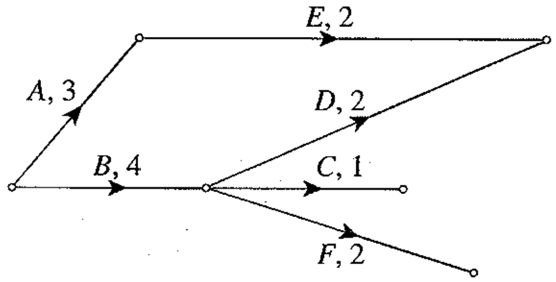
c. It was discovered that the immediate predecessors of activity H were incorrect. The correct predecessors of activities G, H, I and J are shown in the following table.

Activity	Immediate predecessor(s)	Duration
G	C	4
H	D, E, G	2
I	F	unknown
J	H	6

Use the information in the table above to complete the network below by drawing the activities G, H, I and J.



2 marks

d. How many extra hours are required to complete the project after correcting the immediate predecessors of activity H?

2 marks

e. Calculate the maximum number of hours that activity D can be delayed without delaying the overall completion time.

1 mark

f. For each hour that the project completion time is reduced, the company saves \$1000. Reducing activity G will reduce the project completion time. It costs \$450 per hour to reduce activity G.

How much money should be spent on reducing activity G?

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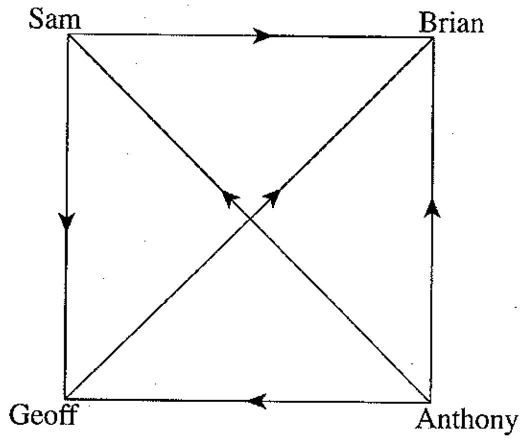
1 mark

Question 4

The four computer assistants used the computers to access internet games. They played a tournament where each player played each of the other three players once. The results are represented in the directed graph below.



Taking into consideration, one-step, two-step and three-step dominances, which player was ranked third in the tournament?

2 marks
Total 15 marks

END OF MODULE 5

Module 6: Matrices

Question 1

Get-rite is an ecological advisory company. One of their products is a series of guides on ecologically responsible building. Various brands of water conservation systems are investigated in various environments. The results are summarised in the table below.

Brand name	Water captured per month (litres per 100 mm rainfall)	Proportion water use reduction
Weather-watch	250	0.3
Irrigator	150	0.4
Dry days	300	0.2
Eco-bounty	250	0.1

a. Produce a 4×2 matrix, A, that displays water capture and usage reduction for the four brands listed.

1 mark

b. Matrix $B = \begin{bmatrix} 1.5 & 0.7 & 0.9 & 1.6 \\ 600 & 480 & 900 & 890 \end{bmatrix}$ gives the expected monthly rainfall in hundreds of millimetres (first row) and the water use (second row) for four particular households.

i. Calculate the matrix product AB.

ii. Describe the nature of the information that matrix AB provides. Describe what each row represents and what each column represents.

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iii. If the total amount of water saved (either through tank capture or reduced usage) is the only consideration, which of the four systems performs the best in mean savings? Provide numerical justification for your choice.

iv. Would the matrix product BA be defined and meaningful? If so, explain what its physical significance would be.

1 + 2 + 1 + 2 = 6 marks

Question 2

Get-rite also produce a garden watering system that uses soil moisture detectors to determine where watering should occur. The garden is divided into three regions: dry (D), watered (W) and over-watered (O). The following are the results of testing.

- Of regions that are dry, 10% remain dry, 5% are considered over-watered and the remainder are classified as watered in the following week.
- Of regions that are regarded as watered, 12% are considered dry, 8% are considered over-watered and the remainder are considered as watered in the following week.
- Of over-watered regions, 40% remain over-watered, 5% are considered dry and 55% are regarded as watered in the following week.

The initial state of the garden is $S_0 = \begin{bmatrix} 0.1 \\ 0.2 \\ 0.7 \end{bmatrix} \begin{matrix} D \\ W \\ O \end{matrix}$

a. Produce a transition matrix, T, that can be used to determine the proportions of each garden state category in successive years.

3 marks

b. For how many weeks does the system need to be operating before at least 75% of the sections are regarded as watered?

2 marks

c. It is decided that it would be more useful to have a transition matrix that made monthly predictions rather than weekly ones.

