Student Name: ______________________________

FURTHER MATHEMATICS

Units 3 & 4 – Written examination 2

2009 Trial Examination

Reading time: 15 minutes
Writing time: 1 hour and 30 minutes

QUESTION & ANSWER BOOK

Structure of book

Core		
Number of questions	*Number of questions to be answered*	*Number of marks*
2	2	15
Module		
Number of modules	*Number of modules to be answered*	*Number of marks*
6	3	45
		Total 60

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 29 pages.

Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

SECTION A - Core

Instructions for Core Questions

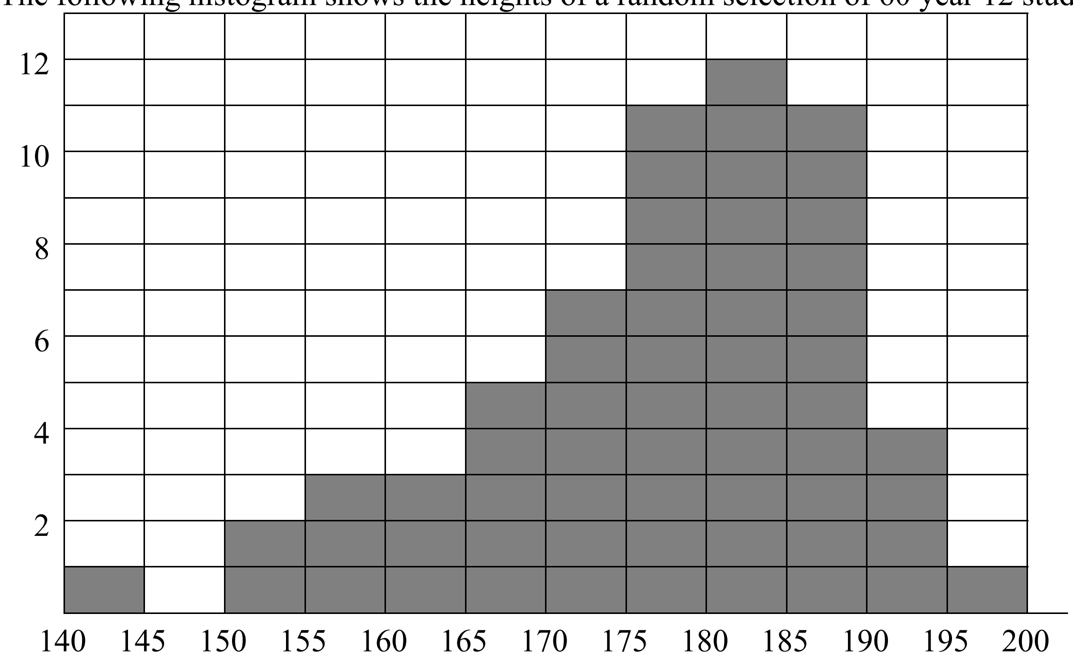
Answer **all** questions in the space provided.

You need not give numerical answers as decimals unless instructed to do so.

Alternative forms may involve, for example, π, surds or fractions.

Question 1

The following histogram shows the heights of a random selection of 60 year 12 students.



a. Describe the shape of the histogram.

1 mark

b.

i. How many students are over 175 cm?

ii. What percentage of students lie between 155cm and 180cm tall?

1 + 1 = 2 marks

SECTION A – continued

Question 2

A scorecard from a 20 twenty cricket match is shown below.

Name	Dismissal	Runs	Balls faced
D Smith	Caught Bradding Bowled Warnook	10	7
M Pollock	LBW Warnook	8	8
S Phillips	Run Out (Taylor)	67	35
A Williams	Bowled Walker	30	15
S Palmer	Caught Bradding Bowled Walker	22	12
N Phillips	Not Out	62	31
A Brown	Bowled Fielding	2	4
X Hogan	Run Out (Bryson)	0	1
L Morgan	LBW Warnook	4	3
S Wright	Bowled Taylor	2	3
A Barnes	Not Out	1	1
	Total	**208**	**120**

a. Suggest a suitable graph to display the frequency of the different modes of dismissal (bowled, LBW, Caught, Run Out etc) and explain why this is a suitable graph to use.

___

___

2 marks

b. When we look at the number of runs scored and the number of balls faced, which variable is the dependant variable?

