INSIGHT

Trial Exam Paper

2009

FURTHER MATHEMATICS

Written examination 2

Worked solutions

This book presents:

- worked solutions, giving you a series of points to show you how to work through the questions.
- tips and guidelines.

SECTION A

Core: Data Analysis

Question 1

The data shows the heights of the males and females who reached the 4th round of the Australian Open tennis tournament in 2009.

Male (cm)	Female (cm)
185	178
183	171
193	178
180	173.5
191	183
188	173
185	175
188	180
188	169
180	170
183	164
188	180
198	175
198	165
196	180
185	176

Table 1

Complete the table below. Express your answers to 2 decimal places.

	Mean	**Standard deviation**
Male	188.06	
Female		5.52

1 + 1 = 2 marks

Worked solution

(s_x) (for male) = 5.79

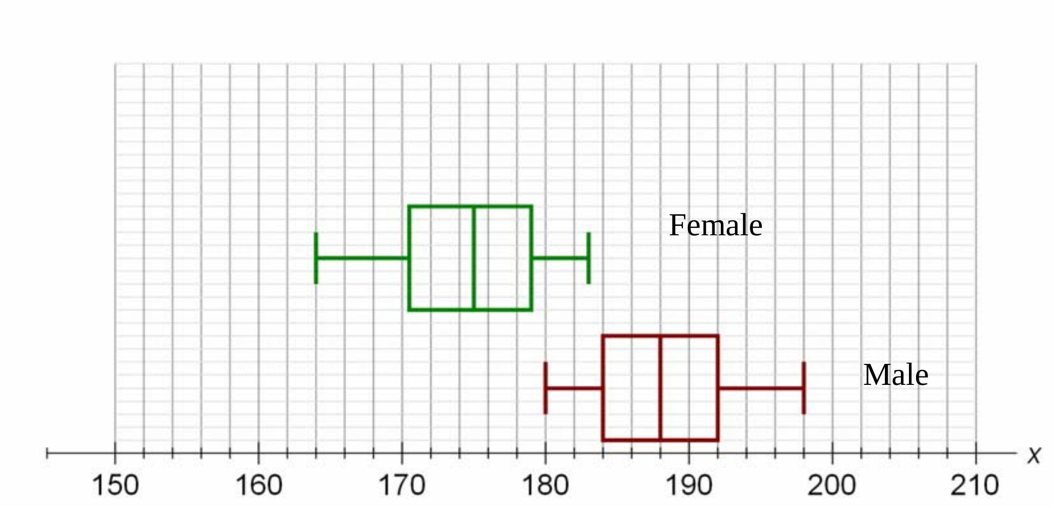
$(\bar{x})$ (for female) = 174.41

The result is obtained by carrying out a 1 variable statistics analysis on the calculator to find mean $(\bar{x})$ and standard deviation (s_x).

Mark Allocation

- 1 mark for each correct answer, rounding to 2 decimal places.

Below are boxplots for the male and female heights.



a. The range of the male tennis players is

1 mark

Worked solution

Range = Max – Min

Range = 198 – 180

Range = 18

Mark Allocation

- 1 mark for the correct answer.

b. Explain the similarities and differences between the male and female heights for the above sample.

2 marks

Worked solution

Similarities: Both distributions have a similar range – male (18cm) and female (19cm). They are each roughly symmetrically distributed and each has a similar IQR – male (8) and female (8.8).

Differences: Males have a greater centre, and 75% of the male data set is greater than the maximum value on the female distribution.

Mark Allocation

- 1 mark for stating a difference and 1 mark for stating a similarity between the 2 data sets.

c. Calculate the standardised score for a player who is 182cm correct to 2 decimal places if the player is:

i. Male

1 mark

Worked solution

Male

$$z = \frac{score - mean}{standard\ deviation}$$

$$z = \frac{182 - 188.06}{5.79}$$

$$z = -1.05$$

ii. Female

1 mark

Worked solution

Female

$$z = \frac{score - mean}{standard\ deviation}$$

$$z = \frac{182 - 174.41}{5.52}$$

$$z = 1.375$$

$$= 1.38$$

Mark Allocation

- 1 mark for each z score calculation. Consequential marks are available for using the incorrect mean and standard deviation from Question 1 a).

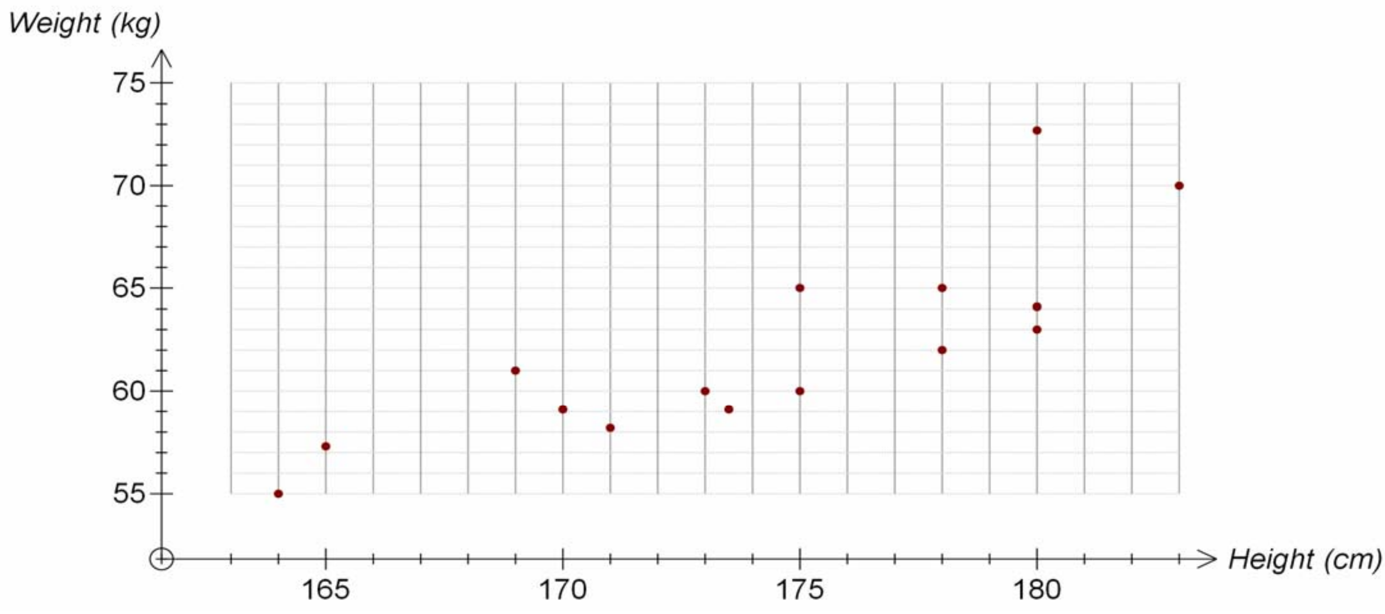
Question 2

The table below shows the weight and height of female tennis players who reached the 4th round of the Australian Open tennis tournament.

Player	Height (cm)	Weight (kg)
A	178	62
B	171	58.2
C	178	65.0
D	173.5	59.1
E	183	70.0
F	173	60.0
G	175	60.0
H	180	72.7
I	169	61
J	170	59.1
K	164	55
L	180	64.1
M	175	65
N	165	57.3
O	180	63
P	176	68.2

Table 2

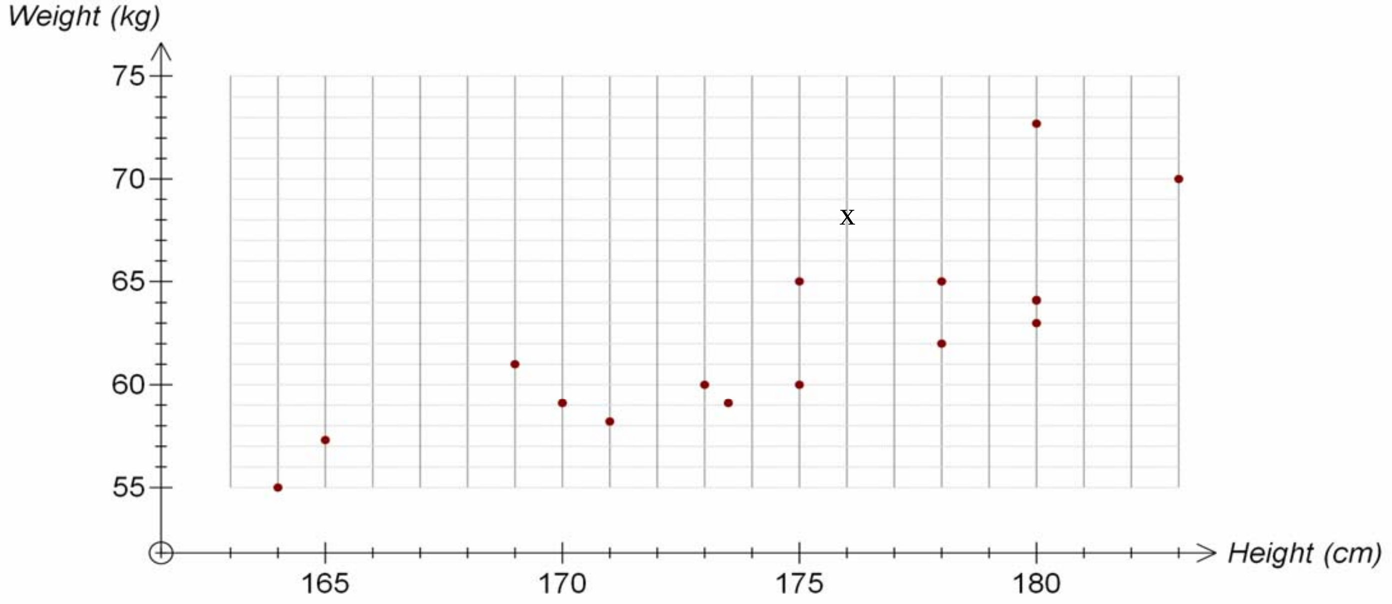
A scatterplot for the data in Table 2 is shown below:



a. Complete the scatterplot by adding player P with a cross (X).