If it is assumed that a month can be approximated to four weeks, determine this matrix to three decimal places.

1 mark

d. Is there a maximum proportion of the garden that is regarded as watered if this system is allowed to continue indefinitely? If so, what is this level?

2 marks

Total 15 marks

END OF QUESTION AND ANSWER BOOKLET

Neap:

Trial Examination 2008

VCE Further Mathematics Units 3 & 4

Written Examination 2

Formula Sheet

Detach this formula sheet during reading time.
This formula sheet is provided for your reference.

FURTHER MATHEMATICS FORMULAS

Core: Data analysis

standardised score:	$z = \frac{x - \bar{x}}{s_x}$
least squares line:	$y = a + bx$ where $b = r\frac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$
residual value:	residual value = actual value − predicted value
seasonal index:	seasonal index = $\frac{\text{actual figure}}{\text{deseasonalised figure}}$

Module 1: Number patterns

arithmetic series:	$a + (a + d) + \ldots + (a + (n - 1)d) = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$
geometric series:	$a + ar + ar^2 + \ldots + ar^{n-1} = \frac{a(1 - r^n)}{1 - r}, r \neq 1$
infinite geometric series:	$a + ar + ar^2 + ar^3 + \ldots = \frac{a}{1 - r}, \lvert r \rvert < 1$

Module 2: Geometry and trigonometry

area of a triangle:	$\frac{1}{2}bc\sin A$
Heron's formula:	$A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a + b + c)$
circumference of a circle:	$2\pi r$
area of a circle:	πr^2
volume of a sphere:	$\frac{4}{3}\pi r^3$
surface area of a sphere:	$4\pi r^2$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a cylinder:	$\pi r^2 h$
volume of a prism:	area of base × height
volume of a pyramid:	$\frac{1}{3}$ area of base × height

Pythagoras' theorem: $c^2 = a^2 + b^2$

sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$

Module 3: Graphs and relations

Straight line graphs

gradient (slope): $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

equation: $y = mx + c$

Module 4: Business-related mathematics

simple interest: $I = \dfrac{PrT}{100}$

compound interest: $A = PR^n$ where $R = 1 + \dfrac{r}{100}$

hire purchase: effective rate of interest $\approx \dfrac{2n}{n+1} \times$ flat rate

Module 5: Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Module 6: Matrices

determinant of a 2×2 matrix: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

inverse of a 2×2 matrix: $A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$

END OF FORMULA SHEET

Neap:

Trial Examination 2008

VCE Further Mathematics Units 3 & 4

Written Examination 2

Suggested Solutions

SECTION A – DATA ANALYSIS – CORE MATERIAL

Question 1

a. **i.** 32 A1

ii. $\frac{(70+85+30+7+17+32)}{6} = \frac{241}{6}$

$= 40.1\dot{6}$

$= 40.2$ A1

iii. Enter war memorial figures into L1, and use Stat → Calc → 1-varStat

$\sigma_x = 94.11$ A1

b. **i.** $\frac{45}{32.25} = 1.395$

$\frac{52}{35} = 1.486$

$\frac{61}{41.75} = 1.461$

$\frac{1.395 + 1.486 + 1.461}{3} = 1.447$

$= 1.45$ A2

ii.

	1st quarter	2nd quarter	3rd quarter	4th quarter
Seasonal index	1.45	0.84	0.73	0.98

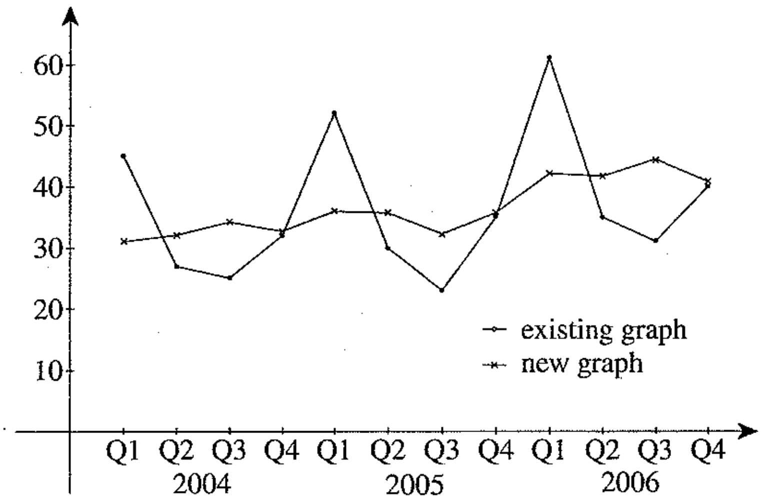
A1

iii.