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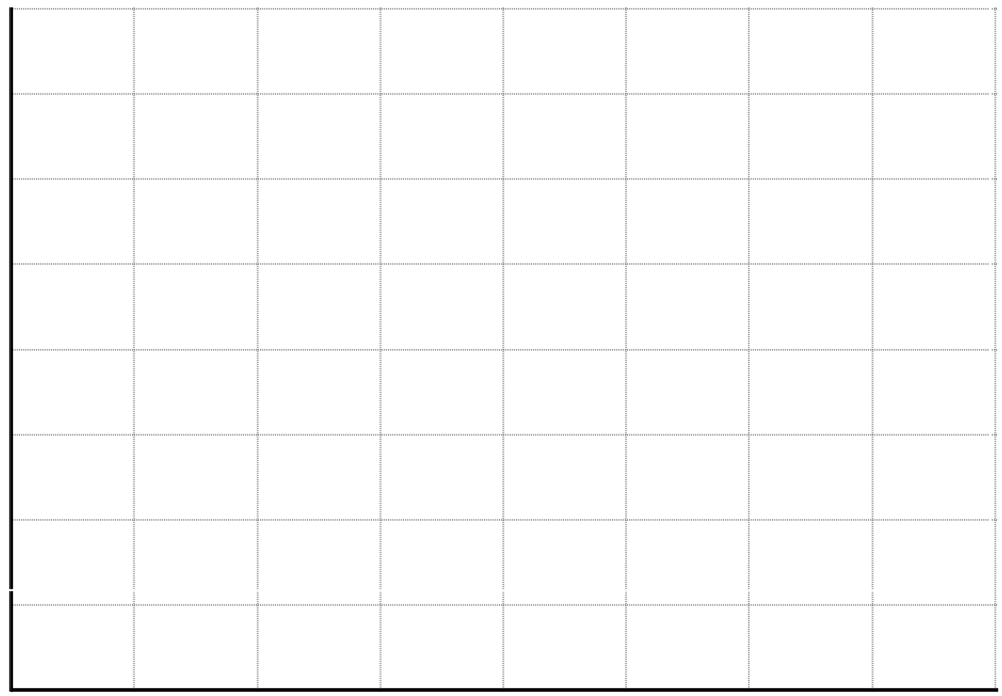
1 mark

SECTION A – Question 2 – continued
TURN OVER

c. On the axes below, draw a scatterplot for the number of runs scored and the number of balls faced.



3 marks

d. Using your graphics calculator, find the equation of the least squares regression line that relates the number of runs scored to the number of balls faced and draw it on the scatterplot above. Give the coefficients correct to two decimal places.

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2 marks

SECTION A – Question 2 – continued

e. On the axes below construct a residual plot to test the linearity of the data.

2 marks

f. Does your residual plot suggest that a linear relationship is suitable? If so, what features of the residual plot tell you that a linear relationship is suitable? If not, suggest a suitable transformation.

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2 marks

Total 15 marks

END OF SECTION A

TURN OVER

SECTION B - Modules

Instructions for Module Questions

Select **three** modules and answer **all** questions in the space provided.

You need not give numerical answers as decimals unless instructed to do so.

Alternative forms may involve, for example, π, surds or fractions.

Module 1: Number Patterns

Question 1

Bill and Sarah plan to ride their bikes a distance of 3430 km, from Melbourne to Perth. They plan on riding 140 km each day.

a. How many kilometres have they travelled at the end of the third day?

1 mark

b. The number of kilometres left to travel after n days riding can be written as

$$D = 3430 - a \times n$$

The value of $a =$ ☐

1 mark

c. How many days, to one decimal place, will it take for Bill and Sarah to ride from Melbourne to Perth?

1 mark

Tom and Sally also decide to ride their bikes from Melbourne to Perth. They left the same day as Bill and Sarah, but instead of riding the same distance each day, they decided that they would steadily increase the distance they ride by 10 km each day.

Tom and Sally rode 100km on their first day.

d. After how many days will Tom and Sally catch up with Bill and Sarah?

3 marks

Module 1: Number Patterns - continued

TURN OVER

Question 2

Jo and Steve plan to ride their motorcycles 3430 kilometres from Melbourne to Perth.

They decide that the number of kilometres they will ride each day will be determined by the rule

$$D_{n+1} = 0.8D_n + 100$$

On the third day, Jo and Steve travel 350 km.

a. How far do Jo and Steve ride on the fifth day?