1 mark

Worked solution



Placing a cross at (176, 68.2).

Mark Allocation

- 1 mark for the correct answer, on the 176 line and in between 68 and 68.5.

The least squares regression equation is:

Weight = 0.7 × Height – 60.06

Pearson's product moment correlation co-efficient is 0.805.

b. On average, for each extra cm increase in height, the least squares regression equation predicts that there will be an increase of ☐ kg in weight.

1 mark

Worked solution

The statement pertains to the gradient of the least squares regression line. Therefore the solution is 0.7.

Mark Allocation

- 1 mark for the correct answer.

c. To the nearest whole percent, ☐ % of the variation in weight can be explained by the variation in height.

1 mark

Worked solution

The statement pertains to the coefficient of determination (r^2).

$r^2 = (0.805)^2$

$r^2 = 0.65 = 65\%$

Mark Allocation

- 1 mark for the correct answer.

d. Complete the residual analysis table below by finding the missing predicted and residual values, correct to 2 decimal places:

Player	Height (cm)	Predicted Weight	Residual value
A	178	64.54	
B	171		–1.44
C	178	64.54	0.46
D	173.5	61.39	2.29
E	183	68.04	1.96
F	173	61.04	–1.04
G	175	62.44	–2.44
H	180	65.94	6.76
I	169	58.24	2.76
J	170	58.94	0.16
K	164	54.74	0.26
L	180	65.94	–1.84
M	175	62.44	2.56
N	165	55.44	1.86
O	180		–2.94
P	176	63.14	

2 marks

Worked solution

Calculation of the Predicted value

Weight = 0.7 × Height – 60.06

Predicted weight = 0.7 × 171 – 60.06

Predicted weight = 59.64

Weight = 0.7 × Height – 60.06

Predicted weight = 0.7 × 180 – 60.06

Predicted weight = 65.94

Calculation of the residual values

Residual = Actual value – predicted value

Residual = 62 – 64.54

Residual = –2.54

Residual = Actual value – predicted value

Residual = 68.2 – 63.14

Residual = 5.06

Mark Allocation

- 2 marks for 4 correct values rounded to 2 decimal places. 1 mark for 1, 2 or 3 correct values.

Tip

- *When a regression analysis is carried out, the calculator will store a list of the residuals. The values could have been read off the calculator. Also, by using the TRACE function, the predicted values can be found.*

e. Explain why the sum of the residual column will be close to 0.

1 mark

Worked solution

The definition of the least squares regression is that the sum of the squares of the residuals is minimised.

Mark Allocation

- 1 mark for the correct answer.

Question 3

a. In an attempt to improve the predictive power of the model, an inverse *y* transformation is carried out.

Find the equation of the least squares regression line, in terms of the variables, after the transformation has been carried out.

1 mark

Worked solution

$$\frac{1}{Weight} = -0.0002 \times Height + 0.0476$$

Mark Allocation

- 1 mark for the correct answer. Check for use of the correct variables, especially use of $\frac{1}{Weight}$.

b. Explain whether the transformation has improved the predictive model. Use appropriate statistics to support your answer.

1 mark

Worked solution

The inverse *y* transformation has slightly improved the predictive power of the model. The absolute value of *r* has increased slightly and the coefficient of determination has increased slightly as well. Predictions made using the transformed data will be marginally more accurate. However, other transformations need to be considered.

Mark Allocation

- 1 mark for a reasonable explanation, referring to the correlation coefficient and the coefficient of determination.

END OF SECTION A

SECTION B

Module 1: Number Patterns

Question 1

Prue is a keen runner. To prepare for an upcoming fun run, she runs laps of the local oval. On the first day she runs 4 laps, on the second day she runs 7 laps and on the third day she runs 10 laps.

a. If this sequence continues, how many laps will Prue run on the 7th day?

1 mark

Worked solution

The sequence increases by 3 each day. Therefore, on the 7th day, Prue will run 22 laps.

Mark Allocation

- 1 mark for the correct answer.

b. The sequence above, t_n, can be described in terms of n as: $t_n = 1 + d \times n$

The value of d is [ ]

1 mark

Worked solution

The value of d is 3, as it represents the common difference between the successive terms.

Mark Allocation

- 1 mark for the correct answer.

c. On what day will Prue first run 31 laps?

1 mark

Worked Solution

$31 = 1 + 3n$

$30 = 3n$

$n = 10$

She will first run 31 laps on Day 10.

Mark Allocation

- 1 mark for the correct answer.

d. Find how many laps Prue will run in total in between the 8th and 12th days, inclusive.

2 marks

Worked solution

$S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{12} - S_7$

$= (\frac{12}{2}(2\times 4 + (12-1)3)) - (\frac{7}{2}(2\times 4 + (7-1)3))$

$= 246 - 91$

$= 155 \text{ laps}$

Mark Allocation

- 1 method mark for calculation of the sum of the first 12 days.
- 1 mark for the correct answer.

Ramon is also training for the fun run with the number of laps, C, that he runs each day in terms of *n* following the sequence $C_n = -3n + 46$

e. Find the day on which Prue will first run more laps than Ramon.

1 mark

Worked solution

Either

Using a table of values with one rule in each list gives the 8th day as the first time that Prue will run more laps than Ramon.

Prue		Ramon	
Day	Laps	Day	Laps
1	4	1	43
2	7	2	40
3	10	3	37
4	13	4	34
5	16	5	31
6	19	6	28
7	22	7	25
8	**25**	**8**	**22**
9	28	9	19
10	31	10	16
11	34	11	13
12	37	12	10

Or

Solve for n

$-3n + 46 < 3n + 1$

$-6n < -45$

$n > 7\frac{1}{2}$

Therefore, by the 8th day, Prue will be running more laps than Ramon.

Mark Allocation

- 1 mark for the correct answer.

Question 2

Stephen rides his bicycle to deliver parcels. The numbers of parcels that he has delivered in the first 3 months of his job are 100, 120 and 144 respectively.

a. Show mathematically that the sequence described is a geometric sequence.

1 mark

Worked solution

$$\frac{144}{120} = \frac{120}{100} = 1.2$$

Therefore as there is a common ratio between consecutive terms, the sequence is geometric.

Mark Allocation

- 1 mark for the correct answer. Some attempt must be made to express the terms as a fraction to generate a common ratio.

b. Write the difference equation which describes this pattern.

1 mark

Worked solution

First term t_1=100

Common ratio = 1.2

Difference equation is $t_{n+1} = 1.2t_n, t_1 = 100$

Mark Allocation

- 1 mark for the correct answer. Must define the first term of the sequence.

c. If this pattern continues, calculate the number of deliveries in the 6th month, rounded to the nearest whole delivery.

1 mark

Worked solution

The *n*th term of a geometric sequence is $T_n = ar^{n-1}$

$$T_6 = 100 \times 1.2^{6-1}$$
$$= 248.832$$
$$= 249 \ (\textit{rounded})$$

Mark Allocation

- 1 mark for the correct answer.

d. In the latest contract negotiation, Stephen needs to be delivering more than 360 parcels in a month to achieve a bonus. At his current rate of deliveries, in which month of delivery will Stephen achieve his bonus?

1 mark

Worked solution

$T_n = a \times r^{n-1}$

$100 \times 1.2^{n-1} > 360$

$1.2^{n-1} > 3.6$

It is possible to use trial and error to solve the value of *n*.

Or, with the use of logs, we can also solve for *n*.

$$\log_{10}(1.2^{n-1}) > \log_{10}(3.6)$$
$$(n-1)\log_{10}(1.2) > \log_{10}(3.6)$$
$$n-1 > \frac{\log_{10}(3.6)}{(\log_{10} 1.2)}$$
$$n > \frac{\log_{10}(3.6)}{\log_{10}(1.2)} + 1$$
$$n > 8.03$$

Stephen will deliver 360 parcels in the 9th month.

Mark Allocation

- 1 mark for the correct answer.

Question 3

The value of a property over the first 3 years is given below:

Year 1	$230 500
Year 2	$283 500
Year 3	$339 680

A difference equation that gives the value of the property in a particular year is in the form of

$V_{n+1} = b + aV_n,\ V_1 = 230\,500$

a. Determine the value of *a* and *b*.

2 marks

Worked solution

$V_{n+1} = b + aV_n,\ V_1 = 230\,500$

$V_2 = b + aV_1$

$283500 = b + 230500a$

$V_3 = b + aV_2$

$339680 = b + 283500a$

$283500 = b + 230500a$

$339680 = b + 283500a$

Solving simultaneously for a

283500 = b + 230500a (1)

339680 = b + 283500a (2)

(2) – (1)

56180 = 53000a

a = 1.06

Substitute in (1)

$283500 = b + 230500 \times 1.06$

$283500 = b + 244330$

$b = 39170$

Mark Allocation

- 1 mark each for finding the value of *a* and *b*.

b. What will the value of the property be in the 5th year, to the nearest dollar?

1 mark

Worked solution

Year 1	\$230 500
Year 2	\$283 500
Year 3	\$339 680
Year 4	$\$339\ 680 \times 1.06 + \$39\ 170 = \$399\ 230.80$
Year 5	$\$399\ 230.80 \times 1.06 + \$39\ 170 = \$462\ 354.65$

The value of the property in the 5th year is \$462 354.65

= \$462 355 to the nearest dollar.

Mark Allocation

- 1 mark for the correct answer.

c. During which year will the value of the investment reach triple its original value?

2 marks

Worked solution

The value that the investment will need to reach to triple its original value is \$691 500. By use of iteration, shown in the table below, the value will reach triple its original value during the 8th year.