	1st quarter	2nd quarter	3rd quarter	4th quarter
2004	31.0	32.1	34.2	32.7
2005	35.9	35.7	31.5	35.7
2006	42.1	41.7	42.5	40.8

A1

iv.



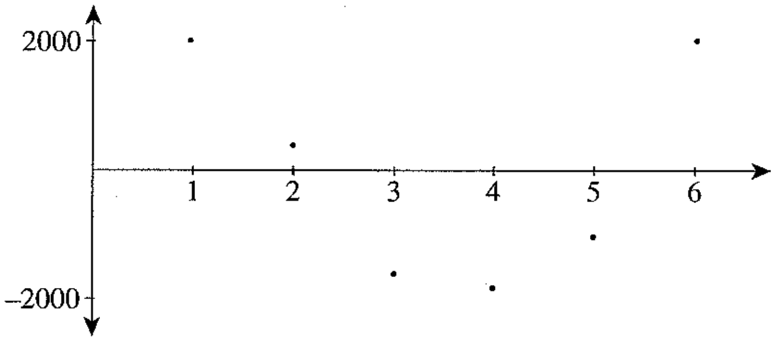
A1

v. Deseasonalising has smoothed the curve and shows a slight positive trend. A1

c. **i.** r = 0.91 using stat → calc → linreg A1

average visitors per day = – 4466 + 2422.86 × year A1

ii.



A1

Plot shows a pattern and does not support the assumption of linearity. A1

iii. x^2 transf. $r = 0.97$

$\log_x$ transf. $r = 0.79$

x^2 gives the best transformation

average visitors per year = $-1483.3 + 362.5 \times (\text{year})^2$

SECTION B – MODULES

Module 1: Number patterns

Question 1

a. The lengths and widths of the bricks form a geometric sequence, where the first term is 1.0 and the common ratio is 0.8.

$l_4 = ar^3$

$= 1.0(0.8)^3$

$= 0.512$

Thus 512 mm.

$w_4 = 0.4(0.8)^3$

$= 0.2048$ A1

Thus 205 mm.

Both l_4 and w_4 required for 1 mark

b. $0.01 = 1.0(0.8)^{n-1}$

$0.01 = 0.8^{n-1}$

$\log_{10}(0.01) = (n-1)\log_{10}(0.8)$ M1

$-2 = -0.9691(n-1)$

$20.638 = n - 1$

$n = 21.638$

$l_{21} = 0.0115$

$l_{22} = 0.00922$

Therefore, the 22nd brick is the last, so 19 more bricks are required. A1

Alternatively, trial and error may be used to find the term in the sequence l_n that is closest to 0.01.

For full marks, evidence that various terms have been calculated is required.

c. The distance travelled by the ant is a geometric sequence, where $a = 1.32$. This question requires finding the sum of the first 22 terms. M1

$S_{22} = \dfrac{1.32(1-0.8^{22})}{1-0.8}$ M1

$= \dfrac{1.32(0.9926)}{0.2}$

$= 6.552$ A1

d. **i.** The area of the first brick is 0.4 m^2. The common ratio of areas is 0.64, i.e. the length common ratio squared. A1

ii.
$$S_\infty = \frac{a}{1-r}$$
$$= \frac{0.4}{1-0.64}$$
$$= \frac{10}{9}\text{ m}^2$$
M1

area = length × width
$$= 1.32 \times 1.056$$
$$= 1.39392\text{ m}^2$$

The percentage of region enclosed by bricks $= \dfrac{\left(\frac{10}{9}\right)}{1.39392} \times 100 = 79.7\%$ A1

Question 2

a.
$$P_n = a + (n-1)d$$
$$P = 1800 - (n-1)350$$ A1
$$P = 2150 - 350n$$

Either the expanded or unexpanded version of the answer is acceptable.

b. This is arithmetic. The sequence F_n is actually the sum of n years of fees.

$$F_n = \frac{n}{2}[2a + (n-1)d]$$ M1
$$F_n = \frac{n}{2}[3600 - 350(n-1)]$$
$$F_n = n(1975 - 175n)$$ A1
$$= 1975n - 175n^2$$

Either the expanded or unexpanded version of the answer is acceptable.

c. Firstly determine when the fee is due to become negative.

$$0 = 1800 - 350n$$
$$350n = 1800$$
$$n = 5.14$$

So the first five years of fees are to be paid.