1 mark

b. Show that the number of kilometres Jo and Steve ride each day does not follow a geometric or arithmetic sequence.

1 mark

c. How far did Jo and Steve ride on the first day?

2 marks

Module 1: Number Patterns – continued

Question 3

Samantha and Ron plan to drive the 3430 kilometres from Melbourne to Perth, towing their caravan.

a. Samantha and Ron plan to slowly reduce the distance they travel each day by 10% of the distance travelled the previous day. Write the difference equation that connects day *n* with day *n*-1.

1 mark

b. If Samantha and Ron traveled 400 km on the first day, how many kilometers would they travel on the 5th day?

1 mark

c. Samantha and Ron want to complete their journey in 10 days. How many kilometers (to 2 decimal places) will Samantha and Ron need to travel on the first day in order to make it from Melbourne to Perth according to their driving plan?

3 marks

Total 15 marks

End of Module 1 – Number Patterns

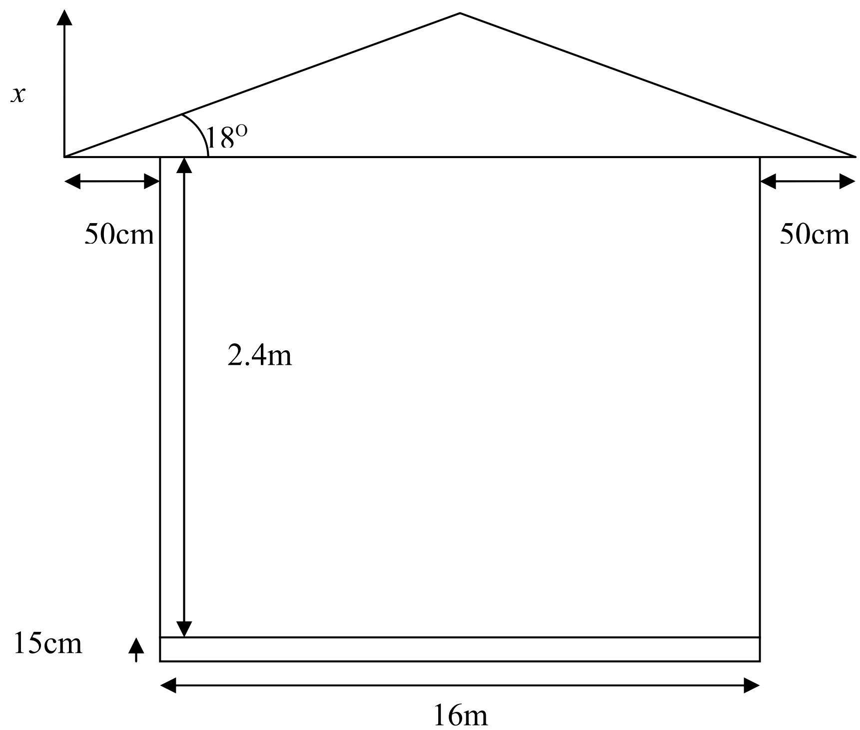
TURN OVER

Module 2: Geometry and Trigonometry

Question 1

Paul and Elaine are looking to build their dream home in a brand new estate, Duck Pond Springs. This new, exclusive estate, situated in the breeding grounds of a wide variey of ducks, has strict regulations about the houses that can be built. All houses must have a 30cm slab that sits at least 15cm above the ground, buildings must be no taller than 5.5m, they must have eaves of 50cm and roofs must have a minimum pitch of 18^{O}.

Paul and Elaine have chosen a rectangular house that is 16m wide and 20m long. They love high ceilings and wish to have 2.4 metre ceilings.



a. Find the height of the pitched roof, x, to the nearest centimetre.

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1 mark

b. Will Paul and Elaine be able to build their dream home with 2.4m ceilings?