Year 1	230500
Year 2	283500
Year 3	339680
Year 4	399230.8
Year 5	462354.6
Year 6	529265.9
Year 7	600191.9
Year 8	675373.4
Year 9	755065.8

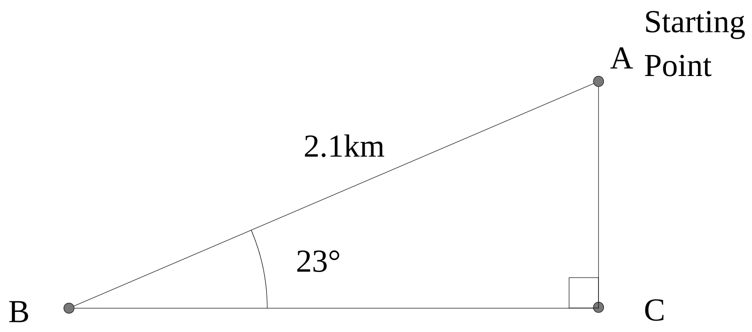
Mark Allocation

- 1 mark for showing working using iteration to produce a table of values.
- 1 marks for the correct answer.

Module 2: Geometry and Trigonometry

Question 1

Peter's running club, the Jennali Joggers, maps out the following course for its annual fun run. The runners begin at Point A, then run to Point B, then turn and run due east to Point C, then return to Point A to finish the run.



a. Find the magnitude of angle BAC.

1 mark

Worked solution

Angle sum of a triangle is 180°

23 + 90 + Angle BAC = 180

Angle BAC = 67°

Mark Allocation

- 1 mark for the correct answer (degrees symbol not required).

b. Find the total length of the course to 1 decimal place.

2 marks

Worked solution

Side AC

$$\sin 23^{\circ} = \frac{AC}{AB}$$

$$\sin 23^{\circ} = \frac{AC}{2.1}$$

$$AC = \sin 23^{\circ} \times 2.1$$

$$AC = 0.821\ km \quad (3\ decimal\ places)$$

Side BC

Using Pythagoras

$a^2 + b^2 = c^2$

$0.821^2 + b^2 = 2.1^2$

$b^2 = 3.737$

$b = 1.933km$

2.1 + 1.933 + 0.821 = 4.854 km

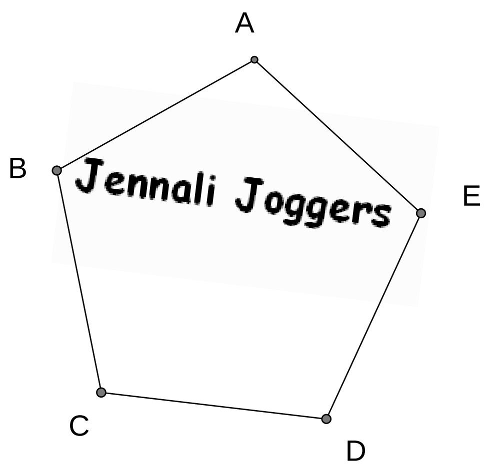
Total length of course = 4.9 km (1 decimal place)

Mark Allocation

- 1 mark for calculating one of the 2 sides using Pythagoras' Theorem or Trigonometric ratios.
- 1 mark for the correct answer.

Question 2

Peter is organising the club badges for the new members of the club. The design of the badge is shown below. It is in the shape of a regular pentagon:



a. Find the size of angle ABC.

1 mark

Worked solution

Interior angle of an n-sided polygon

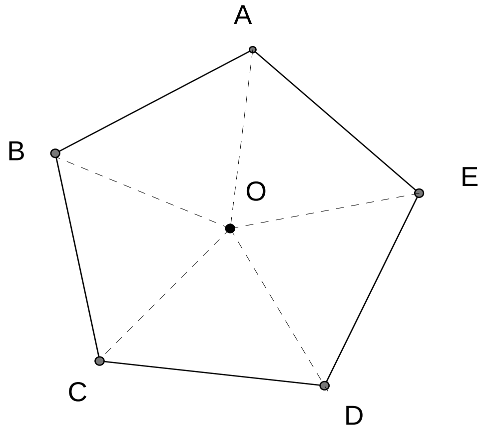
$$A = \frac{(n-2)\times 180}{n}$$

$$A = 108^\circ$$

Mark Allocation

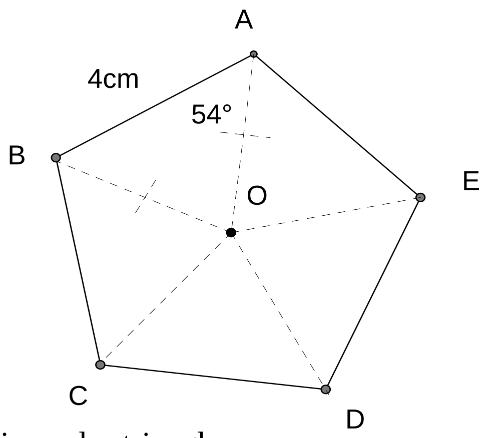
- 1 mark for the correct answer.

b. Given that each side has a length of 4 cm, show that the length of line BO is 3.4 cm.



1 mark

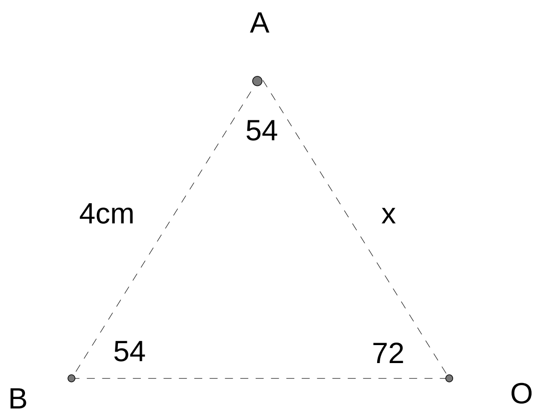
Worked solution



Triangle AOB is an isosceles triangle

BAO = 54° and AOB = 72°

Using the Sine rule



$$\frac{a}{SinA} = \frac{b}{SinB}$$

$$\frac{4}{Sin72} = \frac{x}{Sin54}$$

$$x \times \sin 72 = \sin 54 \times 4$$

$$x = \frac{\sin 54 \times 4}{\sin 72}$$

$$x = 3.4$$

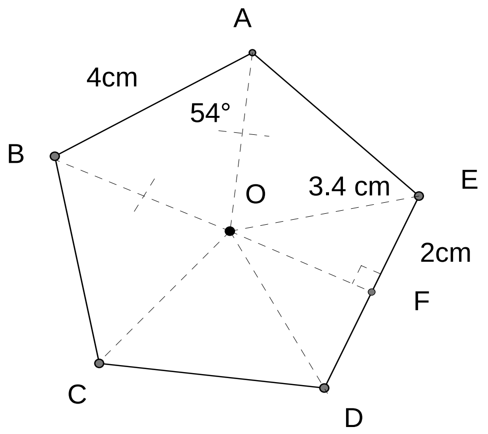
Mark Allocation

- 1 mark for the correct answer.

c. Find the area of the club badge (give your answer correct to one decimal place).

2 marks

Worked solution



The pentagon is made up of 5 isosceles triangles. Calculate the height of the right angle triangle OEF.

$OE^2 = OF^2 + EF^2$

$3.4^2 = OF^2 + 2^2$

$OF = \sqrt{3.4^2 - 2^2}$

OF = 2.75

Area = $5 \times \frac{1}{2} \times b \times h$

$= 5 \times \frac{1}{2} \times 4 \times 2.75$

$= 27.5 cm^2$

OR

area of triangle = ½ *ab* sin C so total area = 5 × ½ × 3.4 × 3.4 sin 72 = 27.5 (the angle at O is 360/5 = 72)

Mark Allocation

- 1 mark for the calculation of the height of each triangle and dividing the pentagon into triangles.
- 1 mark for final answer.

Tip

- *Heron's formula could also be used as all three sides of the triangle are known.*

d. If the diagram is a 1: 2.5 model, what is the area of the badge in real life?

1 mark

Worked solution

The badge in real life is 2.5 times bigger than the model; therefore the area is $(2.5)^2$ bigger than the model.

$$Area = 27.5 \times 2.5^2$$

$$Area = 172\ cm^2$$

Mark Allocation

- 1 mark for the correct answer.

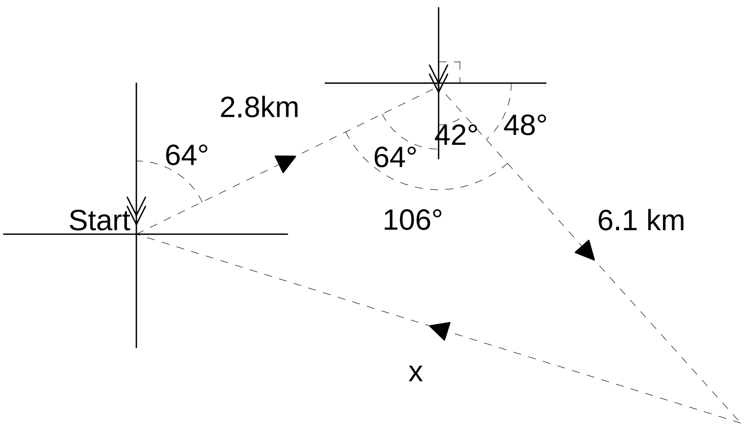
Question 3

In preparation for the fun run, Peter runs on a bearing of 064 T for 2.8 km to the base of a cliff, then changes direction and walks on a bearing of 138T for a further 6.1 km to the edge of a lake.

a. Find how far Peter will have to run to return to his starting point, in kilometres, to 2 decimal places.

1 mark

Worked solution



Using the Cosine Rule

$$a^2 = b^2 + c^2 - 2bc \times CosA$$

$$x^2 = 2.8^2 + 6.1^2 - 2 \times 2.8 \times 6.1 \times Cos106^\circ$$

$$x^2 = 54.466$$

$$x = 7.38\ km$$

Mark Allocation

- 1 mark for the correct answer.

b. Find the bearing Peter will have to run to return to his starting point, to the nearest minute.