$$F_5 = 1975(5) - 175(25)$$
$$= 9875 - 4375$$
$$= \$5500$$ A1

d. $P_{n+2} = 0.5(P_n + P_{n+1}) - 100,\ P_1 = 1500,\ P_2 = 1200$ A1

e. $P_3 = 0.5(1500 + 1200) - 100 = 1250$

$P_4 = 0.5(1200 + 1250) - 100 = 1125$ A1

Module 2: Geometry and trigonometry

Question 1

a. $c = \pi \times d$

$d = \frac{c}{\pi}$

$d = \frac{48.2}{\pi}$

$d = 15.342536\ldots$

$d \cong 15.34$ cm A1

b. $\text{area} = \pi \times r^2$

$= \pi \times \left(\frac{15.3425365}{2}\right)^2$

$= 185.0651577$

$\text{area} \cong 185 \text{ cm}^2$ A1

c. $\angle LKM = 45°$ A1

d. distance KL = radius of circle

$= \frac{\text{diameter}}{2}$

$= \frac{\left(\frac{c}{\pi}\right)}{2}$

$= \frac{\left(\frac{48.2}{\pi}\right)}{2}$

$= 7.67126\ldots \cong 7.67$ cm A1

e. surface area of one pole = circumference × height

$= 48.2 \times 80$

$= 3856 \text{ cm}^2$ M1

surface area of 12 poles $= 12 \times 3856$

$= 46\ 272$

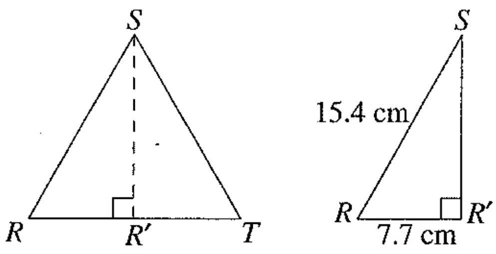
total surface area $\cong 46\ 300 \text{ cm}^2$ A1

f. **i.** perimeter $= 6 \times$ radius

$= 6 \times 7.7$

$= 46.2$ cm A1

ii.



$a^2 = h^2 - b^2$

$= 15.4^2 - 7.7^2$ M1

$= 177.87$

$a = 13.33679\ldots$

distance $UV = 7.7 + 13.33679 + 7.7$

$= 28.73679\ldots$

$\cong 28.7$ cm A1

Question 2

a. $\angle DEF = 18° + 90° + 26°$

$= 134°$ A1

b. $a^2 = b^2 + c^2 - 2bc \times \cos(A)$

$= 145^2 + 55^2 - 2 \times 55 \times 145 \times \cos(134°)$

$= 35129.801\ldots$

$a = 187.42945\ldots$

$a \cong 187$ metres A1

c. $\cos(A) = \dfrac{b^2 + c^2 - a^2}{2bc}$

$= \dfrac{55^2 + 187.429456^2 - 145^2}{2 \times 55 \times 187.429456}$

$= 0.83084839\ldots$

$A = 33.814\ldots$ M1

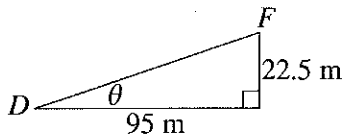
$A \cong 34°$

bearing $\overrightarrow{FD} = 180° + 26° + 34°$

$= 240°$ True A1

Question 3

The scale is 1 : 1000, so the horizontal distance between *D* and *F* is $9.5 \times 1000 = 9500$ cm = 95 m. M1
The vertical distance between *D* and *F* is $55 - 35 + 2.5 = 22.5$ m.



$$\frac{\text{opposite}}{\text{adjacent}} = \tan(\theta)$$

$$\frac{22.5}{95} = \tan(\theta)$$

$\theta = 13.3245\ldots$

$\theta = 13°$ A1

Module 3: Graphs and relations

Question 1

a. 9 minutes — A1

b. The runner is slowest (i.e. on Hill street) between $t = 3$ and $t = 5$. Therefore, the duration is 2 minutes. — A1

c. average speed $= \dfrac{\text{distance}}{\text{time}}$,

where $t = 2$ minutes $= \dfrac{1}{30}$ hours and distance = 0.2 km (from graph)