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1 mark

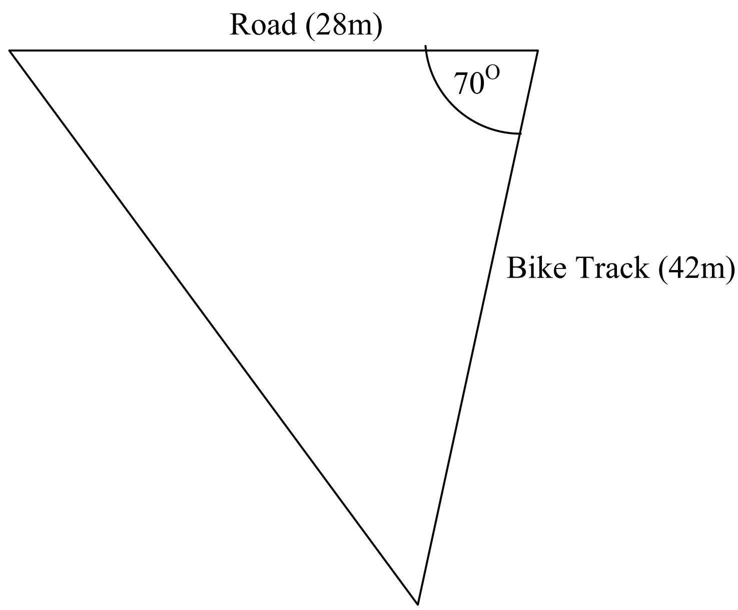
Module 2: Geometry and Trigonometry - Question 1- continued

c. What is the maximum pitch (correct to the nearest degree) that Paul and Elaine can have on their roof and remain under the 5.5m building height?

1 mark

Question 2

Paul and Elaine have their eye on a corner block that has a bike track running along one edge of the property. The frontage (length of the block along the road) is 28m, the length of the block alongside the bike track is 42m and the angle formed between the road and the bike track is 70^{O}.



a. Find the length of the other boundary of the property.

1 mark

Module 2: Geometry and Trigonometry - Question 2 - continued

TURN OVER

b. Find the area of the block of land to the nearest square metre.

1 mark

Paul and Elaine want to make sure that their dream house will fit on their block. They make a scale diagram of the house and block. The scale factor of the model is 1:500.

c. Find the area of the scale model of the block of land to the nearest square centimetre.

2 marks

d. The interior of Paul and Elaine's house has the shape of a cuboid (disregarding the internal walls). Find the volume of the model of the interior of Paul and Elaine's house giving the answer correct to the nearest cm^3.

2 marks

Question 3

Paul and Elaine were able to build their dream house on their block and they are just putting the final touches on their house. There are two television transmission towers in the vicinity of the estate which provide the television coverage and Paul is installing the antenna on the top of the roof. He wants to know in which direction to point the antenna. The bearing of the house (H) from tower A is 232^{O}T, the bearing of the house from tower B is 275^{O}T and the bearing of tower B from tower A is 132^{O}T. The distance between tower A and tower B is 135km.

a. Draw a fully labelled diagram showing the position of the house and the two transmission towers. Include the value of all internal angles and the known side length.

2 marks

b.

i. Show that $\angle ABH = 37^{\circ}$.

ii. Find the distance from the house to each of the two transmission towers, correct to two decimal places.

1 + 2 = 3 marks

Module 2: Geometry and Trigonometry – Question 3 – continued

TURN OVER

b. On what bearing should Paul direct the antenna to get the best reception (shortest distance to the transmission tower)?