2 marks

Worked solution

Using the Sine Rule:

$$\frac{a}{sin\ A} = \frac{b}{sin\ B}$$

$$\frac{2.8}{\sin A} = \frac{7.38}{sin\ 106^\circ}$$

$$\sin A = \frac{2.8 \times \sin 106^\circ}{7.38}$$

$$\sin A = 0.3647$$

$$A = \sin^{-1}(0.3647)$$

$$A = 21.4^\circ$$

$$21.4^\circ + 42^\circ = 63.4^\circ$$

$$360^\circ - 63.4^\circ = 296.6^\circ$$

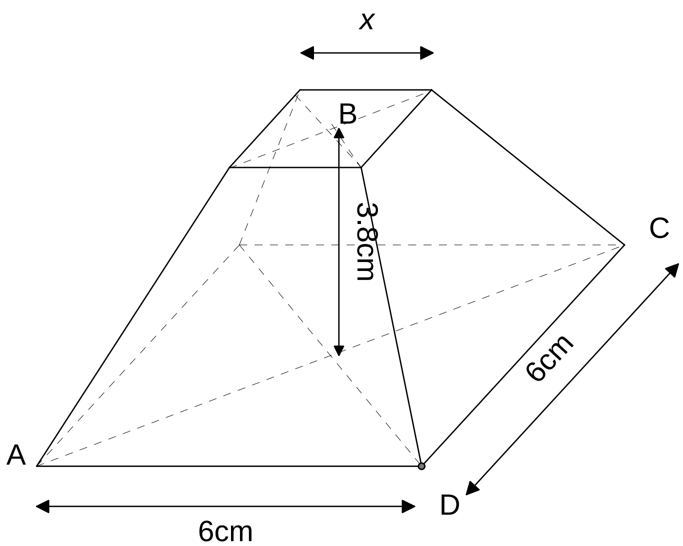
Bearing is $296.6^\circ T$ or $296^\circ 37'$

Mark Allocation

- 1 mark for calculating the missing angle in the triangle.
- 1 mark for the correct bearing.

Question 4

The trophy that is to be given to the first place runner is shown below. A square based pyramid has been removed from the top of the trophy. The removed section was 1.4 cm in height and had a volume of 5.6 cm^3.

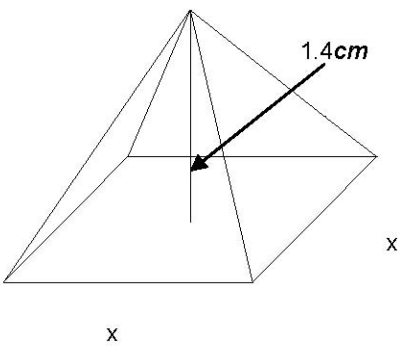


a. Find the value of x, correct to one decimal place.

1 mark

Worked solution

Removed square based pyramid.



Volume = 5.6 cm^3

Volume = $\frac{1}{3} \times Area\,of\,base \times 1.4$

$5.6 = \frac{1}{3} \times x^2 \times 1.4$

$x^2 = 12$

$x = \sqrt{12}$

$x = 3.5$ cm

Mark Allocation

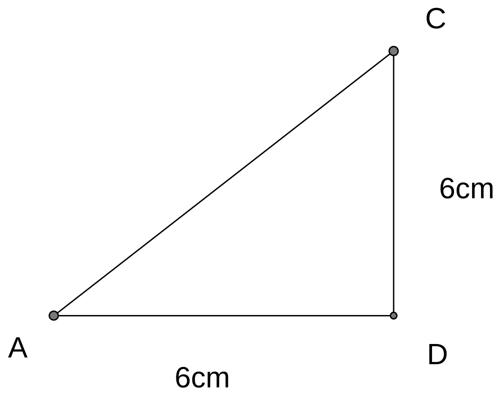
- 1 mark for the correct answer.

b. Find the value of angle ABC to the nearest degree.

2 marks

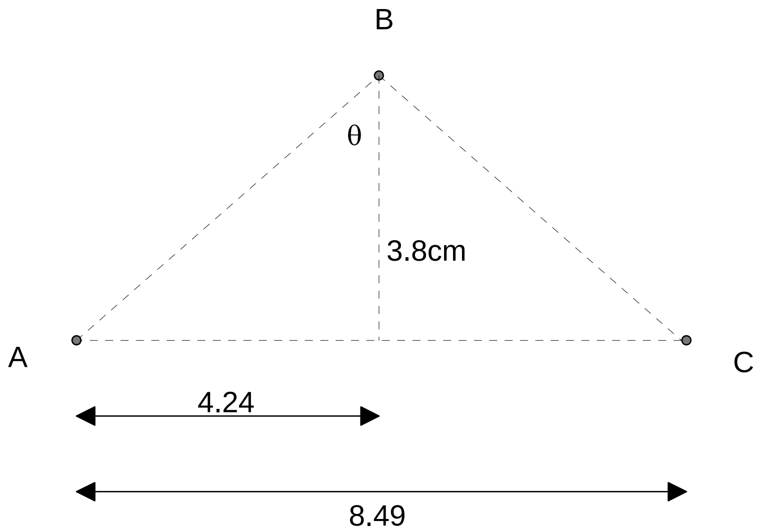
Worked solution

Distance AC



$AC^2 = 72$

$AC = 8.49$



$\tan\theta = \frac{4.24}{3.8}$

$\theta = 48.13$

$2\theta = 96.26^\circ$

$\therefore \angle ABC = 96^\circ$

Mark Allocation

- 2 marks for the correct answer.
- Method mark available for correctly calculating the distance AC.

c. Find the current volume of the trophy to the nearest cm^3.

1 mark

Worked solution

Volume = $\frac{1}{3} \times \textit{Area of the base} \times \textit{Height} - (\textit{volume of removed pyramid})$

$V = \frac{1}{3} \times 6^2 \times (3.8 + 1.4) - 5.6$

$V = 62.4 - 5.6$

$V = 56.8\ cm^3$

$V = 57 cm^3$ (to the nearest cm^3)

Mark Allocation

- 1 mark for the correct answer.

Module 3: Graphs and Relations

Question 1

The table of values below gives the intake of money at a local market against the number of hours after the opening time of 9.00 am.

Hours (x)	Money intake (y)
2	\$2
3	\$6.25
4	\$16
5	\$31.25

The relationship between hours (x) and money intake (y) is $y = kx^3$.

a. Find the value of k.

1 mark

Worked solution

Taking the first co-ordinate point (2, 2)

$2 = k \times 2^3$

$2 = k \times 8$

$k = \frac{2}{8} = \frac{1}{4}$

Checking with the last co-ordinate point (5, 31.25)

$31.25 = k \times 5^3$

$31.25 = k \times 125$

$k = \frac{31.25}{125}$

$k = \frac{1}{4}$

Mark Allocation

- 1 mark for the correct answer.

b. After how many hours did the money intake reach $250?

1 mark

Worked solution

$y = \frac{1}{4}x^3$

$250 = \frac{1}{4}x^3$

$1000 = x^3$

$x = 10$

After 10 hours, the money intake will be $250.

Mark Allocation

- 1 mark for the correct answer.

Question 2

The Back Yard Pottery Company makes mugs to sell at the local market. The cost of producing the mugs is a fixed cost of $380 plus $8 for each mug made.

a. Find the total cost of producing 35 mugs in a week.

1 mark

Worked solution

Cost = $35 \times 8 + 380$
$= \$660$

Mark Allocation

- 1 mark for the correct answer.

b. Write an equation for the cost (C) of producing *x* mugs.

1 mark

Worked solution

$C = 8x + 380$

Mark Allocation

- 1 mark for the correct answer.

c. Write an equation to calculate the profit (P) in terms of the number of mugs (*x*).

1 mark

Worked solution

Profit = Revenue – Cost

$P = 15x - (8x + 380)$

or

$P = 7x - 380$

Mark Allocation

- 1 mark for the correct answer. Either solution is acceptable.

d. Find the number of mugs that would need to be sold to achieve a profit of $100.