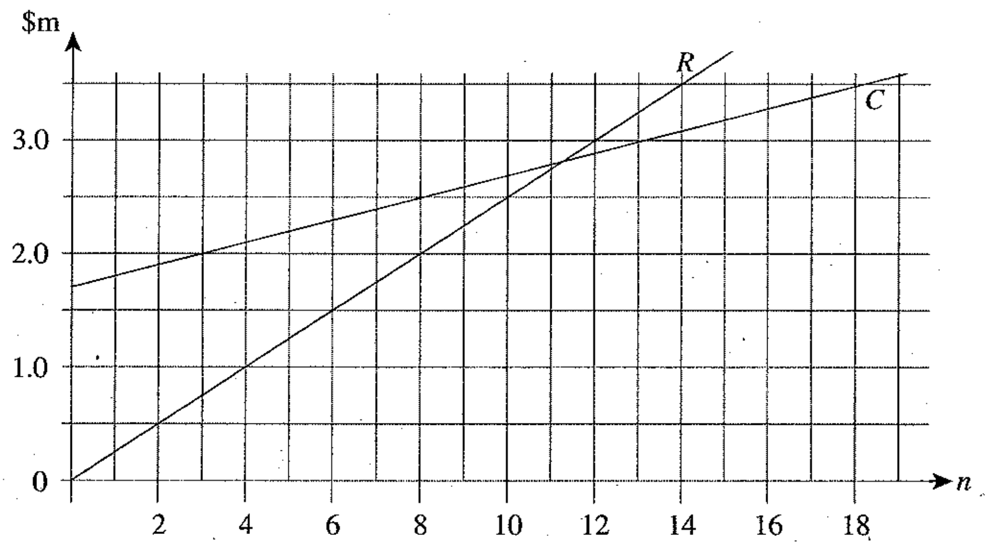
average speed $= \dfrac{0.2}{\left(\dfrac{1}{30}\right)} = 6$ km/h — A1

Question 2

a. $C = 1.7 + 0.1n$ — A1

b. $R = 0.25n$ — A1

c.



A1

Requires both lines clearly marked

d. $n = 11$ represents a loss since the line C is higher. $n = 12$ is the least value of n for which R is greater. Therefore, twelve events are required in order to make a profit. — A1

Question 3

a. The data in the question is summarised in the table below.

	Maximum event per hour		**Events capacity**
	Location A	**Location B**	
track	3	2	24
field	2	4	20

M1

$3x + 2y \geq 24$

$2x + 4y \geq 20$

$x \leq 9$

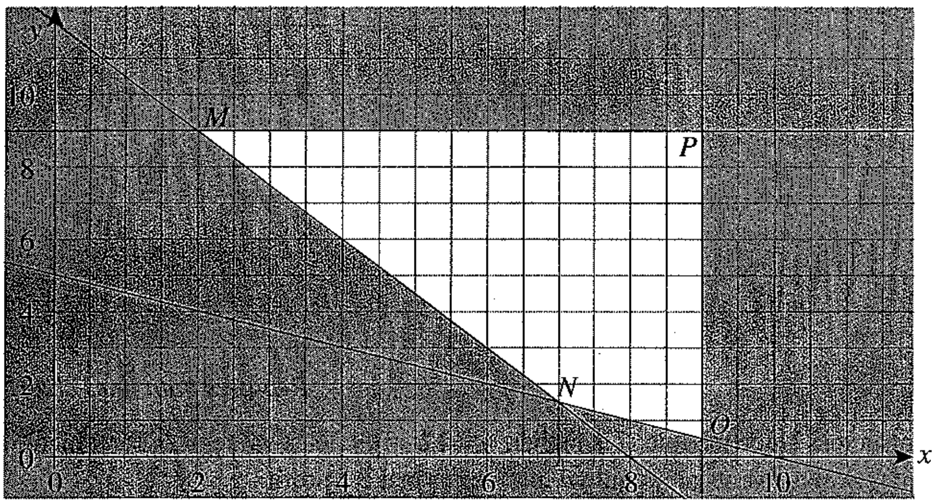
$y \leq 9$

A1

b.

$2x + 4y = 20$	$2x + 3y = 24$
y-intercept, $x = 0$	y-intercept, $x = 0$
$4y = 20$	$3y = 24$
$y = 5$	$y = 8$
$(0,5)$	$(0,8)$
x-intercept, $x = 0$	x-intercept, $x = 0$
$2x = 20$	$2x = 24$
$x = 10$	$x = 12$
$(10,0)$	$(12,0)$

M1



The unshaded region is required.

A2

1 mark for correct boundaries
1 mark for correct shading

c. **i.** $C = 50x + 70y$ A1

ii. We need to check the critical points that form the boundary of the required region.

at M, $y = 9$

$3x + 18 = 24$

$3x = 6$

$x = 2$

$M(2,9)$

at O, $x = 9$

$18 + 4y = 20$

$4y = 2$

$y = 0.5$

$O(9,0.5)$

at N

$3x + 2y = 24$ ...1

$2x + 4y = 20$ ...2

$(1) \times 2 : 6x + 4y = 48$ ...1a

$(1a) - (2) : 4x = 28$

$x = 7$

(sub in 1)$21 + 2y = 24$

$y = 1.5$

$N(7,1.5)$

Location of critical points method. A1

Calculating the cost (C) at each of these points.