1 mark

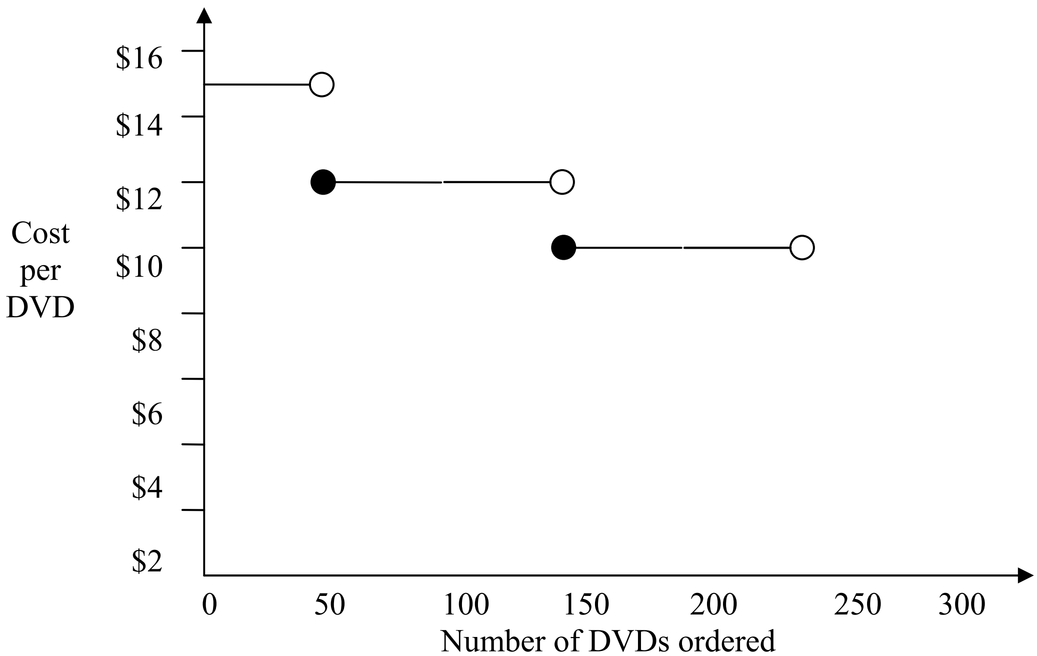
Total 15 marks

End of Module 2: Geometry and Trigonometry

Module 3: Graphs and Relations

Question 1

Carl runs a DVD production company. The price he charges per DVD depends on the number of copies he is asked to produce. The cost per DVD is shown in the graph below for purchases up to, but not including 250 DVDs.



a. How much would it cost per DVD if a client wanted 125 copies of a DVD?

1 mark

b. What would the total cost be if a client ordered 150 copies of a DVD?

1 mark

c. What minimum number of DVDs must a client purchase to get them for $12 a DVD?

1 mark

d. If a client orders more than 250 but less than 300 DVDs the cost is $8 per DVD. If they order more than 300 DVDs the cost is $7 per DVD. Draw this information on the graph.

1 mark

Module 3: Graphs and Relations – continued

TURN OVER

Question 2

Carl's production company has been given the opportunity to purchase the rights to DVD film from the Cannes Film Festival. The cost to buy the rights is \$20000 and the production and royalties cost per DVD is \$3. Carl will charge retail outlets \$20 per DVD.

a. What is the rule that relates the total cost (C) with the number of DVDs produced (x)?

____

1 mark

b. What is the rule that relates the revenue (R) with the number of DVDs produced (x)?

____

1 mark

c. What is the minimum number of DVDs Carl will need to sell in order to make a profit?

____

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2 marks

Module 3: Graphs and Relations - continued

Question 3

Carl has just upgraded his equipment to produce Blue-Ray DVDs.

Let x be the number of thousands of normal DVDs produced in a week
y be the number of thousands of Blue-Ray DVDs produced in a week

- In order to satisfy demand, he must produce at least 10000 normal DVDs and 8000 Blue Ray DVDs.
- It takes 2 hours to produce 1000 normal DVDs.
- It takes 2.5 hours to produce 1000 Blue-Ray DVDs.
- The company can only run the machines for 50 hours in a week.

a. Write down the three inequalities produced by this situation.

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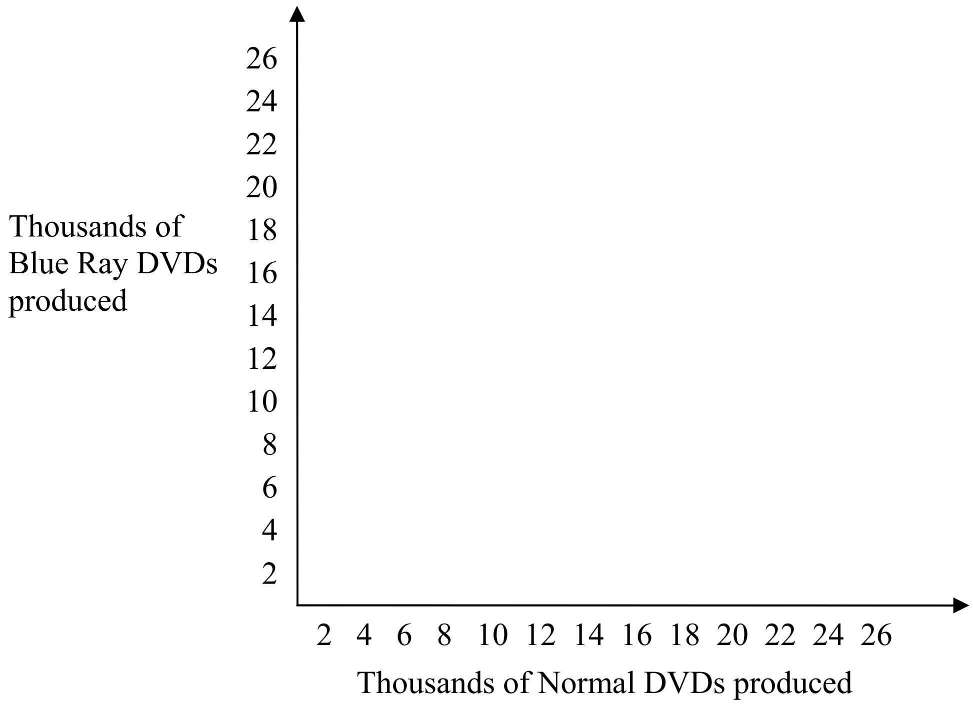
3 marks