1 mark

Worked solution

If $\text{Profit} = 7x - 380$

$100 = 7x - 380$

$480 = 7x$

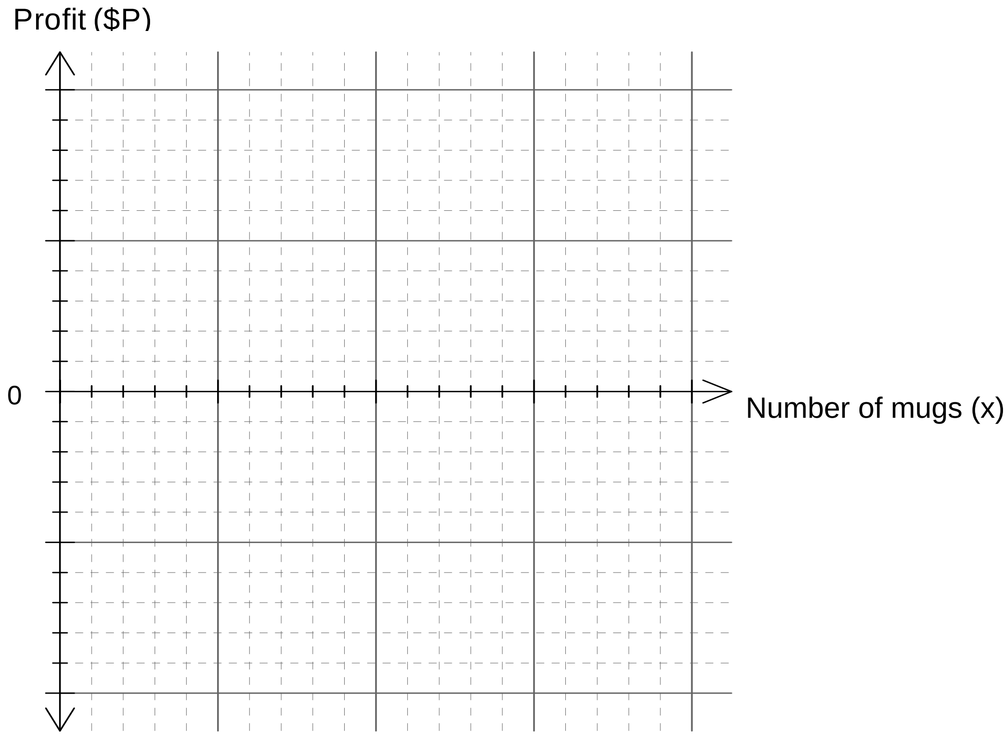
$x = 68.57 \text{ mugs}$

$\therefore 69 \text{ mugs}$

Mark Allocation

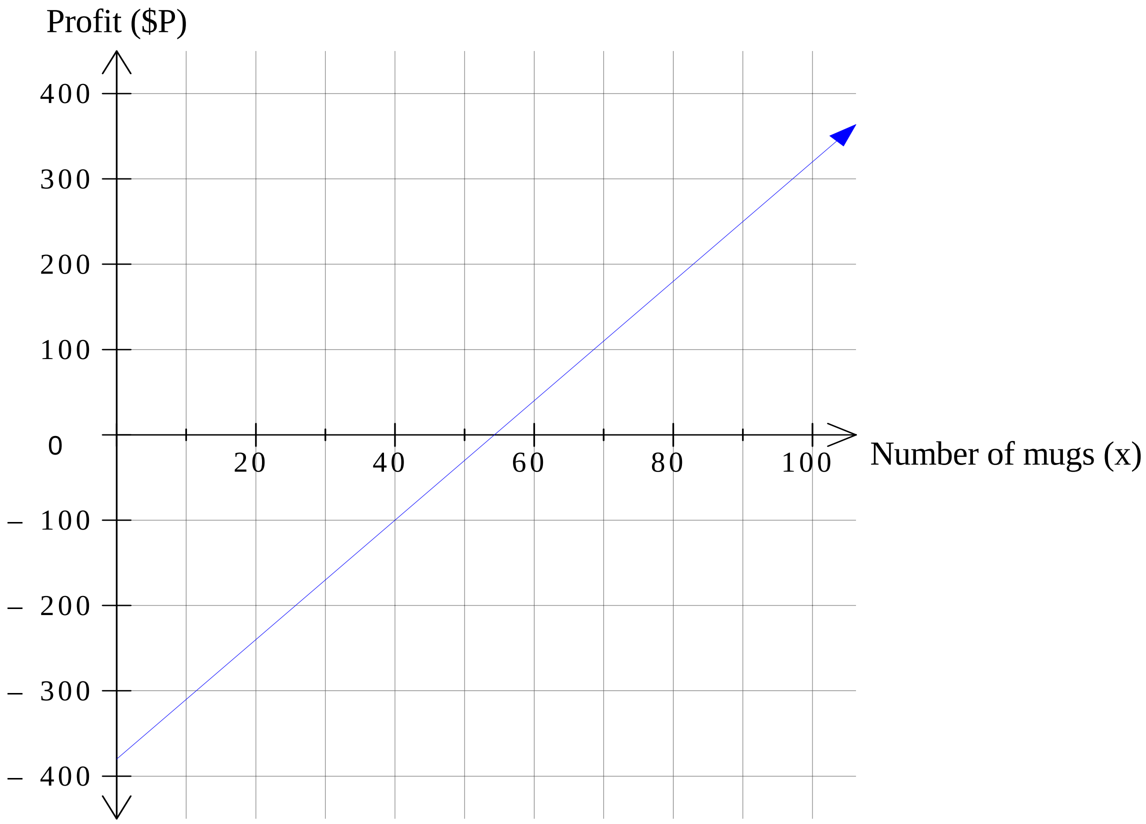
- 1 mark for the correct equation.

e. On the axis below, draw the graph of the rule for the total profit for selling *x* mugs. Include an appropriate scale.



1 mark

Worked solution



Mark Allocation

- 1 mark for the correct graph.

f. Find the number of mugs that would need to be sold to break even.

1 mark

Worked solution

Reading from the graph, the value is approximately 55 mugs.

$\text{Profit} = 7x - 380$

Let Profit = 0

$7x - 380 = 0$

$7x = 380$

$x = 54.28$

Therefore, 55 mugs will need to be sold to break even.

Mark Allocation

- 1 mark for the correct answer.

Question 3

The pottery company has decided to start to manufacture ceramic vases. They are to come in two sizes: small and large. To meet demand, the manufacturer has agreed to produce a maximum of 74 vases. They have also decided that they have enough material to make no more than 50 small vases and 35 large vases, and they are to manufacture at most three times as many small vases as large vases. The manufacturer must make at least one vase of each type.

Let x be the number of small vases and y be the number of large vases.

a. Using the above information, generate the 3 remaining constraints.

Constraint 1 $y \le 35$

Constraint 2 $x \ge 1$

Constraint 3 $y \ge 1$

Constraint 4 [ ]

Constraint 5 [ ]

Constraint 6 [ ]

3 marks

Worked solution

Constraint 1 $y \le 35$

Constraint 2 $x \ge 1$

Constraint 3 $y \ge 1$

Constraint 4 $x + y \le 74$

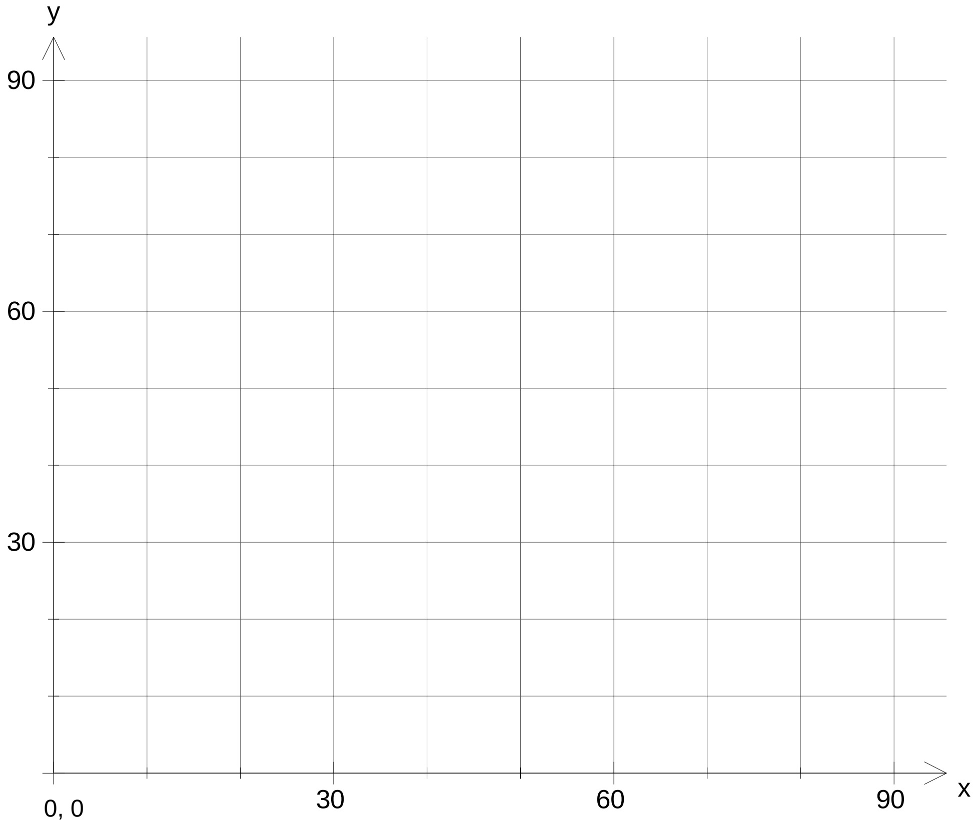
Constraint 5 $x \le 50$

Constraint 6 $x \le 3y$

Mark Allocation

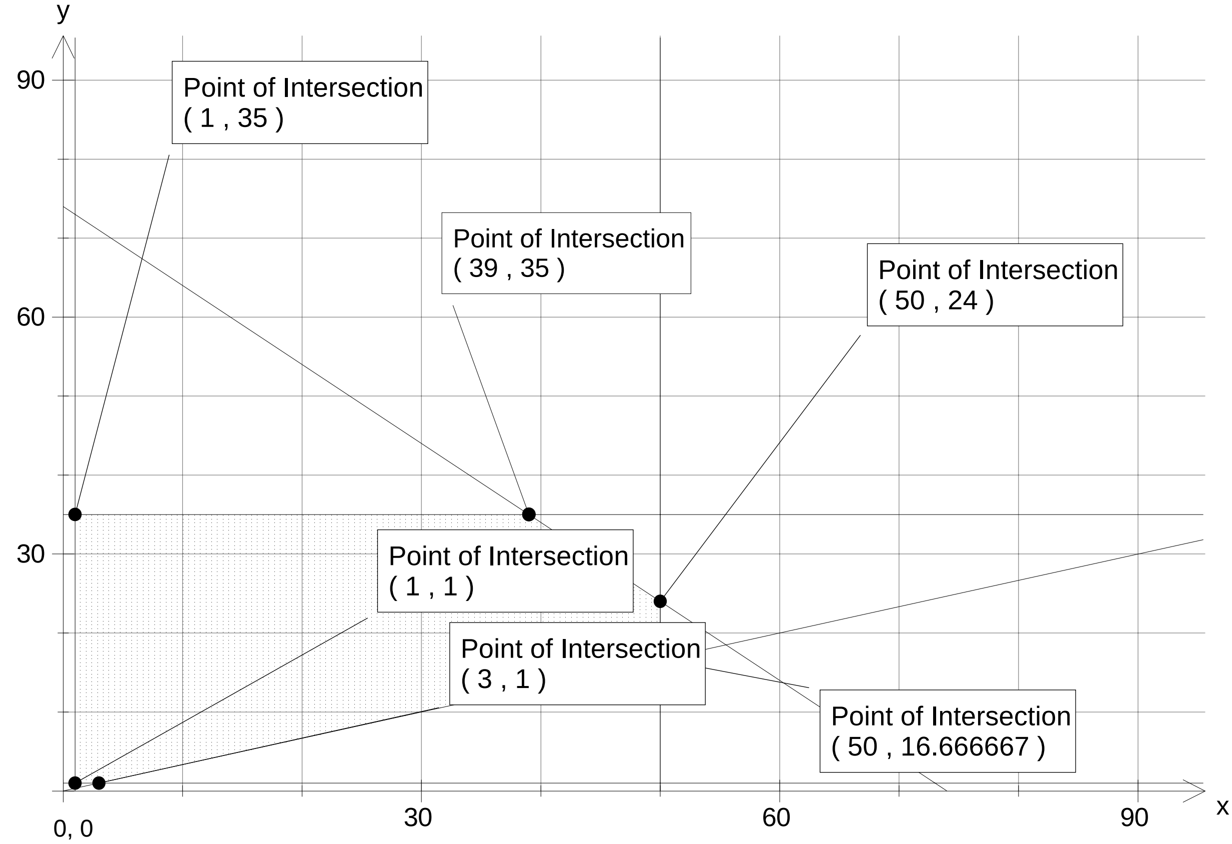
- 1 mark for each correct constraint.

b. On the set of axes below, graph the 5 constraints and show the feasible region.