at M, $C = 100 + 630 = 730$

at N, $C = 350 + 105 = 455$

at O, $C = 450 + 35 = 485$

at P, $C = 450 + 630 = 1080$

The cheapest is \$455, when location A is used for 7 hours and location B for 1.5 hours. A1

Module 4: Business-related mathematics

Question 1

a. $340 - 40 = 300$

$I = \frac{Prt}{400}$

$= \frac{300 \times 8.1 \times 3}{100}$

$= \$72.90$ A1

b. $\frac{372.90}{36} = \$10.36$ per month A1

c. $10.36 \times 36 + 40 = \$412.96$ or $300 + 72.90 + 40 = \$412.90$ A1

d. $r_e = \frac{8.1 \times 2 \times 36}{36 + 1} = 15.76\%$ A1

e. $340 \times 0.65^3 = \$93.37$ A1

Question 2

a. $X = 3800 + 750 = 4550$ A1

$Y = 5400 + 1000 = 6400$ A1

b. **i.** $\frac{5.1}{12} = 0.425$ A1

ii. January, February and March minimum balance = 3800

$\therefore I = \frac{3800 \times 0.425 \times 3}{100}$

$= \$48.45$ A1

iii. April minimum balance $= 3800 \times \frac{0.425}{100} \times 1 = 16.15$

May minimum balance $= 2550 \times \frac{0.425}{100} \times 1 = 10.84$

June minimum balance $= 5400 \times \frac{0.425}{100} \times 1 = 22.95$

interest = \$98.39 A1

$\therefore$ balance = \$5498.39 A1

Question 3

a. **i.** Using a graphics calculator, use the TVM Solver:

```
N=48
I%=8.5
PV=-25000
PMT=616.21
FV=0
P/Y=12
C/Y=12
PMT:END BEGIN
```

The payment would be \$616.21 per month. A1

ii. $616.21 \times 48 = \$29\ 578.08$ A1

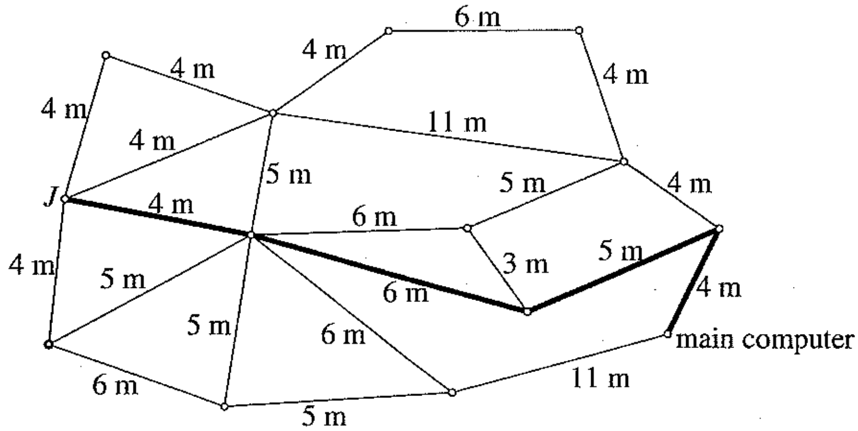
iii. $\$29\ 578.08 - 6500 = \$23\ 078.08$ A1

b. $(200 \times 12) + (0.25 \times 18\ 000) = \$6900$ A1

Module 5: Networks and decision mathematics

Question 1

a. i.

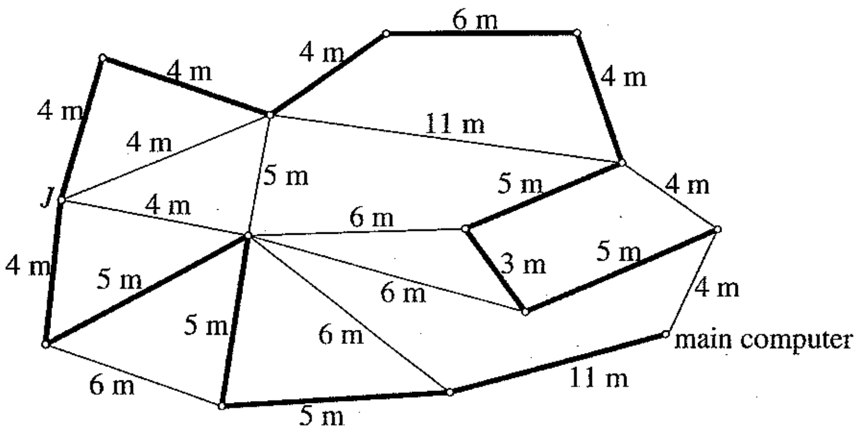


A1

ii. shortest path = 4 + 5 + 6 + 4 = 19 metres A1

b. i. Hamilton path A1

ii.