Module 3: Graphs and Relations – Question 3 – continued
TURN OVER

b. Draw the inequalities on the axes below and identify corner points.



2 marks

Carl can make \$15 profit on the normal DVDs he produces and \$25 profit on the Blue-Ray DVDs he produces.

c. What is the maximum profit he can make under these constraints and how many of each type of DVD should he produce?

2 marks

Total 15 marks

End of Module 3: Graphs and Relations

Module 4: Business-Related Mathematics

Question 1

Frank wants to buy a new computer to run his business. He has found that the computer he wants is sold in three different stores.

a. The first store is charging $2800 for the computer and has an interest free finance deal over 12 months. Frank needs to pay a 20% deposit and the balance in 12 equal monthly instalments

i. How much will Frank need to pay as the deposit?

ii. How much will Frank need to pay each month?

1 + 1 = 2 marks

b. A second store is selling the same computer using a finance package which requires a deposit of $500 and weekly instalments of $50 for one year. Which finance package should Frank use and how much would he save?

2 marks

c. A third store has the computer advertised for $2950, but is offering a 17.5% discount if he pays by cash. What is the cash price of the computer?

1 mark

Module 4: Business Related Mathematics - Question 1 – continued

TURN OVER

d. The discounted price of the computer from the third store includes the 10% GST. Find the value of the GST paid on the discounted computer.

1 mark

Question 2

Frank decides that he wants to upgrade his whole computer network, which would cost another \$22000. He finds a business loan that has an interest rate of 7.9% for 7 years.

a. How much are the repayments Frank would need to make if he chose to pay the loan off in monthly instalments?

1 mark

b. How much would Frank save over the life of his loan if he chose to pay the loan off in weekly instalments?

2 marks

Frank decides to pay his loan off in weekly instalments, but after 2 years he receives a government rebate of \$5000. He uses the money to reduce the loan.

c. If Frank continued to pay the same amount each week, when would he fully repay the loan? Give your answer in years and weeks.

2 marks

Module 4: Business Related Mathematics - Question 2 – continued

d. How much interest would Frank save if he chose to make this early repayment?

1 mark
2

The network Frank has purchased has a scrap value of $5000. He is given two options to depreciate the value of the network over time for tax reasons.

Option 1: flat rate of 6.5% per year.

Option 2: reducing balance rate of 11% per year.

e. Which depreciation scheme would allow Frank to claim the costs of the network on tax in the shortest amount of time (that is: the first one to reach scrap value)?

3 marks
Total 15 marks

End of Module 4 : Business Related Mathematics
TURN OVER

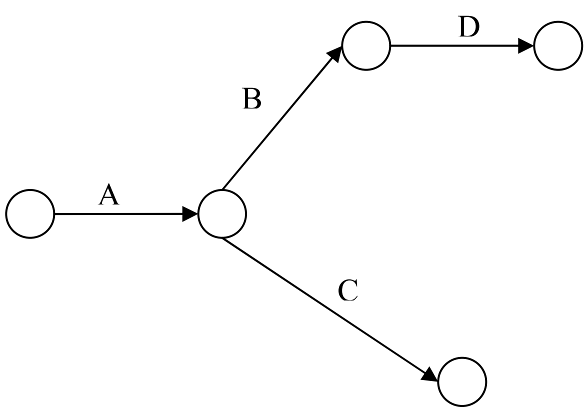
Module 5: Networks and Decision Mathematics

Question 1

A company is building a new rail network to link 7 popular tourist attractions. There are 8 tasks that are required to complete the railway. The activities, the number of weeks each activity will take and the predecessors are shown in the table below.