2 marks

Worked solution



Mark Allocation

- 1 mark for the correct equations. Consequential mark available for correct graphs of incorrect constraints.
- 1 mark for the identification of the correct feasible region.

c. The profit on the large vase is $10 and the profit on the small vase is $18. Write an objective function for the manufacturer's profit, P dollars.

1 mark

Worked solution

$P = 18x + 10y$

Mark Allocation

- 1 mark for the correct answer.

d. Find the number of each type of vase that the manufacturer should make in order to make a maximum profit, while still satisfying the constraints.

1 mark

Worked solution

Considering the 4 points from the graph, as the vertices of the feasible region, the values are substituted into the objective function to find the maximum value.

Points	Objective Function (P)	Value
(39,35)	$18 \times 39 + 10 \times 35$	$1052
$(50, 16\frac{2}{3})$	$18 \times 50 + 10 \times 16\frac{2}{3}$	$1066.67
(50,24)	$18 \times 50 + 10 \times 24$	**$1140**
(1,35)	$18 \times 1 + 10 \times 35$	$368

(Note: (1,1) and (3,1) have not been considered, as the value of the objective function at this point is clearly less than the others listed.)

Therefore, the combination that will provide the maximum value is 50 small vases and 24 large vases.

Mark Allocation

- 1 mark for the correct answer.

Module 4: Business-related Mathematics

Question 1

The local hardware store, Nuts and Bolts, has their yearly sale. A drill that was normally $425 is reduced to $395.

a. What is the percentage discount? Write your answer to one decimal place.

1 mark

Worked solution

$$\%\ discount = \frac{discount}{original\ amount} \times 100$$

$$= \frac{425 - 395}{425} \times 100$$

$$= 7.1\%$$

Mark Allocation

- 1 mark for the correct answer.

b. What will be the value of the drill in 3 years, to the nearest dollar, if the reducing balance depreciation rate is 30% (given that the original value is taken as the discounted price)?

1 mark

Worked solution

Reducing balance depreciation

Using TVM to solve for the final value:

N = 3
I = –30%
PV = –395
PMT = 0
FV = 135.485 (ALPHA Solve)
P/Y = 1
C/Y = 1

Value = $135

Mark Allocation

- 1 mark for the correct answer.

Question 2

Two employees of Nuts and Bolts, Peter and Sam, each negotiate different contracts for the purchase of their car. Sam's car costs $21 000. He pays a deposit of $3 000 and makes 40 monthly repayments of $700 using a hire-purchase contract.

Peter's car costs only $15 000. His hire-purchase agreement is a deposit of $1 000, with 24 equal monthly repayments. He will pay a flat rate of interest of 21% per annum.

a. Calculate the total interest that Sam will pay.

1 mark

Worked solution

Interest = (Deposit + repayments) – cost of purchase

$= \$3000 + (40 \times \$700) - \$21000$

$= \$10000$

Mark Allocation

- 1 mark for the correct answer.

b. Calculate the total amount that Peter will pay for his car.

1 mark

Worked solution

Balance of loan = $15 000 – $1 000

= $14 000

Interest = $\dfrac{PRT}{100}$

$= \dfrac{14000 \times 21 \times 2}{100}$

$= \$5880$

Total cost of the car

$15 000 + $5 880 = $20 880

Mark Allocation

- 1 mark for the correct answer.

c. Which employee is paying a higher rate of interest for his loan? Show working to justify your choice.

2 marks

Worked solution

Peter's flat rate = 21%

Sam's flat rate:

Principal of loan = $18 000 Time = $\frac{40}{12}$ years Interest = $10 000

Interest = $\frac{PRT}{100}$

$$10000 = \frac{18000 \times R \times \frac{40}{12}}{100}$$

$R = 16.7\%$

Therefore, Peter's flat rate is 4.3% higher than Sam's.

Mark Allocation

- 1 mark for stating that Peter had a higher flat rate than Sam.
- 1 mark for a justification, showing working to support your answer.

d. Determine the effective interest rate per annum for Peter's contract. Give your answer correct to one decimal place.

1 mark

Worked solution

Effective interest rate = $\frac{2n}{n+1} \times \textit{flat rate}$

$= \frac{2 \times 24}{24+1} \times 21$

$= 40.3\%$

Mark Allocation

- 1 mark for the correct answer.

Question 3

Another employee of Nuts and Bolts, Phillipa, has been saving for a holiday. She has $6 500 saved and is considering a number of options.

a. The first option is a term deposit of 12 months that pays 8.1% per annum with the interest payable at the end of the month. What will be the value of her investment at the end of the year, to the nearest dollar?

2 marks

Worked solution

Compound interest

$$A = P(1+\frac{r}{100})^n$$

$$A = 6500(1+\frac{\frac{8.1}{12}}{100})^{12\times 1}$$

$$A = \$7\,046$$

Mark Allocation

- 2 marks for the correct answer.

b. Phillipa's holiday is going to cost $9 000. She would like to reach her target by the end of the year. A second option is to invest the money into a management fund that pays interest at the end of each month and earns 5.7% p.a. How much will she need to contribute to the fund each month, to the nearest cent, so that she can pay for her holiday?

2 marks

Worked solution

Using the TVM solver and solve for PMT:

N = 12
I = 5.7%
PV = –6500
PMT = –172.0715039 (ALPHA Solve)
FV = 9000
P/Y = 12
C/Y = 12

Therefore, monthly contributions of $172.07 are required.

Mark Allocation

- 2 marks for the correct answer.

Question 4

Nuts and Bolts have been approached by a local charity to set up a perpetuity in the form of a scholarship. They decide on the following arrangement:
An investment of $120 000 will be offered. The perpetuity is to provide $4 500 per year to disadvantaged students from the local area.

a. What will the interest rate be, if compounded monthly, that will require the perpetuity to make the yearly payment?

2 marks

Worked solution

Using TVM Solver to solve for the rate:

N = 1
I = 3. 687050044 % = 3.69 % (ALPHA Solve)
PV = –120 000
PMT = 4 500
FV = 120 000
P/Y = 1
C/Y = 12

Mark Allocation

- 2 marks for the correct answer.

b. How much will the yearly grant be increased, to the nearest dollar, if the decision is made to compound the amount daily rather than monthly (considering that the interest rate will not change)?

2 marks

Worked solution

Using TVM Solver to solve for the PMT

N = 1
I = 3. 68705004 %
PV = –120 000
PMT = 4506.8 (ALPHA Solve)
FV = 120 000
P/Y = 1
C/Y = 365

Therefore, the amount available each year will be increased to $4507 should the compounding period change to daily.

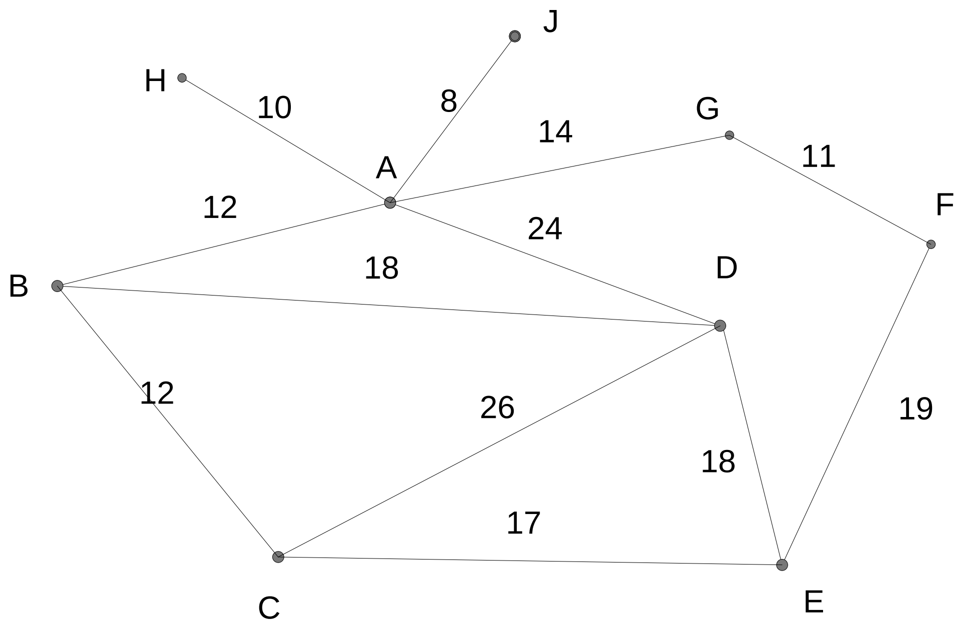
Mark Allocation

- 2 marks for the correct answer.

Module 5: Networks and Decision Mathematics

Question 1

Beth is a sales representative for a local food company. As part of her job, each week she travels to the towns in the network below. The paths between each town represent the roads that can be travelled. The numbers represent the distance between each town in kilometres.



a. Find the sum of the degrees of the vertices in the network.

1 mark

Worked solution

The degree to the number of edges joining a vertex.

A – 5 B – 3, C – 3, D – 4, E – 3, F – 2, G – 2, H – 1, J – 1
Therefore, the sum of the degrees is 24.