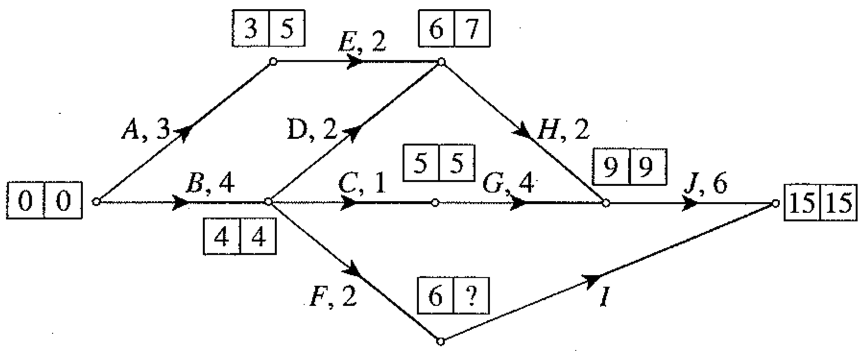
There are other journeys that Kylie could take. A1

Question 2

If Geoff is allocated to the keyboard, Anthony must be allocated to the processor since there is no other option. Sam and Brian remain, and since Brian cannot be allocated to the motherboard, Sam must perform the motherboard task. A1

Question 3

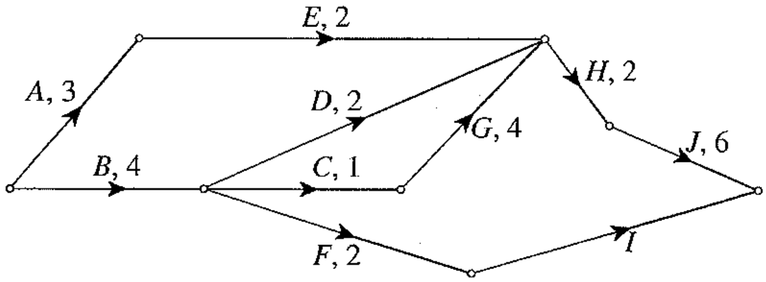
a.



B, *C*, *G*, *J* A1

b. $4 + 1 + 4 + 6 = 15$ hours A1

c.



A2
1 mark for activity G
1 mark for activities H, I and J

d. original critical path = $4 + 1 + 4 + 6 = 15$ hours

revised critical path = $4 + 1 + 4 + 2 + 6 = 17$ hours M1

Therefore, 2 extra hours are required to complete the project. A1

e. latest finish time of activity $D = 9$ hours

earliest start time of activity $D = 4$ hours

duration of activity $D = 2$ hours

maximum delay = $(9 - 4) - 2 = 3$ hours A1

f.

Reduction of activity *G* (hours)	Cost ($)	Critical Path	Project completion time (hours)
0	0	*B, C, G, H, J*	17
1	450	*B, C, G, H, J*	16
2	900	*B, C, G, H, J*	15
3	1350	*B, C, G, H, J* and *B, D, H, J*	14 14
4	1800	*B, D, H, J*	14

From the table, it can be seen that $1350 should be spent as this reduces the project completion time to 14 hours. Spending $1800 does not generate any further reduction in project completion time. A1

Question 4

$$D_1 = \begin{matrix} & S & B & G & A \\ S & 0 & 1 & 1 & 0 \\ B & 0 & 0 & 0 & 0 \\ G & 0 & 1 & 0 & 0 \\ A & 1 & 1 & 1 & 0 \end{matrix} \qquad D_2 = \begin{matrix} & S & B & G & A \\ S & 0 & 1 & 0 & 0 \\ B & 0 & 0 & 0 & 0 \\ G & 0 & 0 & 0 & 0 \\ A & 0 & 2 & 1 & 0 \end{matrix} \qquad D_3 = \begin{matrix} & S & B & G & A \\ S & 0 & 0 & 0 & 0 \\ B & 0 & 0 & 0 & 0 \\ G & 0 & 0 & 0 & 0 \\ A & 0 & 1 & 0 & 0 \end{matrix}$$

$$D_{1+2+3} = \begin{matrix} S \\ B \\ G \\ A \end{matrix} \begin{bmatrix} 2+1+0 \\ 0+0+0 \\ 1+0+0 \\ 3+3+1 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 1 \\ 7 \end{bmatrix}$$