Activity	*Time (weeks)*	*Predecessors*
A	4	-
B	2	A
C	3	A
D	3	B
E	2	D
F	6	C
G	4	E, F
H	2	G

a. Use this information to complete the network below by including activities E, F, G and H and placing the times for each task below each activity.



2 marks

b. Identify the critical path and determine the time that the project will take.

____

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1 mark

Module 5: Networks and Decision Mathematics - Question 1 – continued

c. The company have been given a deadline of 20 weeks to finish the task. What is the maximum time that task D can be delayed, without making the project run over time?

1 mark

It is possible for the company to reduce the time of certain tasks by paying extra staff to work on the project. A table of the costs for reducing the time taken per week and the minimum time the task can be shortened to is shown below.

Activity	Cost to reduce activity	Minimum time needed to complete task
C	\$2000 per week	2 weeks
E	\$3000 per week	1 week
F	\$5000 per week	3 weeks
G	\$3000 per week	3 weeks

d. The tourism board has decided to award a bonus of \$5000 if the railway is completed within 18 weeks and \$20000 if it is completed within 15 weeks.

i. Assuming all activities go to plan, what is the minimum extra cost to complete the project within 18 weeks?

ii. Assuming all activities go to plan, what is the minimum extra cost to complete the project within 15 weeks?

iii. What should the company do in order to maximise the profit from the railway?

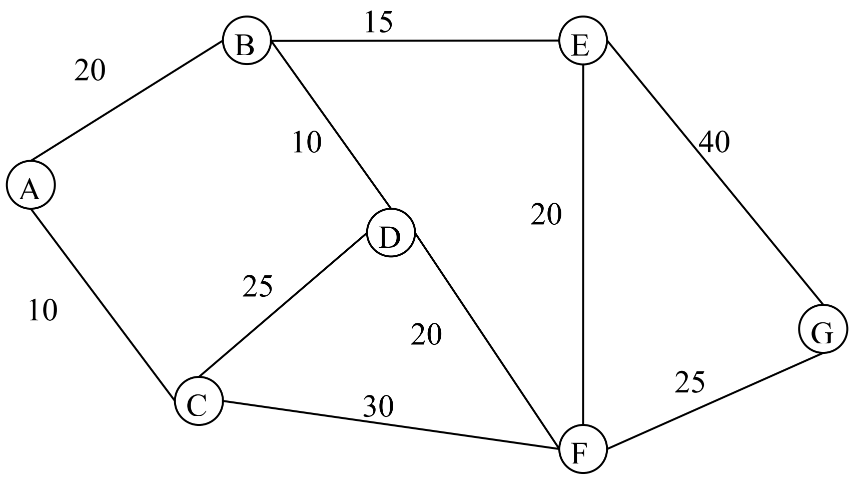
1 + 1 + 1= 3 marks

Module 5 : Networks and Decision Mathematics – continued

TURN OVER

Question 2

A diagram of the railway, showing the time it takes to get from one attraction to the next, in minutes, is shown below.



a. What is the minimum time a tourist could take to get from attraction A to attraction G using the rail network?

____

1 mark

b.

i. During non-peak tourist times they plan to run a reduced service. Draw the minimum spanning tree of this network that would show the links required to get people to all attractions in the quickest time, using the fewest rail links.

ii. Find the length of the minimum spanning tree.