Mark Allocation

- 1 mark for the correct answer.

b. Beth is to travel from Town A to Town E. List the shortest path that Beth could take.

1 mark

Worked solution

The Path A–B–C–E is the shortest path from A to E (41 km).

Mark Allocation

- 1 mark for the correct answer.

c. What is the distance of the minimum spanning tree?

1 mark

Worked solution

The tree, HA – JA – AB – BD – BC – CE – AG – GF will give the minimal spanning tree of length 102 km.

Mark Allocation

- 1 mark for the correct answer.

d. During a particular day, Beth wishes to visit each town on the network to check with her suppliers without visiting the same town more than once. She does not need to start and finish at the same vertex.

i. What mathematical term describes Beth's task?

1 mark

Worked solution

Beth's task is a Hamiltonian Path.

Mark Allocation

- 1 mark for the correct answer.

ii. This task is not possible for Beth to complete. If she were to start at Vertex H, what is the maximum number of towns she could visit while attempting this task, without visiting the same town more than once?

1 mark

Worked solution

The maximum number of towns she will be able to visit, starting at Vertex H, while attempting a Hamiltonian Path, is 8. If she returns to Vertex A, she has visited one town more than once.

Mark Allocation

- 1 mark for the correct answer.

Question 2

The purpose of Beth’s visit is to promote 4 new products. Towns A, B, C and D will sell one product each. They have each quoted Beth the cost of marketing each product in their town.

	Product 1	Product 2	Product 3	Product 4
A	$145	$140	$110	$160
B	$260	$300	$230	$130
C	$270	$250	$200	$130
D	$180	$210	$180	$170

a. Find a possible matching between town and products so that the total cost to Beth is at a minimum.

2 marks

Worked solution

Reduce each row by subtracting the smallest number from each row. This produces the following table:

$35	$30	$0	$50
$130	$170	$100	$0
$140	$120	$70	$0
$10	$40	$10	$0

Now, reduce each column by repeating:

$25	$0	$0	$50
$120	$140	$100	$0
$130	$90	$70	$0
$0	$10	$10	$0

$25	$0	$0	$50
$120	$140	$100	$0
$130	$90	$70	$0
$0	$10	$10	$0

An optimal allocation cannot be made. A zero is an indication that an optimal allocation can be made. Only 3 lines need to be used to cover all zeros.

A further step is required – the Hungarian Algorithm.

Select the smallest number that is not covered, i.e. 70.

Subtract this number from anything uncovered; add this number to anything that the lines intersect.

$25	$0	$0	$120
$50	$70	$30	$0
$60	$20	$0	$0
$0	$10	$10	$70

Now an allocation can be made.

$25	**$0**	$0	$120
$50	$70	$30	**$0**
$60	$20	**$0**	$0
$0	$10	$10	$70

Product 1 D
Product 2 A
Product 3 C
Product 4 B

Mark Allocation

- 2 mark for the correct answer. 1 mark for at least 2 allocations being made correctly.

b. State the minimum cost.

1 mark

Worked solution

As the allocation has now been made, the minimal cost is the sum of the allocation:

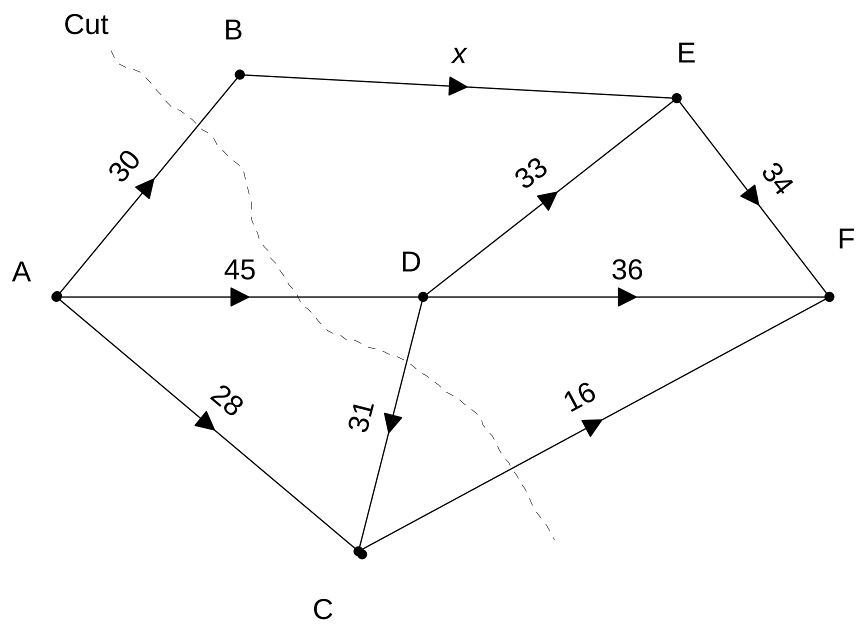
180 + 140 + 200 + 130 = $650

Mark Allocation

- 1 mark for the correct answer.

Question 3

The network below shows the path through a National Park, with the values on the edges representing the number of people that are allowed on the track due to safety concerns.



a. What is the value of the cut shown above?

1 mark

Worked solution

The value of the cut is 30 + 45 + 16 = 91 people.

(Note – the cut of value 31 is an example of backward flow – the flow is from the left to right of the cut and is not included in the sum of a cut.)

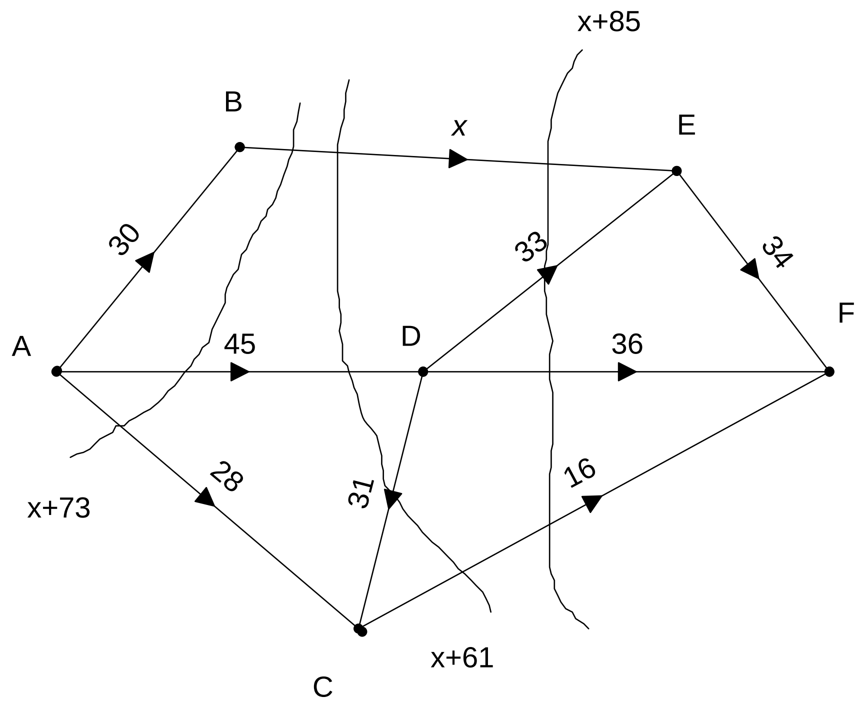
Mark Allocation

- 1 mark for the correct answer.

b. Given that the maximum flow of the network is 70, find the value of *x*.

1 mark

Worked solution



The value of each cut that intersects edge BE is shown on the network. Maximum flow is the minimum cut on a directed network. Therefore, as $x + 61$ is the minimum cut, and the value of the maximal flow is 70, then the value of x is 9.

Mark Allocation

- 1 mark for the correct answer.

Question 4

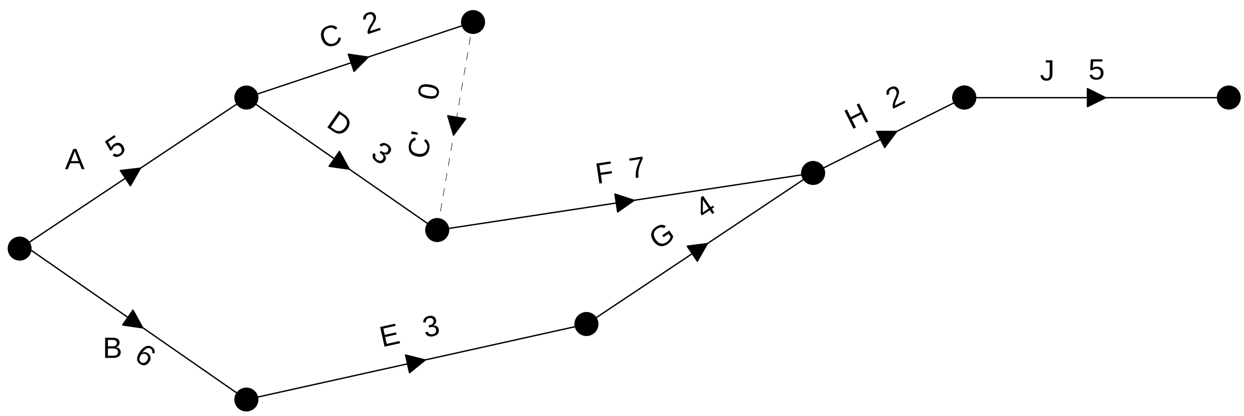
While building one of the paths, the builders need to complete a number of activities in order to construct the tracks. The immediate predecessors and activity times (in days) are shown in the following table.

Activity	Predecessor	Duration
A	-	5
B	-	6
C	A	2
D	A	3
E	B	3
F	C, D	7
G	E	4
H	G, F	2
J	H	5

a. Complete the network diagram below, labelling each activity and the duration.

2 marks

Worked solution



Mark Allocation

- 1 mark for the correct answer. Must include the dummy activity C' of duration 0.

b. Give the float time for Activity C.

1 mark

Worked solution

Float time = EFT – EST – Activity Time

Activity C can be delayed by 1 day without the whole project being delayed.