Final rankings – 1st Anthony
2nd Sam
3rd Geoff
4th Brian

M1

Geoff was ranked third. A1

Module 6: Matrices

Question 1

a. $A = \begin{bmatrix} 250 & 0.3 \\ 150 & 0.4 \\ 300 & 0.2 \\ 250 & 0.1 \end{bmatrix}$ A1

b. **i.** $AB = \begin{bmatrix} 250 & 0.3 \\ 150 & 0.4 \\ 300 & 0.2 \\ 250 & 0.1 \end{bmatrix} \begin{bmatrix} 1.5 & 0.7 & 0.9 & 1.6 \\ 600 & 480 & 900 & 890 \end{bmatrix}$

$= \begin{bmatrix} 555 & 319 & 495 & 667 \\ 465 & 297 & 495 & 596 \\ 570 & 306 & 450 & 658 \\ 435 & 223 & 315 & 489 \end{bmatrix}$ A1

ii. The matrix AB shows the total water saved for each combination of brand and household. A1

Each row represents a different brand and each column represents a separate household. A1

iii. For Weather-watch, mean savings $= \dfrac{(555 + 319 + 495 + 667)}{4} = 509$ litres

For Irrigator, mean savings $= \dfrac{(465 + 297 + 495 + 596)}{4} = 463.3$ litres

For Dry-days, mean savings $= \dfrac{(570 + 306 + 450 + 658)}{4} = 496.0$ litres

For Eco-bounty, mean savings $= \dfrac{(435 + 223 + 315 + 489)}{4} = 365.5$ litres

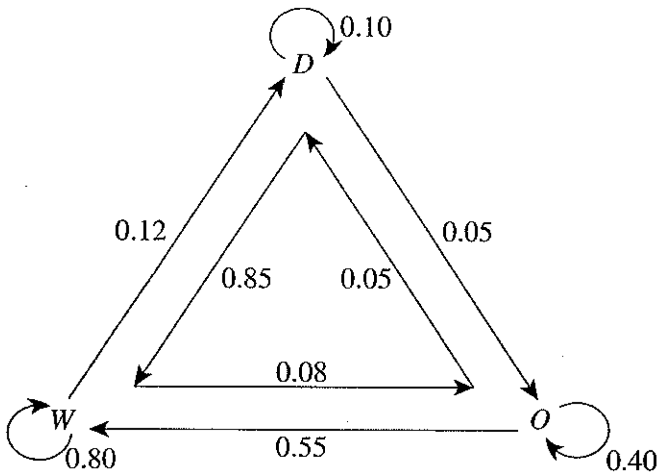
Weather-watch offers the best mean savings. A1

iv. Matrix BA is defined as the number of columns of B matches the number of rows of A. A1

Consideration of the first element in the first row of the matrix reveals that BA is not meaningful. The element is the sum of the rainwater captured for household one by brand one, household two by brand two, household three by brand three and household four by brand four. Thus BA makes no sense. A1

Question 2

a.



The diagram above shows the transitions between the three states of watered (W), dry (D) and over-watered (O).

M1

Any other similar diagram or description is satisfactory

$$T = \begin{bmatrix} 0.10 & 0.12 & 0.05 \\ 0.85 & 0.80 & 0.55 \\ 0.05 & 0.08 & 0.40 \end{bmatrix}$$

A2

1 mark for one incorrect matrix element

b. Determine the proportions in each category for each week until the 75% watered is reached.

$$S_1 = TS_0 = \begin{bmatrix} 0.069 \\ 0.630 \\ 0.301 \end{bmatrix} \quad S_2 = TS_1 = \begin{bmatrix} 0.098 \\ 0.728 \\ 0.174 \end{bmatrix} \quad S_3 = TS_2 = \begin{bmatrix} 0.106 \\ 0.761 \\ 0.133 \end{bmatrix}$$

M1

It takes three weeks to achieve the 75% watered state in the garden. A1

c. For one month, the matrix is T^4, which is the weekly transformation applied four times instead of once.

$$M = T^4 = \begin{bmatrix} 0.110 & 0.110 & 0.108 \\ 0.779 & 0.778 & 0.769 \\ 0.111 & 0.112 & 0.123 \end{bmatrix}$$

A1

d. Check the proportions after a large number of months to see if they remain constant.

$$M^{20}S_0 = \begin{bmatrix} 0.110 \\ 0.777 \\ 0.113 \end{bmatrix}$$

$$M^{21}S_0 = \begin{bmatrix} 0.110 \\ 0.777 \\ 0.113 \end{bmatrix}$$

M1

Since these results are constant, we can say that a steady state is achieved. It involves the watering level reaching a maximum value of 0.777. A1