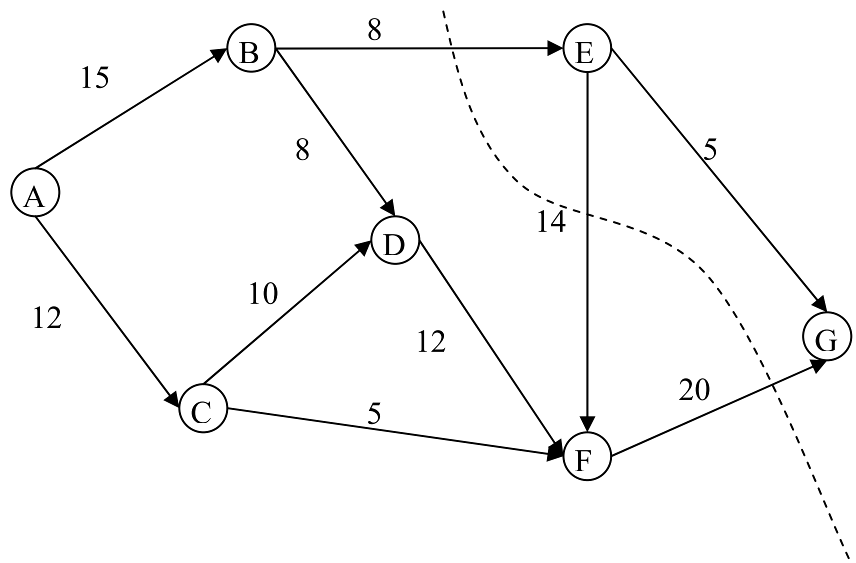
____

1 + 1 = 2 marks

Module 5 : Networks and Decision Mathematics – continued

Question 3

A free concert is being held at attraction G. The network below shows the number of tickets available on each link that would allow tourists to get the necessary connections to get to the free concert.



a. Find the capacity of the cut shown on the network diagram.

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1 mark

b. On the network diagram, draw the minimum cut for the network and thus find the maximum number of people that can get to the free concert using the available train tickets.

___

2 marks

c. 5 extra tickets have just become available for the E-G train trip. Does this change the maximum number of people that can get to the concert? If so, how many more people can get to the concert? If not, why does this not have an effect?

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2 marks

Total 15 marks

End of Module 5: Networks and Decision Mathematics

TURN OVER

Module 6: Matrices

Question 1

A restaurant, Applebox, has a fixed price of $12 for entrées, $20 for main meals and $8 for desserts. Table One ordered 2 entrées, 3 mains and 2 desserts. Table Two ordered 1 entrée, 3 mains and 3 desserts. Table Three ordered 4 entrées, 6 mains and 6 desserts. Table Four ordered no entrées, 2 mains and 2 desserts.

a. Create a 3×1 matrix, *C*, that represents the price of entrées, main meals and desserts

1 mark

b. Create a matrix, *O*, that represents the orders of the four tables.

1 mark

c. Find the matrix product *OC* and explain what this product means.

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2 marks

d. Explain why the matrix product *CO* does not exist.

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1 mark

Module 6: Matrices - continued

Question 2

A new restaurant, Baskerville, which only sells main meals and desserts, has just opened in direct competition to Applebox. Baskerville charges a set price for main meals and desserts. Table One ordered 3 mains and 2 desserts for a total of $90. Table Two ordered 6 mains and 6 desserts for a total of $204.

a. Write down this situation as a matrix equation in the form $AX=C$ where x represents the price of an entrée and y represents the price of a main meal.

1 mark

b. Show all working out to find the value of the matrix A^{-1}, the inverse of the above matrix, A.

1 mark

c. What is the cost of an entrée and a main meal respectively at Baskerville? Show all working.

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2 marks

Module 6: Matrices – continued

TURN OVER

Question 3

A third restaurant, The Club, has also opened, causing even greater competition to Applebox and Baskerville. The owners of Applebox have conducted significant market research and have found that:

72% of the people that eat at Applebox will return to Applebox the next week.
16% of the people that eat at Applebox will eat at Baskerville the next week.
12% of the people that eat at Applebox will eat at The Club the next week

80% of the people that eat at Baskerville will return to Baskerville the next week
14% of the people that eat at Baskerville will eat at Applebox the next week
6% of the people that eat at Baskerville will eat at The Club the next week

84% of the people that eat at The Club will return to The Club the next week
12% of the people that eat at The Club will eat at Applebox the next week
4% of the people that eat at The Club will eat at Baskerville the next week

Enter this information into a transition matrix *T*, expressing the percentages as decimals.

2 marks

a. In the first week that statistic*s* were taken, 2000 diners went to Applebox, 1500 diners went to Baskerville and 800 diners went to The Club. Write this information in the form of a column matrix D_0.

1 mark

Module 6: Matrices – Question 3 - continued

b. Find the number of diners you would expect to be at each restaurant after three weeks (D_3).

3 marks
Total 15 marks

END OF QUESTION AND ANSWER BOOK