Mark Allocation

- 1 mark for the correct answer.

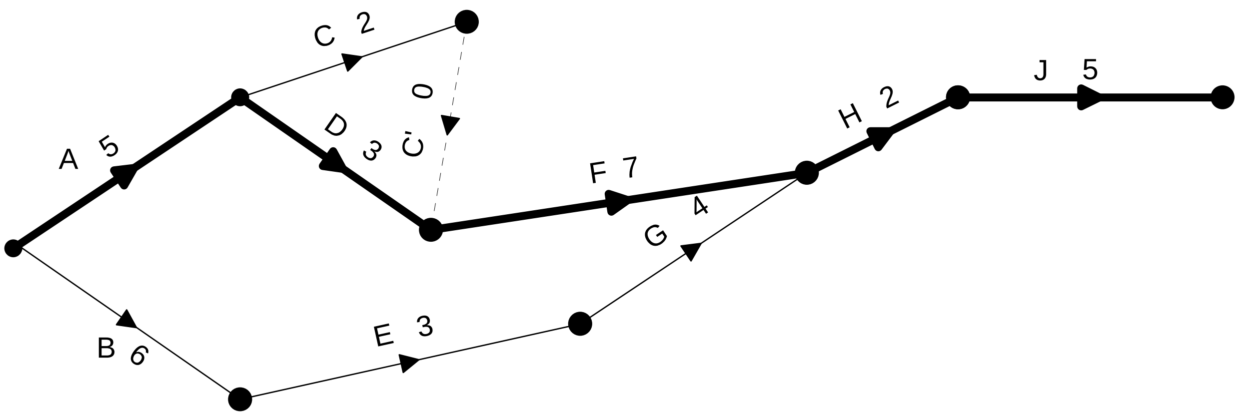
c. Find the critical path.

1 mark

Worked solution

Using forward and backward scanning the critical path is:

A – D – F – H – J



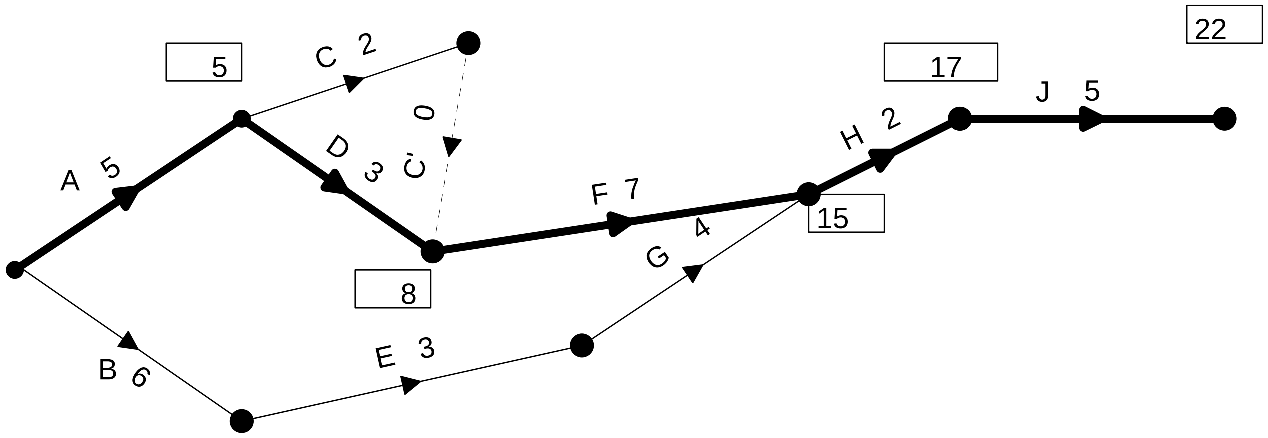
Mark Allocation

- 1 mark for the correct answer.

d. What is the earliest time in which all activities can be completed?

1 mark

Worked solution



The project can be completed in 22 days in total.

Mark Allocation

- 1 mark for the correct answer.

Module 6 – Matrices

Question 1

The matrix, G, represents the marks achieved by 4 students taking 3 separate tests. Each test is out of 10.

$$\begin{bmatrix} 2 & 3 & 1 \\ 1 & 4 & 7 \\ 6 & 5 & 1 \\ 7 & 9 & 2 \end{bmatrix}$$

a. Find element (3, 2) and interpret the result.

1 mark

Worked Solution

The element (3, 2) is the value in the third row and the second column. This value is 5.
Interpretation – this shows student number 3's results on the 2nd test.

Mark Allocation

- 1 mark for the correct answer.

Matrix G is multiplied by a second matrix H: $\begin{bmatrix} 0 \\ 1.05 \\ 0 \end{bmatrix}$

b. Prove that the product matrix GH is defined.

1 mark

Worked Solution

GH is defined because there is the same number of columns in Matrix G as there are rows in matrix H.

Mark Allocation

- 1 mark for the correct answer.

c. Solve the product matrix, GH, and explain your result.

2 marks

Worked Solution

$$G \times H = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 4 & 7 \\ 6 & 5 & 1 \\ 7 & 9 & 2 \end{bmatrix} \times \begin{bmatrix} 0 \\ 1.05 \\ 0 \end{bmatrix} = \begin{bmatrix} 3.15 \\ 4.2 \\ 5.25 \\ 9.45 \end{bmatrix}$$

The product matrix is an increase of 5% in the second Test.

Mark Allocation

- 1 mark for the correct product matrix and 1 mark for the interpretation of the resulting matrix.

Question 2

The matrix A is given as $\begin{bmatrix} 2 & c-3 \\ 5 & c \end{bmatrix}$

a. Find the determinant of matrix A.

1 mark

Worked Solution

Det (A) = $ad - bc$

$= 2c - 5(c-3)$

$= 2c - 5c + 15$

$= -3c + 15$

or

$= -3(c-5)$

Mark Allocation

- 1 mark for the correct answer.

b. Find the inverse of matrix A in terms of *c*.

2 marks

Worked Solution

$$A^{-1} = \frac{1}{\det(A)}\begin{bmatrix} c & -(c-3) \\ -5 & 2 \end{bmatrix}$$

$$= \frac{1}{-3c+15}\begin{bmatrix} c & -(c-3) \\ -5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{c}{-3c+15} & \frac{-(c-3)}{-3c+15} \\ \frac{-5}{-3c+15} & \frac{2}{-3c+15} \end{bmatrix}$$

Mark allocation.

- 1 mark for correctly substituting the determinant into the inverse. 1 mark for the correct inverse.

c. Find the value of *c* if the matrix A is to be singular.

1 mark

Worked solution

For the matrix to be singular, the determinant must equal zero.

Therefore,

$-3c+15=0$

$-3c=-15$

$c=5$

Mark Allocation

- 1 mark for the correct answer.

d. The result of the sum of matrix A and matrix B is $\begin{bmatrix} -2 & c \\ 2 & 2c+4 \end{bmatrix}$. Find matrix B.

1 mark

Worked solution

$$\begin{bmatrix} 2 & c-3 \\ 5 & c \end{bmatrix} + \begin{bmatrix} w & x \\ y & z \end{bmatrix} = \begin{bmatrix} -2 & c \\ 2 & 2c+4 \end{bmatrix}$$

$2+w=-2$

$5+y=2$

$c-3+x=c$

$c+z=2c+4$

$w=-4$

$y=-3$

$x=3$

$z=c+4$

Matrix B is $\begin{bmatrix} -4 & 3 \\ -3 & c+4 \end{bmatrix}$

Mark Allocation

- 1 mark for the correct answer.

Question 3

a. Express the following simultaneous equations in matrix form:

$3x+2y+4z=26$

$7x+y=24$

$7x+2z=32$

1 mark

Worked solution

$$\begin{bmatrix} 3 & 2 & 4 \\ 7 & 1 & 0 \\ 7 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 26 \\ 24 \\ 32 \end{bmatrix}$$

Mark allocation

- 1 mark for the correct matrix representation.

b. Solve the values of *x*, *y* and *z* to two decimal places.

2 marks

Worked solution

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 & 2 & 4 \\ 7 & 1 & 0 \\ 7 & 0 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 26 \\ 24 \\ 32 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3.44 \\ -0.08 \\ 3.96 \end{bmatrix}$$

$x = 3.44$, $y = -0.08$, $z = 3.96$

Mark allocation

- 1 mark for the correct matrix calculation using the inverse matrix and 1 mark for the correct values of *x*, *y* and *z*.

Question 4

In a survey of local road users in a large town, 25% of drivers are currently using the new tollway regularly to divert around a busy intersection.

Eight percent of the surveyed drivers plan to start using the tollway the following year, while 6% of the current users plan to stop using the tollway the following year, due to the high cost involved.

a. Represent this situation using a transition matrix.

1 mark

Worked solution

Transition matrix

	From using	From not using
To using	0.94	0.08
To not using	0.06	0.92

Mark allocation

- 1 mark for the correct matrix representation.

b. The owners of the tollway believe that in 8 years, if the pattern continues, at least 45% of the local road users will regularly be using their tollway. Support or reject their claim using appropriate matrix calculations.

2 marks

Worked solution

The initial state matrix (S_0) is a (2×1) matrix $\begin{bmatrix} 0.25 \\ 0.75 \end{bmatrix}$ with the first row representing the drivers using the tollway and the second row representing the drivers not using the tollway.

To find the state after 8 years, the calculation is as follows:

$$S_8 = T^8 \times S_0$$

$$S_8 = \begin{bmatrix} 0.94 & 0.08 \\ 0.06 & 0.92 \end{bmatrix}^8 \times \begin{bmatrix} 0.25 \\ 0.75 \end{bmatrix}$$

$$S_8 = \begin{bmatrix} 0.475 \\ 0.525 \end{bmatrix}$$

Therefore, 47.5% of the drivers will be using the tollway in 8 years' time based on the information. This supports the owners' claim that in 8 years, 45% of the drivers will be using the new tollway.

Mark allocation

- 1 mark for the correct matrix solution and 1 mark for justifying the solution.

END OF SOLUTIONS BOOK